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GRAMMAR SCHOOL BOOK

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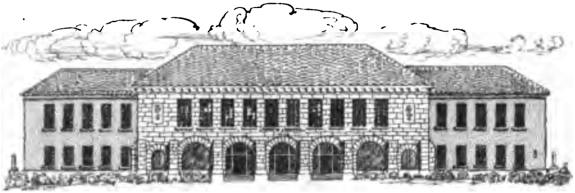

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WENTWORTH-SMITH MATHEMATICAL SERIES

ESSENTIALS OF ARITHMETIC

GRAMMAR SCHOOL BOOK

BY
GEORGE WENTWORTH
AND
DAVID EUGENE SMITH

GINN AND COMPANY
BOSTON · NEW YORK · CHICAGO · LONDON
ATLANTA · DALLAS · COLUMBUS · SAN FRANCISCO

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PREFACE

This work furnishes the material for a thorough study of the great essentials of arithmetic in the last four years of the elementary school. In general, in the schools of this country the work of Grade V consists of a review of the fundamental operations and a thorough study of the fractions and denominate numbers used in practical life. The central feature, in substantially all schools, is common fractions. The needs of this grade are met in Chapters I–III.

In Grade VI the central feature is usually decimal fractions, the other topics being practical measurements and an introduction to percentage. These are the topics discussed in Chapters IV–VI. Percentage and its applications is taken up more thoroughly in Chapter IX, and the few schools which do not care to devote any time to percentage in Grade VI may omit Chapter VI and the few examples requiring percentage in Chapters VII and VIII.

In general, the work of Grade VII is a thorough study of percentage and its applications, while Grade VIII considers arithmetic from the standpoint of the pupil's relation to the commercial, civic, and industrial world which he is about to enter. These two great features have been emphasized in the general plan of this book.

Having always advocated the elimination of obsolete matter from mathematics, and having repeatedly urged the reduction of the number of separate topics and the avoidance of subjects in which the child has no immediate or remote interest, the authors take pleasure in presenting a series of books containing only those essentials of arithmetic which are needed in common life. More complete courses are offered in other books, but the teacher who sympathizes with the modern trend in education will find in this series all of arithmetic that the pupil must thoroughly know.

The arrangement of the book is topical, so that a pupil stays long enough with a subject at one time to acquire that feeling of mastery which is his right and privilege. Along with this sequence of topics, however, are two new features which are noteworthy. One is the Little Examinations, a brief series of tests covering each chapter in turn; the other is the Review and Drill section, also placed near the end of each chapter, and furnishing a cumulative review of all preceding work. By the aid of these features a teacher may be assured that the pupil is kept refreshed upon those essentials of computation without which he cannot hope to succeed.

In the theory of the work the authors have no more sympathy with the idea that a pupil should be told to do a thing in a certain way, with no knowledge of why this way is the right one, than they have with the notion that he must explain every operation with all the care that a textbook writer would show. They believe that every process should be learned with an appeal to the pupil's understanding, and that thereafter it should become entirely mechanical; and in this way each operation has been presented in this book.

As to the applications, the effort has been to select the problems with reference to the life and interests of our people to-day. Groups of related problems are frequently given, and the isolated problems are, in general, typical of those which the pupil will need in the ordinary walks of life. The applications are more numerous than is usually the case in textbooks, but these applications are not permitted to exclude the abstract drill work without which no pupil has ever become a good computer. To balance adequately the abstract and the concrete, the drill work and the applied problem, the review work and the new material, has been one of the earnest endeavors of the authors in the preparation of this series.

The authors hope that their effort to prepare a perfectly usable textbook for Grades V–VIII, free from all eccentricities, will prove helpful to teachers and pupils alike.

GEORGE WENTWORTH
DAVID EUGENE SMITH

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BRIEF TITLES OF RELATED PROBLEMS

In Grades I and II no problem should be given which involves more than a single step. In Grade III only an occasional problem involving two steps, and this of the simplest kind, should be attempted. In Grade IV the problem involving two steps is gradually introduced. In general, in the primary grades groups of related problems should be of the simplest character, and the relation should not be such as to require the pupil to hold many details in mind. This was the policy followed in the primary book of this series.

In Grades V and VI more attention may properly be given to narrative problems of various kinds, those which are grouped about some incident or interest. These should be introduced occasionally, the dependence of one problem upon another being generally avoided, but the chief work of these years must necessarily be computation. In Grades VII and VIII related problems, those in which the work of one or two pages centers about some special interest, become more prominent, and this feature is seen in this book.

The following are the brief titles of some of the related problems to be found in the following pages:

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ESSENTIALS OF ARITHMETIC

GRAMMAR SCHOOL BOOK

CHAPTER I

FUNDAMENTAL OPERATIONS

1. What you have Learned of Arithmetic. You have already learned much about arithmetic. You have learned to count and to write numbers, at least to a million; how to add, subtract, multiply, and divide whole numbers, or integers; how to solve many problems; how to keep simple accounts; and how to do some easy work with fractions.

You have also learned how to write United States money, how to measure things, and how to use the ordinary tables needed in making these measurements. Besides all this, you have seen that you have need for arithmetic at the store, in your games, in reading, and in many other ways.

2. What there is still to Learn. There is, however, much more to learn about the uses of arithmetic, and more to learn about how to work with numbers. Every one of us is interested in what the world is doing and in our country and its products. We all like to know about our farms, our mines, our forests, our great industries, the accounts that we must sometimes keep, and the customs of business life, and we find much of interest about these matters in arithmetic. We shall now learn how to work with larger numbers, and how to solve problems about our individual interests and about the great industries of the country.

3. Numerals. Symbols used to represent numbers are called *numerals.* The common numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are called *Arabic*, or *Hindu-Arabic, numerals.* The symbol 0 is called *zero*, *naught*, or *cipher*, or by the name of the letter *o.*

Little attention is now given to the distinction between *notation* (the writing of numbers) and *numeration* (the reading of numbers) or to the memorizing of such definitions as those given on this page.

Schools should recognize the modern reading of numbers in telephoning, as "three–nine–six–o" for 3960, and in rapidly comparing entries in bookkeeping, as "nineteen seventy-five point fifty" for $1975.50.

4. Period. A group of figures into which a number is separated for ease in reading is called a *period.*

We separate into periods by commas or, in print, by spaces, thus: 14,375,286, 14 375 286, $756,283.75, and $756 283.75.

In separating into periods, begin at the right unless there is a decimal point, in which case begin at the decimal point. The left-hand period may have one, two, or three figures. We do not separate into periods the part at the right of the decimal point. Periods are not commonly used in written numbers of four figures, in dates, or in telephone and car numbers. In this book we shall use them only with numbers of five figures or more.

5. Place Value. The value of a period or figure depends upon the place it occupies. Therefore each has a *place value.*

For example, in the number 5427, the 7 is in *units' place* or *units' order*, and means 7 ones; the 2 is in *tens' place* or *tens' order* and means 2 tens; and so on.

6. Reading Numbers. In the number 62,124,706,871 we have four periods. In each period there are three orders: units, tens, and hundreds. We read this number "sixty-two billion, one hundred twenty-four million, seven hundred six thousand, eight hundred seventy-one."

$$\overbrace{62}^{\text{Billions}},\overbrace{124}^{\text{Millions}},\overbrace{706}^{\text{Thousands}},\overbrace{871}^{\text{Units}}$$

ORAL EXERCISE

1. What name do you give to 3 tens? to 10 tens?

2. What name do you give to 1000 thousands?

3. Looking at your fingers, do you see why people came to count by tens? Why was it?

4. Read these numbers:

296　　10,000　　354,200　　4,246,384
1296　　21,236　　400,576　　5,027,050
5280　　34,007　　148,392　　15,125,000
9999　　90,009　　900,009　　75,248,309

5. What advantage has our method of writing nine (9) over the method the Romans used, as shown on a clock face (IX)?

6. In the number 234, what place is occupied by the 4? by the 3? by the 2?

7. In the number 123,456,789, what name is given to the period 789? to the period 456? to the period 123?

WRITTEN EXERCISE

Write the following in words:

1. 15,756.
2. 24,908.
3. 137,005.
4. 230,230.
5. 500,005.
6. 999,999.
7. 2,342,756.
8. 31,427,642.
9. 141,141,141.

Write the following in figures:

10. Four million, twenty-five thousand, five.
11. Seventy-seven thousand, five hundred eighty-six.
12. Ninety-eight thousand, seven hundred fifty-four.
13. One hundred six thousand, seven hundred twenty.
14. Two hundred million, fourteen thousand, sixty-eight.

7. Roman Numerals. Roman numerals are used chiefly for numbering chapters, for dates, and on clocks. They consist of seven capital letters, as follows:

Letters,	I	V	X	L	C	D	M
Values,	1	5	10	50	100	500	1000

The first nine numbers are written thus:

I, II, III, IV or IIII, V, VI, VII, VIII, IX.

The tens are written thus:

X, XX, XXX, XL, L, LX, LXX, LXXX, XC.

The hundreds are written thus:

C, CC, CCC, CD, D, DC, DCC, DCCC, DCCCC or CM.

The numbers from eleven to nineteen are written thus:

XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX.

The following are examples of other numbers:

XXIII = 23 XCVIII = 98 CLXVI = 166
XXXVI = 36 LXXVI = 76 CCCLIX = 359
XLVIII = 48 LXXXII = 82 MDCCXCV = 1795
MDCCCCXXII or MCMXXII = 1922

WRITTEN EXERCISE

Write in common numerals:

1. XI.
2. LXIV.
3. LXXX.
4. XXVI.
5. LXVI.
6. DCCX.
7. LXXII.
8. XCVII.
9. MDCX.
10. LXXV.
11. XXXIV.
12. MDCCCCXX.

Write in Roman numerals:

13. 31.
14. 175.
15. 42.
16. 1916.
17. 125.
18. 1919.
19. 150.
20. 1928.

WRITTEN EXERCISE

1. Write in Arabic numerals: MDCLXIX, MCCCXLIV.

2. Write in Roman numerals: 49, 79, 94, 96, 99, 146.

3. Write in Roman numerals the number of the present year; of last year; of fifty years ago.

4. Write in common figures the number twenty-one million, one thousand, four hundred seventy-five.

5. Write in common figures the number one billion, one hundred seven million, one thousand, one.

6. Write in common figures the number of Ex. 5 decreased by 42,675; by 12,245; by 295,001.

Write in words:

7. 127,007.

8. 236,036.

9. 417,000.

10. 2,176,144.

✓**11.** 3,981,426.

12. 5,237,826.

13. 12,123,123.

14. 42,307,400.

15. 100,000,000.

Write in common numerals:

16. Six million, six; seven million, twelve.

17. Seventy-seven million, seventy-seven.

18. Five hundred fifty-five thousand, five.

19. Seven hundred seventeen thousand, one hundred one.

20. Six hundred sixty-six thousand, eighteen.

21. One hundred million, five hundred seventy-five.

Write in Roman numerals:

22. 240. **23.** 375. **24.** 126. **25.** 1930.

Write in common numerals:

26. MC. **27.** MDC. **28.** MDI. **29.** MCII.

8. Addition. The process of finding the number which is equal to two or more numbers taken together is called *addition.* The numbers added are called *addends* and the result is called the *sum.*

9. Unit. A standard by which we measure or count is called a *unit.*

Thus, in speaking of $4 the unit is $1; in speaking of 5 eggs the unit is 1 egg, but in speaking of 10 dozen eggs the unit is 1 dozen eggs; in speaking of 40 thousand bricks the unit is a thousand bricks; in counting 1, 2, 3, 4, and so on, the unit is 1; in counting by thousands the unit is 1000; and in measuring long distances the unit may be 1 mile.

10. Like Numbers. Numbers applied to the same unit are called *like numbers.*

Thus $5 and $17 are like numbers, and 27 and 36 are also like numbers.

Only like numbers can be added, and the sum is like the addends.

That is, we cannot add $2 and 3 yd. or 7 lb. and 4 books. If we add $12 and $13, the sum must be like the addends; that is, it must be $25, not 25 ft. It is true that we can put 4 books with 7 crayons, but we have to think of them only as *things,* and say that we have 11 things.

Teachers may, if they wish, be guided by the note on abstract and concrete numbers given on page 18, where the subject first assumes some importance.

11. How to Add. In adding a column of figures, try to read a column like a word. Try especially to see the 10's, the 5's, or the 15's.

In this column, for example, you should see the two 10's and the 15, and write the answer, 35. However, it is not wise to skip numbers in order to group by 10's, because you may forget the numbers that you have skipped and thus make a serious error.

$$\begin{array}{r} \left.\begin{array}{r}4\\2\\9\end{array}\right\} \\ \left.\begin{array}{r}8\\2\end{array}\right\} \\ \left.\begin{array}{r}7\\3\end{array}\right\} \\ \hline 35 \end{array}$$

12. Check. As a *check* or *proof* of the work, add the column from the top downwards, if you first added it upwards.

WRITTEN SPEED TEST

Copy the numbers in lines 1 to 8 inclusive, and add; then add the columns numbered 9 to 16 inclusive:

	9.	10.	11.	12.	13.	14.	15.	16.
1.	3	4	6	4	3	3	9	2
2.	4	4	1	9	8	1	4	3
3.	2	9	5	3	4	8	9	5
4.	4	7	4	9	4	8	2	8
5.	8	3	3	4	1	3	7	6
6.	8	2	2	1	9	7	3	6
7.	9	5	4	2	8	1	6	3
8.	6	4	8	4	3	9	9	2

Proceed in the same way for lines numbered 17 to 24 inclusive, and for columns numbered 25 to 32 inclusive:

	25.	26.	27.	28.	29.	30.	31.	32.
17.	5	9	8	7	3	2	6	8
18.	2	5	9	2	8	5	7	6
19.	9	7	6	5	4	6	9	7
20.	2	7	4	1	7	6	4	9
21.	7	7	5	8	8	3	9	4
22.	1	3	6	6	2	8	4	7
23.	2	4	6	3	9	5	8	7
24.	4	2	3	8	5	7	9	6

Such tests are of great value, especially if the pupils keep records of their speed and accuracy and review the exercise from time to time to see how the records improve.

13. Addition of Larger Numbers. You have already learned, in primary arithmetic, how to add numbers of two or more figures, so that we need consider only one simple example. To add 67, 86, and 34, we write them with units under units, and so on.

$$\boxed{\begin{array}{r}67\\86\\\underline{34}\\187\ \text{sum}\end{array}}$$

Then $4 + 6 + 7 = 17$, or 1 ten + 7 units. We therefore write the 7 in the units' column and add the 1 to the tens.

Then 1 ten + 3 tens + 8 tens + 6 tens = 18 tens, or 1 hundred + 8 tens. We therefore write the 8 in the tens' column and the 1 in the hundreds' column.

The sum is 187.

Arrange units of the same order in the same column.

Add each column by itself, beginning at the right, and write the sum, if it is less than 10, below the column added.

If the sum of any column is 10 or more, write the unit figure below the column added, and add the tens with the next column.

WRITTEN SPEED TEST

Copy the numbers in lines 1 to 6 inclusive, and add; then add the columns numbered 7 to 14 inclusive, and time yourself:

	7.	**8.**	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
1.	12	20	23	14	86	77	80	78
2.	6	10	15	17	83	85	90	94
3.	7	9	12	23	87	88	91	93
4.	13	15	14	36	42	23	31	46
5.	21	31	21	41	28	17	16	20
6.	42	24	33	23	14	23	41	26

ORAL EXERCISE

Look at each example and state only the sum:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
17	27	37	67	97	29	42
2	2	22	72	32	10	30
8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
15	21	32	10	21	11	12
13	12	14	6	10	11	10
15.	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**
20	33	42	81	16	15	17
25	12	63	40	10	10	20
22.	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**
21	32	41	10	21	31	40
35	70	88	73	20	31	50
29.	**30.**	**31.**	**32.**	**33.**	**34.**	**35.**
75	31	41	21	32	50	51
32	92	21	36	40	60	62
36.	**37.**	**38.**	**39.**	**40.**	**41.**	**42.**
41	23	20	21	20	30	32
62	40	42	42	96	40	43
10	10	20	20	10	50	54

This is a type of the rapid oral work that should be given daily in arithmetic. From three to five minutes of such rapid work in each recitation will do much to remove the charge that pupils are usually slow and inaccurate. The most important part of arithmetic is found in the four fundamental operations. Particular attention should be given to oral drill in addition by endings. The pupil should think of 15 + 13 as 15 + 10 + 3, or 25 + 3, and of 25 + 17 as 25 + 10 + 7, or 35 + 7.

WRITTEN EXERCISE

See how long it takes to copy and add each of these columns, and to check each result. Write the time with the results.

1.	2.	3.	4.
$147.50	$748.06	$375.42	$298.04
35.70	126.53	104.41	127.63
61.80	39.40	630.67	701.96
130.50	86.51	49.60	872.37

5.	6.	7.	8.
$359.68	$496.48	$396.04	$982.69
298.76	127.92	72.63	604.05
400.92	600.00	129.87	81.97
640.32	503.52	871.64	631.83
701.24	872.08	280.93	400.00

9.	10.	11.	12.
$1482.63	$4761.42	$3147.29	$4268.75
689.74	3917.63	2873.41	3942.35
2908.37	5238.58	4192.64	2065.08
8517.37	6082.37	128.46	5095.02
9310.26	4098.07	3029.00	2711.40
7091.63	5901.93	7081.00	1029.60

13.	14.	15.	16.
$3246.48	$2763.48	$2983.96	$1430.00
4139.72	298.43	4198.73	280.00
863.41	4200.75	2299.80	7557.75
96.37	8342.66	4763.81	263.48
2800.42	528.93	989.95	2986.75
437.00	2743.66	2263.48	3945.50

14. Subtraction. The operation of taking one number from another is called *subtraction.*

The number thus obtained is called the *difference* between the two given numbers; that is, the difference between 563 and 127 is 436. The difference is sometimes called the *remainder.*

15. Subtrahend. The number which is subtracted is called the *subtrahend.*

563 minuend
127 subtrahend
436 difference

16. Minuend. The number from which we subtract is called the *minuend.* The minuend is the sum of the difference and the subtrahend.

Subtraction may also be defined as the operation of finding the number which when added to the smaller of two numbers makes the larger. Since a great deal of subtraction is "making change," where the work is based on addition, the latter definition is often preferred.

The teacher is advised to spend but little time in asking for such definitions as those given above, since the terms are rarely used in business. The business man says, "*Deduct* $10 and pay me the *balance*," or "*Take out* $10 and pay me the *rest*," instead of "*Subtract* $10 and pay me the *difference*."

The teacher should lead the pupil to see that 100 — 37 is the sum of 90 — 30 and 10 — 7, or 63.

ORAL EXERCISE

Subtract the following numbers:

1.	17	27	16	26	13	43	15	75
	3	3	2	2	1	1	2	2

2.	27	27	36	36	43	43	85	85
	10	13	20	22	30	31	40	42

3.	67	48	96	78	57	39	75	87
	21	23	35	46	27	18	66	78

17. How to Subtract. If one village has 1632 inhabitants and another has 756, and we wish to find how many more persons live in the first village than in the second, we need to subtract 756 from 1632. We cannot subtract 6 from 2, but we can subtract 6 from 12, for 12 – 6 = 6. We have now used 10 of the 30, leaving 20. We cannot subtract 50 from 20, but 120 – 50 = 70. We have now used 100 of the 1600, and 1500 – 700 = 800. Therefore the result is 876.

$$\boxed{\begin{array}{r}1632\\ \underline{756}\\ 876\end{array}}$$

Check. Since 876 + 756 = 1632, the work is evidently correct.

WRITTEN EXERCISE

Subtract the following numbers:

1.	**2.**	**3.**	**4.**	**5.**
1796	1796	1796	1746	1746
384	394	398	399	587
6.	**7.**	**8.**	**9.**	**10.**
3234	3234	3234	3234	3234
872	815	845	1856	2978
11.	**12.**	**13.**	**14.**	**15.**
4368	5721	2933	4203	4307
299	4186	1768	2192	2358
16.	**17.**	**18.**	**19.**	**20.**
3628	4023	5127	5962	6014
969	3648	3675	3183	4235
21.	**22.**	**23.**	**24.**	**25.**
6271	8123	8207	8190	9235
4632	4637	5620	6391	7687

18. Making Change. If you owe 70¢ to a merchant and give him $1, he says, "70 and 5 are 75, and 25 more makes a dollar," at the same time laying down 5¢ and 25¢. He thus sees that $1 - $0.70 = $0.30.

Teachers may use real or toy money in teaching this subject, but pupils are now old enough to imagine the coins, as they will do in actual business methods.

ORAL EXERCISE

Make change for $1, when you owe:

1. 25¢ 50¢ 96¢ 85¢ 63¢ 39¢
2. 35¢ 40¢ 94¢ 87¢ 37¢ 29¢

Make change for 50¢, when you owe:

3. 25¢ 22¢ 36¢ 17¢ 27¢ 43¢
4. 35¢ 32¢ 38¢ 16¢ 19¢ 24¢

Make change for 25¢, when you owe:

5. 10¢ 8¢ 7¢ 6¢ 21¢ 13¢
6. 12¢ 18¢ 17¢ 16¢ 19¢ 15¢

Make change for $2, when you owe:

7. $1 $1.25 $1.40 $1.45 $1.57
8. $1.50 $1.75 $1.60 $1.85 $1.78

Imagine yourself buying goods at a store and handing the clerk the first amount given, the goods costing the second amount. State the amount of change due you in each case:

9. $1; 72¢.
10. $2; $1.37.
11. $2; $1.67.
12. $3; $2.17.
13. $4; $3.41.
14. $5; $1.27.
15. $5; $2.40.
16. $10; $6.55.
17. $5; $3.05.
18. $5; $1.70.
19. $10; $7.05.
20. $10; $6.05.

WRITTEN EXERCISE

See how long it takes to copy these numbers and to subtract and check. Write the time with the last result:

1. 42,350 21,175	**2.** 14,762 9,987	**3.** 36,431 21,596	**4.** 50,000 29,872
5. 63,527 41,962	**6.** 27,998 19,999	**7.** 60,000 48,762	**8.** 70,900 56,907
9. $175.62 81.98	**10.** $129.87 67.98	**11.** $208.09 98.78	**12.** $400.12 69.84
13. $170.08 69.99	**14.** $370.42 91.64	**15.** $209.18 87.69	**16.** $302.78 62.95
17. $802.65 69.81	**18.** $200.00 93.74	**19.** $608.75 35.82	**20.** $672.93 98.94
21. $409.98 23.49	**22.** $682.70 93.81	**23.** $200.00 187.60	**24.** $325.34 128.92
25. $429.81 149.92	**26.** $308.72 129.85	**27.** $298.05 109.72	**28.** $408.00 129.60

Teachers will do well to return to these efficiency tests occasionally, comparing the records of the pupils with their earlier records.

19. Cash Accounts. In keeping a cash account it is customary to write the receipts on the left side (called the *debit* side), and the expenditures on the right side (called the *credit* side), thus:

	19—					*19—*					
a	*May*	*4*	*Cash on hand*	*$1*	*50*	*May*	*5*	*Paid for ball*		*20*	*g*
b		*5*	*From father*		*20*		*6*	*Ball game*		*25*	*h*
c		*6*	*From mother*		*10*		*6*	*Marbles*		*15*	*i*
d		*7*	*Earned, errands*		*25*		*8*	*Balance*	*1*	*45*	*j*
e				*2*	*05*				*$2*	*05*	*k*
f	*May*	*8*	*Balance*	*$1*	*45*						

Here *a* shows cash on hand when this page of the account is begun; *b*, *c*, and *d* are receipts from father and mother, and for errands; *e* is the sum of these items on the day that the account is added to see how it stands. On the right side *g*, *h*, and *i* are bills paid. To find how much is now on hand we subtract the sum of 20¢, 25¢, and 15¢, or 60¢, from $2.05, and we find that the remainder (in accounts called the *balance*) is $1.45, which is written on the right, at *j*, and on the left, at *f*. To check the work, *g*, *h*, *i*, and *j* are added, and the sums, at *e* and *k*, must agree.

WRITTEN EXERCISE

Make out cash accounts and balance them as above:

1. Receipts: April 3, cash on hand, $2.10; April 4, caring for furnace, $0.75; April 7, selling papers, $0.15. Expenses: April 4, picture show, $0.10; April 6, book, $0.45. Balanced on April 8.

2. Receipts: Feb. 6, cash on hand, $135.60; Feb. 6, R. S. Roberts, $75.50; Feb. 7, M. L. Richards, $87.90; Feb. 8, R. P. Jasper, $42.25. Expenses: Feb. 6, rent, $75; Feb. 7, salaries, $60; Feb. 8, coal, $15.30; Feb. 8, account book, $2.25; Feb. 8, telephone rent, $6.50; Feb. 8, gas bill, $3.75. Balanced on Feb. 9.

Make out cash accounts and balance them:

3. Receipts: May 1, cash on hand, $1.25; May 3, earned by errands, 35¢. Expenses: May 3, pencil, 5¢. Balanced May 4.

4. Receipts: Sept. 15, cash on hand, 35¢; Sept. 16, earned by delivering papers, 16¢; Sept. 17, earned by cleaning automobile, 25¢; Sept. 18, earned by errands, 15¢. Expenses: Sept. 16, ball, 18¢; Sept. 18, dog collar, 15¢. Balanced Sept. 19.

5. Receipts: Oct. 7, cash on hand, $2.10; Oct. 7, earned by caring for horse, 30¢; Oct. 8, from Uncle John, $1; Oct. 9, earned by errands, 20¢. Expenses: Oct. 8, football pants, $1.50; Oct. 9, share in football, 50¢; Oct. 10, football shoulder pads, 30¢. Balanced Oct. 11.

6. Receipts: Dec. 4, cash on hand, 62¢; Dec. 4, earned by caring for furnace, 15¢; Dec. 5, earned by cleaning walks, 20¢; Dec. 6, earned by caring for furnace, 15¢. Expenses: Dec. 6, pair of skates, 94¢. Balanced Dec. 7.

7. Receipts: May 1, cash on hand, $36.50; May 2, R. S. Jacobs, $20; May 4, M. R. Randall, $32.60. Expenses: May 3, groceries, $7.86; meat, $3.50. Balanced May 5.

8. Receipts: April 3, cash on hand, $25.30; April 4, J. B. Fuller, $30; April 7, R. H. Bates, $4.30. Expenses: April 4, groceries, $9.32; April 6, meat, $7.28. Balanced April 8.

9. Receipts: Feb. 8, cash on hand, $154.25; Feb. 9, John Smith, $62.75; Feb. 11, M. L. Tilton, $85.44; Feb. 12, J. R. Haskins, $51.10. Expenses: Feb. 9, rent, $75; Feb. 12, clerks, $72; Feb. 15, coal, $15.30; Feb. 17, account book, $1.75; Feb. 22, telephone, $4.50; Feb. 23, gas, $4.25. Balanced Feb. 23.

Problems of this kind should be made by the pupils. The children should also be urged to keep their personal accounts, although these should not, as a rule, be submitted to general class inspection.

10. This boy is comparing his father's cash account with his own. He finds that he received $1.00 from his father on Oct. 15, 20¢ for work on Oct. 16, 35¢ for picking apples on Oct. 17, 15¢ for work on Oct. 18, 25¢ for work on Oct. 19, and 50¢ for some chestnuts that he gathered on Oct. 20. He paid 35¢ for a book on Oct. 16, 5¢ for some marbles on Oct. 17, 20¢ for paper and ink on Oct. 18, 40¢ for a knife on Oct. 20, and on Oct. 21 he put the balance that he had on hand in the bank, where he already had $17.50. Make out the cash account and find how much he then had in the bank.

11. For his father he made out a statement of the farm property, which showed the following items: value of the land and buildings, $7250; standing wood and timber, $1600; wagons and machinery, $750; cows and horses, $1975; oats, corn, and wheat on hand, $490; hay and straw on hand, $230; poultry, $35; sewing machine and household furniture, $325; due from creamery for milk furnished, $72.75; due from Mr. Young for hay, $28.50. The father still owed $2250 on the farm, and there was a grocery bill of $12.80 due. Make a statement like a cash account, showing the value of the property after deducting what was owed on the farm and at the grocery.

20. Multiplication. The process of taking one number as many times as there are units in another is called *multiplication.*

This means multiplication by a whole number. Thus, to multiply \$3 by 4 means that \$3 is taken 4 times.

The number multiplied is called the *multiplicand.*

The number by which the multiplicand is multiplied is called the *multiplier.*

The result of multiplying is called the *product.*

$$\begin{array}{rl} \$325 & \text{multiplicand} \\ \underline{\quad 3} & \text{multiplier} \\ \$975 & \text{product} \end{array}$$

21. Abstract and Concrete Numbers. It is common to speak of a number that does not refer to any particular kind of object or measure as an *abstract number.* It is also customary to speak of a number that refers to some particular kind of object or measure as a *concrete number.*

Thus 4 is commonly called an abstract number, and 4 feet a concrete number.

This distinction is not observed so much as it formerly was, for all *number* is essentially abstract. We may specify the units (feet, yards, apples, etc.), but the *number* is still abstract. The custom of specifying the units in solving a problem is also tending to disappear, the pupil keeping the units in mind just as the business man does when he solves his problems.

For example, consider the following cases:

If one book costs 20¢ and another book costs 30¢, both books cost *as many cents* as $20 + 30$, or 50¢.

If one book costs 20¢ and another book costs 30¢, the second book costs *as many cents* more than the first as $30 - 20$, or 10¢.

If 1 orange costs 5¢, 4 oranges will cost *as many cents* as 4×5, or 20¢.

If 1 orange costs 5¢, one can buy *as many oranges* for 20¢ as $20 \div 5$, or 4 oranges.

If 4 oranges cost 20¢, each orange costs *as many cents* as $20 \div 4$, or 5¢.

If desired, the above method may be used throughout the entire book.

In school, however, it adds to the clearness of the solution if the more important numbers are labeled with their proper units, and therefore this is done in the solutions in this book. For example, in $3 \times \$325 = \$975$, the multiplicand and product are properly given the same label.

22. Nature of the Numbers. It is therefore seen that

The multiplier must be thought of as abstract, and the product is like the multiplicand.

That is, if we have 3 times 17 ft., the 3 is abstract, and the product is feet, like the multiplicand.

23. Multiple. The product of two abstract integers is called a *multiple* of either. For example, 35 is a multiple of 7 and of 5.

24. Factors. The numbers which multiplied together make another number are called its *factors.* For example, the factors of 30 are 2, 3, and 5.

25. Power. The result of taking a number any number of times as a factor is called a *power* of the number.

For example, $2 \times 2 = 4$, and 4 is called the *second* power, or the *square*, of 2, and the product 2×2 is written 2^2. So $2 \times 2 \times 2$, or 2^3, is the *third* power, or the *cube*, of 2.

ORAL EXERCISE

1. State four multiples of 7; of 3; of 9; of 11; of 15.

2. State the factors of 35; of 77; of 21; of 49; of 121.

3. State the squares of 7, 6, 9, 8, 10, 11, 20, 100, 1000.

4. What is the square of 5? the cube of 3? the fourth power of 2? the cube of 5? the cube of 2? the cube of 10?

5. What is the square of 12? the cube of 4? the fourth power of 1? the fifth power of 2?

6. Multiply, in turn, by 2, 3, 4, and so on to 9:

7	4	6	3	8	5	9	2
11	20	21	30	50	70	40	60

26. How to Multiply. You have already learned how to multiply one number by another. For example, to multiply \$635.52 by 215, 3247 by 1305, and 3247 by 1350 we have:

$$\begin{array}{rcrcr}
\$635.52 & \qquad & 3247 & \qquad & 3247 \\
215 & & 1305 & & 1350 \\
\hline
3177\ 60 & & 16235 & & 162350 \\
6355\ 2 & & 9741 & & 9741 \\
127104 & & 3247 & & 3247 \\
\hline
\$136636.80 & & 4237335 & & 4383450
\end{array}$$

In multiplying \$635.52 by 215 we multiply as if the problem were 215 × 63,552¢. Then 5 × 63,552 = 317,760. We next have to multiply 63,552 by 1 ten, and the result is 63,552 tens, which we write so that the 2 is in tens' place. We next have to multiply 63,552 by 2 hundreds, and the result is 127,104 hundreds, which we write so that the right-hand figure is in hundreds' place. Adding, we have 13,663,680 cents, or \$136,636.80.

WRITTEN EXERCISE

Multiply each number by each of the numbers below it in the column; thus, 376 × 243, 405 × 243, and so on:

I	II	III	IV	V	VI
243	307	442	581	226	822
376	476	208	622	642	344
405	921	179	389	379	296
221	330	643	427	496	473
392	407	827	462	577	892
470	920	906	330	640	487
689	821	432	448	807	745
573	139	507	229	488	307

This will give 168 examples, from which the teacher should select only as many as necessary. Upon this whole question of efficiency tests it would be well at this time to read the suggestions on page 41.

WRITTEN EXERCISE

Multiply the following:

1. 24 x 325.
2. 32 x 526.
3. 41 x 578.
4. 53 x 482.
5. 67 x 573.
6. 78 x 409.
7. 89 x 277.
8. 96 x 483.
9. 523 x 832.
10. 624 x 421.
11. 532 x 409.
12. 677 x 505.
13. 752 x 327.
14. 581 x 826.
15. 823 x 983.
16. 435 x 481.
17. 632 x 408.
18. 563 x 481.
19. 408 x 662.
20. 507 x 836.
21. 802 x 906.
22. 532 x 848.
23. 783 x 927.
24. 860 x 342.
25. 83 x 1765.
26. 59 x 2763.
27. 23 x 4126.
28. 58 x 3279.
29. 63 x 4081.
30. 49 x 7756.
31. 82 x 3793.
32. 41 x 8872.
33. 64 x 8709.
34. 33 x 4981.
35. 25 x 4444.
36. 62 x 3436.
37. 83 x 7070.
38. 49 x 6236.
39. 83 x 8192.
40. 54 x 7762.
41. 36 x 8087.
42. 53 x 8129.
43. 66 x 6666.
44. 91 x 3473.
45. 17 x 4646.
46. 33 x 8308.
47. 46 x 12,481.
48. 34 x 36,408.
49. 43 x 21,761.
50. 56 x 29,376.
51. 58 x 42,807.
52. 64 x 80,009.
53. 123 x 1407.
54. 236 x 3143.
55. 823 x 4062.
56. 347 x 8096.
57. 823 x 7743.
58. 629 x 3648.
59. 426 x 3140.
60. 408 x 3776.
61. 567 x 9873.
62. 228 x 4362.
63. 503 x 8742.
64. 621 x 8263.
65. 987 x 7890.
66. 602 x 3348.
67. 672 x 4815.
68. 360 x 4572.
69. 666 x 7883.
70. 565 x 8432.
71. 821 x $248.75.
72. 633 x $336.90.

ORAL EXERCISE

1. Multiply by 10: 10¢, 50¢, $1.20, $3.50.

2. Multiply by 100: 3, 42, 565, $1.50, $21.75.

3. State a short way of multiplying a whole number (integer) by 10; by 100; by 1000; by 10,000.

4. State a short way of multiplying a number representing United States money by 10; by 1000.

27. Multiplying by Powers of 10. *To multiply an integer by 10, annex a zero; to multiply by 100, annex two zeros; to multiply by 1000, annex three zeros.*

To multiply a number representing United States money by 10, move the decimal point one place to the right; to multiply by 100, move the decimal point two places to the right; to multiply by 1000, move the decimal point three places to the right.

Hence to multiply 27 by 40, multiply by 4 and annex 0.

To multiply $25.16 by 300, multiply by 3 and move the decimal point two places to the right.

WRITTEN EXERCISE

Multiply the following:

1. 210 × $346.

2. 800 × 932 ft.

3. 9600 × $4300.

4. 7000 × $15.42.

5. 6520 × $12.50.

6. 320 × $4215.

7. 700 × 864 rd.

8. 4300 × $8250.

9. 9000 × $21.75.

10. 2750 × $31.02.

11. If a railway train goes 48 mi. an hour, how far will it go in 20 hr., at the same rate?

12. If it costs $14.75 a year to educate you, how much does it cost to educate 300 pupils, at the same rate?

ORAL EXERCISE

1. At $4 each, how much will 20 desks cost?
2. At $9 each, how much will 30 tables cost?
3. At $12 each, how much will 20 tables cost?

Multiply the following:

4. 60 × 8 70 × 9 40 × 11 50 × 7
5. 20 × 13 30 × 12 80 × 10 90 × 10

WRITTEN EXERCISE

Multiply, in turn, by 2, 20, and 200:

1. 143 286 409 527 683
2. $1.43 $2.86 $40.90 $52.70 $6.83

Multiply, in turn, by 3, 30, and 300:

3. 482 525 633 840 900
4. $4.82 $52.50 $63.30 $8.40 $9.00

Multiply, in turn, by 40, 400, 50, and 500:

5. 736 821 963 742 893
6. $5.35 $8.93 $7.34 $6.81 $9.75
7. Multiply $234.65 by 9; by 90; by 900; by 982.
8. Multiply $275.40 by 8; by 80; by 800; by 874.
9. Multiply $324.45 by 7; by 35; by 305.
10. Multiply $846.60 by 6; by 60; by 3; by 300.
11. At $85 each, how much will 60 mules cost?
12. At $55 each, how much will 70 cows cost?
13. At $135 each, how much will 80 horses cost?
14. At $575 each, how much will 90 automobiles cost?

ORAL EXERCISE

1. On the blackboard, multiply 68 by 5. Divide 680 by 2 [illegible]

2. On the blackboard, multiply $12.25 by 5. Also divide $122.50 by 2. How do the results compare? Tell a short way of multiplying by 5.

3. Because $5 = 10 \div 2$, to multiply an integer by 5 is the same as to annex 0 and divide by what number?

4. On the blackboard, multiply $42.74 by 50. Also divide $4274 by 2. Compare the results.

5. Because $50 = 100 \div 2$, to multiply by 50 we may first multiply by what number, and then divide by what other number?

6. On the blackboard, multiply $32.56 by 25. Also divide $3256 by 4. Compare the results.

Multiply the following by 5, 50, and 25, in turn:

7. 4   12   16   28   40   80   120

8. 8   24   32   48   88   96   160

28. Aliquot Part. A number that will exactly divide another is called an *aliquot part* of that number.

For example, 2 and 5 are aliquot parts of 10, and 2, 5, 10, 20, 25, and 50 are aliquot parts of 100.

Therefore the aliquot parts of $1 are 2¢, 5¢, 10¢, 20¢, 25¢, and 50¢. To these may be added $33\frac{1}{3}$¢, $16\frac{2}{3}$¢, and certain other aliquot parts which will be considered later.

29. Multiplying by Aliquot Parts. From the above exercise we have the following:

To multiply by 5, multiply by 10 and divide by 2.
To multiply by 50, multiply by 100 and divide by 2.
To multiply by 25, multiply by 100 and divide by 4.

ORAL EXERCISE

[redacted] 5; divide 200 by 2.

2. Multiply 40 by 5; divide 400 by 2.

3. Multiply 4 by 25; divide 400 by 4.

4. At 24¢ each, what will 5 pineapples cost?

5. At 28¢ a yard, what will 5 yd. of ribbon cost?

6. At $80 each, what will 50 ponies cost?

7. State a short way of multiplying by 5; by 50; by 500.

8. How much is 25 × 4? 25 × 8?

9. State a short way of multiplying by 25.

WRITTEN EXERCISE

1. Multiply by 50: 237, 425 ft., $68.50, $987.40.

2. Multiply by 25: 134, 802 ft., $49.60, $384.40.

3. Multiply by 500: 16, 64, 726, $42.80, $124.40.

Multiply the following by 5:

4.	48	74	88	82	448	1264	3642
5.	64	38	78	96	826	2488	8177

Multiply the following by 50:

6.	66	38	96	76	224	2172	4242
7.	88	46	98	42	338	3108	6543

8. Multiply the following by 25:

32	36	64	56	224	4844	4036

9. Multiply, in turn, 50¢, 25¢, 20¢, and 10¢ by:

40	200	160	240	360	480	2800

WRITTEN EXERCISE

1. At $72 an acre, how much must a man pay for 25 acres of land? for 250 acres?

2. At $55 each, how much must a farmer pay for 25 cows? for 50 cows? for 5 cows?

3. At $1425 each, how much will a manufacturer receive for 25 automobiles? for 50 automobiles?

4. If 2 M (2 thousand) feet of pine cost $96.50, how much will 8 M cost?

Pupils should appreciate the importance of recognizing the *relations of numbers.* In this example they should see that 8 M is 4 × 2 M, and should simply multiply $96.50 by 4.

5. If 6 thousand square feet of building paper cost $13.68, how much will 18 thousand square feet cost?

6. If 5 lb. of butter cost $1.50, how much will 15 lb. cost? How much will 25 lb. cost? How much will 50 lb. cost?

7. If a man grinds 35 bu. of corn, each bushel weighing 56 lb., how many pounds does he grind?

8. A farmer adds to his property a field of 65 acres, paying $75 an acre for it. How much does the field cost?

9. A dealer sells 15 automobiles for $13,875. How much does he receive for 45 automobiles at this rate?

10. If 38 cows average 408 lb. of milk apiece a month, how many pounds of milk do the cows produce in a month?

11. If a dealer sells 8 building lots for $13,875, how much does he receive for 24 lots at this rate?

Teachers should accustom the pupils always to estimate an answer in advance and to write it down. Then if they find a great difference between their result and their estimate they have either shown poor judgment or made an error in their work. Many absurd answers can be avoided in this way.

30. Nature of Division. In multiplication two factors are given to find the product. In division the product and one of two factors are given to find the other factor.

That is, in the case of 2 × $5 = $10 we have given the factors 2 and $5, from which to find the product $10.

In division we have given the product $10 and the factor 2, to find the other factor $5; or the factor $5, to find the other factor 2.

Upon the question of abstract and concrete numbers the teacher should consult § 21.

31. Division. If the product and one of two factors are given, the process of finding the other factor is called *division*.

There are two common symbols used in division. Besides the symbol ÷ we use the fraction form. Thus, we may write $15 \div 5 = 3$, or $\frac{15}{5} = 3$. That is, regarded as operations to be performed, $15 \div 5$ and $\frac{15}{5}$ have the same meaning.

32. Dividend. The given product, that is, the number to be divided, is called the *dividend*.

33. Divisor. The given factor, that is, the number by which we divide, is called the *divisor*.

34. Quotient. The required factor, that is, the number found by division, is called the *quotient*.

35. Remainder. Often a divisor does not exactly divide the dividend. Then that part of the dividend which is left is called the *remainder*.

Thus $14 \div 3 = 4$ with remainder 2. The quotient is usually written $4\frac{2}{3}$, meaning 4 with 2 still to be divided into 3 equal parts.

Such terms have long been familiar to you, and they are given at this time only for the purpose of review.

36. Two Kinds of Division. Since 2 × \$4 = \$8, we see that \$8 ÷ \$4 = 2 and \$8 ÷ 2 = \$4. That is,

If the dividend and the divisor are concrete, they must be like numbers, and the quotient is abstract.

If the dividend is concrete and the divisor abstract, the quotient is concrete and like the dividend.

Concerning this distinction, however, the teacher should consult § 21.

37. Check. To check the work in division, we multiply the quotient by the divisor; this product added to the remainder, if any, should equal the dividend.

For example, 45 ÷ 5 = 9; then 5 × 9 = 45. Again, 48 ÷ 5 = 9 with remainder 3; then 5 × 9 = 45, and 45 + 3 = 48.

38. Short Division. When the divisor is so small that the work may be done mentally, the process is called *short division.*

1. Required to divide 342 by 3.

$$\begin{array}{r} 3\,\underline{)\,300+30+12} \\ 100+10+\ \ 4=114 \end{array}$$

If 342 is separated as shown, we can divide each part by 3. This is what we really do mentally. We write the numbers as shown; then 3 is contained in 3 once (that is, 100 times in 300); 3 is contained in 4 once with remainder 1 (that is, 10 times in 40 with remainder 10); this 1 is 10 units, and with the 2 it makes 12. Then 3 is contained in 12 four times. Therefore the quotient is 114. We think, "3 in 3, 1; in 4, 1; in 12, 4."

$$\begin{array}{r} 3\,\underline{)\,342} \\ 114 \end{array}$$

2. Required to divide 42,364 by 7.

Here 7 is not contained in 4, but 7 is contained in 42 six times. Then 7 is not contained in 3, so we write a 0 below the 3. Then 7 is contained in 36 five times with remainder 1, and in 14 twice. Therefore the quotient is 6052.

$$\begin{array}{r} 7\,\underline{)\,42364} \\ 6052 \end{array}$$

3. Required to divide 35,282 by 9.

Here 9 is contained in 35 three times with remainder 8; in 82 nine times with remainder 1; in 18 twice; in 2 no times with remainder 2. Therefore the quotient is 3920 with remainder 2, and this is written 3920$\frac{2}{9}$.

$$\begin{array}{r} 9\underline{)35282} \\ 3920\frac{2}{9} \end{array}$$

4. Required to divide $27.53 by 4.

Dividing as before, we have \$27 ÷ 4 = \$6 with \$3 remaining; \$3.50 ÷ 4 = \$0.80 with \$0.30 remaining; \$0.33 ÷ 4 = \$0.08 with \$0.01 remaining. Therefore the quotient is \$6.88$\frac{1}{4}$.

$$\begin{array}{r} 4\underline{)\$27.53} \\ \$6.88\frac{1}{4} \end{array}$$

ORAL EXERCISE

Divide the following:

1. 48 ÷ 2.
2. 39 ÷ 3.
3. 84 ÷ 4.
4. 65 ÷ 5.
5. 72 ÷ 6.
6. 84 ÷ 7.
7. 96 ÷ 8.
8. 54 ÷ 9.
9. 38 ÷ 2.
10. 54 ÷ 3.
11. 52 ÷ 4.
12. 75 ÷ 5.
13. 78 ÷ 6.
14. 91 ÷ 7.
15. 96 ÷ 6.
16. 33 ÷ 2.
17. 41 ÷ 3.
18. 53 ÷ 4.
19. 63 ÷ 5.
20. 75 ÷ 6.

WRITTEN EXERCISE

Divide the following:

1. 8242 ÷ 2.
2. 4122 ÷ 3.
3. 7004 ÷ 4.
4. 4015 ÷ 5.
5. 4032 ÷ 6.
6. 3073 ÷ 7.
7. 8368 ÷ 8.
8. 4019 ÷ 9.
9. 7555 ÷ 2.
10. 9015 ÷ 3.
11. $38.72 ÷ 4.
12. $92.75 ÷ 5.
13. $41.30 ÷ 6.
14. $99.47 ÷ 7.
15. $65.68 ÷ 8.
16. $83.61 ÷ 9.
17. 81,004 ÷ 4.
18. 39,008 ÷ 5.
19. 41,269 ÷ 6.
20. 80,047 ÷ 7.
21. 42,345 ÷ 8.
22. 76,209 ÷ 9.
23. $35.21 ÷ 2.
24. $72.43 ÷ 3.
25. $61.23 ÷ 4.
26. $72.36 ÷ 5.
27. $82.02 ÷ 6.
28. $42.07 ÷ 7.
29. $75.20 ÷ 8.
30. $73.18 ÷ 9.

39. Dividing by Powers of 10. To divide by 10 is merely to find how many tens there are in the number.

Thus in 1230 there are 123 tens and no units. In 2373 there are 237 tens and 3 units remainder; that is, $2373 \div 10 = 237\frac{3}{10}$.

To divide an integer by 10, cut off the right-hand figure of the dividend. The result is the quotient, and the right-hand figure expresses the remainder.

To divide an integer by any power of 10, cut off as many figures from the right of the dividend as there are zeros at the right of the divisor. The result is the quotient, and the right-hand figures express the remainder.

Thus $12{,}400 \div 100 = 124$; $736{,}117 \div 1000 = 736\frac{117}{1000}$.

40. Dividing by Multiples of Powers of 10. In this case we divide by the power of 10, as in § 39, and then by the number indicating the multiple.

For example, to divide 42,800 by 200, we cut off two zeros from the divisor and the dividend and then divide by 2.

So, to divide 27,407 by 300, we cut off two zeros from the divisor and two figures from the right of the dividend. We then divide by 3, and the quotient is 91 with remainder 1 in the hundreds' place, and to this we join the 07 cut off, obtaining the remainder 107.

$$\begin{array}{r} 2\cancel{0}\cancel{0}\overline{)\,428\cancel{0}\cancel{0}} \\ 214 \\ 3\cancel{0}\cancel{0}\overline{)\,274\cancel{0}\cancel{7}} \\ 91\frac{107}{300} \end{array}$$

WRITTEN EXERCISE

Find the quotient, and the remainder if any:

1. $860 \div 10$.
2. $940 \div 20$.
3. $630 \div 30$.
4. $840 \div 40$.
5. $950 \div 50$.
6. $720 \div 60$.
7. $840 \div 70$.
8. $960 \div 80$.
9. $450 \div 90$.
10. $810 \div 90$.
11. $375 \div 10$.
12. $625 \div 20$.

Find the quotient, and the remainder if any:

13. 625 ÷ 30.
14. 475 ÷ 40.
15. 336 ÷ 50.
16. 527 ÷ 70.
17. 642 ÷ 70.
18. 495 ÷ 80.
19. 873 ÷ 90.
20. 888 ÷ 90.
21. 41,300 ÷ 100.
22. 37,800 ÷ 200.
23. 54,300 ÷ 300.
24. 92,400 ÷ 400.
25. 72,501 ÷ 500.
26. 67,803 ÷ 600.
27. 49,521 ÷ 700.
28. 32,449 ÷ 800.
29. 378,367 ÷ 900.
30. 438,000 ÷ 3000.
31. 228,000 ÷ 4000.
32. 745,000 ÷ 5000.
33. 663,600 ÷ 6000.
34. 569,800 ÷ 7000.
35. 848,623 ÷ 8000.
36. 729,542 ÷ 9000.

37. There being 60 min. in an hour, how many hours are there in 1080 min.?

38. There being 60 sec. in a minute, how many minutes are there in 1680 sec.?

39. At $40 a set, how many sets of harness must a manufacturer sell to receive $1240?

40. A farmer pays $6650 for some land at $70 an acre. How many acres does he buy?

41. A dealer sells some city lots for $44,100 at $700 a lot. How many lots does he sell?

42. A dealer has $2750 with which to buy some village lots at $500 each. What is the greatest number he can buy, and how much money will he have left?

43. An automobile dealer has $112,000 to invest in automobiles at $1200 each. What is the greatest number he can buy for this sum, and how much money will he have left?

44. A ton is 2000 pounds. If a coal pocket contains 597,350 pounds of coal, how many tons does it contain, and how many pounds are left over?

41. Long Division. When the divisor has two or more figures, except in such cases as are given in §§ 39 and 40, it is customary to use a process known as *long division.*

We shall first consider the simplest case, that in which the units' figure of the divisor is 1.

1. Required to divide 7182 by 21.

If all the work is written, it would appear as in the first of these forms, but for practical purposes the second is used.

(1)

$$
\begin{array}{rrl}
 & 342 & \\
21 & \overline{)7182} & \\
 & 6300 & = 300 \text{ times the divisor} \\
 & \overline{882} & \\
 & 840 & = 40 \text{ times the divisor} \\
 & \overline{42} & \\
 & \underline{42} & = 2 \text{ times the divisor}
\end{array}
$$

(2)

$$
\begin{array}{rr}
 & 342 \\
21 & \overline{)7182} \\
 & 63 \\
 & \overline{88} \\
 & 84 \\
 & \overline{42} \\
 & \underline{42}
\end{array}
$$

First we will consider (1), the complete form. We see that 21 evidently is not contained in 7, so there are no thousands in the quotient. The divisor is contained three times in 71, and since the 71 is hundreds, the quotient figure 3 is written over the hundreds' place. Then $300 \times 21 = 6300$, and there remains 882 still to be divided; 21 is contained in 88 four times, and since the 88 is tens, the 4 of the quotient is written over the tens' place. Then $40 \times 21 = 840$, and there remains 42 to be divided; 21 is contained in 42 twice, and 2 is written over the units' place. The quotient is therefore 342. It will be seen that 6300, 840, and 42 make up 342×21, as should be the case.

Consider now the second form, which is always used in practice. Since at any time in this example we need only two figures of the dividend or two figures of such a partial dividend as 882, we may omit the zeros in 6300 and 840, and use only the 88 in the remainder 882, thus making the work shorter.

We may check our result by multiplying 342 by 21. Then we have $21 \times 342 = 7182$, the dividend.

2. Required to divide 8547 by 21.

$$\begin{array}{r} 407 \\ 21\overline{)8547} \\ 84 \\ \hline 147 \\ 147 \\ \hline \end{array}$$

Here we have 85 ÷ 21 = 4, and since the 84 is hundreds, the 4 is written over the hundreds' place. Then 4 × 21 = 84, and when 84 is subtracted, there is a remainder 1. Bringing down one figure as before, we have 14 tens to be divided by 21. Since 21 is not contained in 14, we write 0 in the tens' place to show that there are no tens, and bring down the next figure, 7. Then 147 ÷ 21 = 7, and 7 × 21 = 147, exactly, so there is no remainder.

WRITTEN EXERCISE

Divide the following:

1. 462 ÷ 11.
2. 561 ÷ 11.
3. 693 ÷ 11.
4. 979 ÷ 11.
5. 869 ÷ 11.
6. 968 ÷ 11.
7. 748 ÷ 11.
8. 924 ÷ 11.
9. 273 ÷ 21.
10. 294 ÷ 21.
11. 462 ÷ 21.
12. 693 ÷ 21.
13. 798 ÷ 21.
14. 819 ÷ 21.
15. 987 ÷ 21.
16. 899 ÷ 31.
17. 779 ÷ 41.
18. 816 ÷ 51.
19. 1098 ÷ 61.
20. 1353 ÷ 41.
21. 1633 ÷ 71.
22. 2673 ÷ 81.
23. 3367 ÷ 91.
24. 2331 ÷ 21.
25. 2583 ÷ 21.
26. 3472 ÷ 31.
27. 2121 ÷ 21.
28. 6789 ÷ 31.
29. 8568 ÷ 21.
30. 3317 ÷ 31.
31. 10,731 ÷ 21.
32. 14,322 ÷ 31.
33. 16,461 ÷ 31.
34. 25,297 ÷ 41.
35. 18,879 ÷ 31.
36. 28,741 ÷ 41.
37. 41,004 ÷ 51.
38. 49,410 ÷ 61.
39. 59,696 ÷ 41.
40. 52,582 ÷ 61.
41. 62,267 ÷ 71.
42. 63,048 ÷ 71.
43. 71,280 ÷ 81.
44. 90,090 ÷ 91.
45. 90,909 ÷ 91.
46. 50,337 ÷ 51.
47. 50,898 ÷ 51.
48. 62,937 ÷ 81.
49. 71,928 ÷ 81.
50. 60,939 ÷ 61.
51. 34,587 ÷ 61.
52. 73,629 ÷ 81.
53. 65,448 ÷ 81.
54. 73,619 ÷ 91.

42. Long Division Continued. We shall now consider the more general case of a two-figure divisor.

Required to divide 19,224 by 54.

The divisor is not contained in 1 or in 19, so there are no ten-thousands and no thousands in the quotient. But since 3 × 54 = 162, the divisor is contained three times in 192 with remainder 30. Since this is hundreds, the quotient figure 3 is written over the hundreds' place. Bringing down the next figure of the dividend, 2, we have 302 tens to be divided. Since 5 × 54 = 270, the divisor is contained five times in 302 with remainder 32. Since this is tens, the quotient figure 5 is written over the tens' place. Bringing down the next figure of the dividend, 4, we have 324 still to be divided. Since 324 is exactly 6 × 54, the units' figure is 6 and there is no remainder. Hence the quotient is 356.

$$\begin{array}{r} 356 \\ 54\overline{)19224} \\ 162 \\ \hline 302 \\ 270 \\ \hline 324 \\ 324 \\ \hline \end{array}$$

43. Product Tables. It is sometimes helpful for beginners to prepare a table of products of the divisor, as follows:

2 × 54 = 108
3 × 54 = 162
4 × 54 = 216
5 × 54 = 270
6 × 54 = 324
7 × 54 = 378
8 × 54 = 432
9 × 54 = 486

With such a table made out in advance, we can tell at once, without making several trials, what the next figure of the quotient will be.

WRITTEN EXERCISE

Divide the following:

1. 3848 ÷ 52.
2. 8556 ÷ 92.
3. 6572 ÷ 62.
4. 4599 ÷ 73.
5. 8815 ÷ 43.
6. 2275 ÷ 35.
7. 2793 ÷ 57.
8. 6006 ÷ 77.
9. 2470 ÷ 19.
10. 8580 ÷ 66.
11. 22,311 ÷ 67.
12. 31,950 ÷ 75.
13. 63,204 ÷ 69.
14. 74,907 ÷ 87.
15. 89,001 ÷ 99.
16. 41,890 ÷ 59.
17. 301,184 ÷ 64.
18. 891,891 ÷ 99.

WRITTEN EXERCISE

1. If $1548 is divided equally among a dozen persons, how much is the share of each?

2. If $2193.75 is divided equally among 15 persons, how much is the share of each?

3. If a piece of land containing 289 acres is cut into 17 equal lots, what is the area of each?

4. If the dividend is 420, the quotient 32, and the remainder 4, what is the divisor?

5. If the quotient is 24, the divisor 12, and the remainder 4, what is the dividend?

6. At 260 mi. a day, how long would it take a steamer to go from San Francisco to Guam, 5200 mi.?

7. At a certain temperature sound travels 1132 ft. a second. How long will it take sound to travel 11,320 ft.?

8. The product of two numbers is 98,441, and one of them is 49. What is the other number?

9. A man paid $20,280 for a farm, the price being $65 an acre. How many acres did he buy?

10. A canning factory ships 30,000 cans of fruit in boxes containing 4 dozen cans each. How many boxes does it require?

11. There being 27 cu. ft. in a cubic yard, how many loads (cubic yards) of earth must be removed to excavate 108,675 cu. ft.?

12. One of the routes around the world is 18,424 mi. long. If a man makes the trip in 56 days, what is the average distance traveled per day?

13. If the circumference of a wagon wheel is 15 ft., and there are 5280 ft. in a mile, how many times will the wheel turn in going 9 mi.? how many times in going 25 mi.?

44. Long Division Continued. We shall now consider a case like that of 341,597 divided by 248.

The divisor is not contained in 3 or in 34, so there are no hundred-thousands or ten-thousands in the quotient. But it is contained in 341 once, with remainder 93. Since this is thousands, the quotient figure, 1, is written over the thousands' place. Bringing down the next figure of the dividend, 5, we have 935 hundreds to be divided. It might seem as if the next figure of the quotient would be 4, and here a product table (§ 43) would be helpful; but 4 × 248 = 992, and this is larger than 935. Trying 3 for the next figure, we have 3 × 248 = 744 with 191 remaining. Since we have divided hundreds by 248, the 3 is hundreds and is written over the hundreds' place. Bringing down the next figure, we find that 1919 ÷ 248 = 7 with 183 remaining. Since the 7 is tens, we write it over the tens' place. Then, bringing down the last figure, we have 1837 ÷ 248 = 7 with 101 remaining. Therefore the quotient is $1377\frac{101}{248}$.

$$
\begin{array}{r}
1377\\
248\overline{)341597}\\
\underline{248}\\
935\\
\underline{744}\\
1919\\
\underline{1736}\\
1837\\
\underline{1736}\\
101
\end{array}
$$

We may check the work by adding 101 to the product of 248 × 1377, the result being 341,597, the dividend.

45. General Directions. Therefore, in long division

Write the divisor to the left of the dividend.

Take for the first partial dividend the least number of left-hand figures that will contain the divisor, and write the quotient over the right-hand figure of this partial dividend.

Multiply the divisor by this quotient, and place the product under the partial dividend used.

Subtract this product, and to the remainder annex the next figure of the dividend for a new partial dividend.

Divide as before, and continue this process until all the figures of the dividend have been used.

WRITTEN EXERCISE

Divide the following:

1. 25,585 ÷ 301.
2. 52,373 ÷ 631.
3. 47,971 ÷ 623.
4. 153,397 ÷ 211.
5. 976,950 ÷ 501.
6. 972,471 ÷ 111.
7. 161,784 ÷ 321.
8. 324,360 ÷ 306.
9. 140,850 ÷ 225.
10. 183,750 ÷ 175.
11. 148,555 ÷ 407.
12. 234,901 ÷ 503.
13. 208,575 ÷ 309.
14. 870,672 ÷ 204.
15. 367,848 ÷ 786.
16. 102,465 ÷ 495.
17. 472,932 ÷ 604.
18. 349,206 ÷ 407.
19. 117,440 ÷ 320.
20. 419,640 ÷ 538.
21. 287,712 ÷ 216.
22. 615,000 ÷ 328.
23. 685,542 ÷ 286.
24. 997,108 ÷ 478.
25. 965,496 ÷ 852.
26. 7753 ÷ 69.
27. 8598 ÷ 76.
28. 8585 ÷ 23.
29. 76,452 ÷ 56.
30. 18,753 ÷ 64.
31. 71,797 ÷ 57.
32. 19,140 ÷ 46.
33. 26,523 ÷ 47.
34. 27,559 ÷ 59.
35. 47,078 ÷ 69.
36. 50,602 ÷ 82.
37. 101,160 ÷ 232.
38. 110,370 ÷ 124.
39. 252,150 ÷ 412.
40. 120,830 ÷ 653.
41. 260,380 ÷ 422.
42. 192,892 ÷ 232.
43. 211,428 ÷ 5873.
44. 287,712 ÷ 5328.
45. 313,205 ÷ 8465.
46. 411,075 ÷ 5481.
47. 616,693 ÷ 8009.
48. 965,496 ÷ 9852.
49. 1,599,560 ÷ 2836.
50. 2,402,728 ÷ 6257.

WRITTEN EXERCISE

1. If 11 acres of land are worth $1485, what is the average price per acre? How much are 4 acres worth?

2. If a dealer pays $336 a dozen for suits of ready-made clothes, how much does he pay per suit? for 7 suits?

3. A man pays $1276 for the rent of a house in the city for 11 mo. At this rate, how much does he pay for 5 mo.?

4. A wagon dealer pays $1215 for 9 wagons. At this rate, how much do 4 wagons cost? How much do 7 wagons cost?

5. At the rate of $784 for 8 acres of land, how much will a man have to pay for 3 acres? for 5 acres? for 100 acres?

6. If a stock dealer buys some cattle for $72,012, paying $51 a head, how many head of cattle does he buy?

7. If a jobber sells during the year $82,411 worth of shoes at $61 a case, how many cases does he sell?

8. If a dealer sells $164,598 worth of linseed oil during a year at $42 a barrel, how many barrels does he sell?

9. If a factory sells 3684 ready-made suits of a certain grade for $92,100, what is the price per suit?

10. If a state pays $2,325,798 for constructing 471 mi. of model road, what is the average cost per mile?

11. If it costs $5,236,602 to construct 287 mi. of railroad, what is the average cost per mile?

12. If a company employs 8009 men on the same schedule of wages, and the pay roll for these men is $6,247,020 a year, what is the annual wages of each?

13. If a company employs 250 men on the same schedule of wages, and the pay roll for these men is $270,000 a year, what is the monthly wages of each?

46. Long Division of United States Money. Division of United States money does not involve any new principle. We divide as with other numbers, placing the decimal point in the quotient above that in the dividend.

Required to divide $21,375.12 by 372.

```
        $57.46
372 )$21375.12
      1860
       2775
       2604
        1711
        1488
         2232
         2232
```

Since $2, $21, and $213 cannot be divided by 372 with an integer for a quotient, we divide $2137 by 372, and the quotient is $5 with remainder $277. Since the $2137 is in the tens' place of dollars, we write the $5 over the tens' place of the dividend.

The rest of the division is as in cases already studied. The decimal point in the quotient is placed over that in the dividend, and the quotient is $57.46.

WRITTEN EXERCISE

Divide the following:

1. $973.47 ÷ 111.
2. $573.12 ÷ 199.
3. $1967.50 ÷ 50.
4. $6882.92 ÷ 11.
5. $12,018.30 ÷ 21.
6. $96,899.23 ÷ 23.
7. $36,147.06 ÷ 53.
8. $33,328.80 ÷ 54.
9. $33,747.20 ÷ 64.
10. $39,913.56 ÷ 81.
11. $27,891.25 ÷ 125.
12. $14,225.40 ÷ 420.
13. $734,986.12 ÷ 836.
14. $501,738.80 ÷ 538.
15. $102,465.00 ÷ 2070.
16. $692,356.00 ÷ 8764.
17. $616,693.00 ÷ 8009.
18. $388,591.00 ÷ 9037.

19. A shop has 664 men on its pay roll, each receiving the same wages. For 6 days this pay roll amounts to $9960. How much does each man receive a day?

LITTLE EXAMINATIONS

I. 1. MDXC = (?).
2. 987,654 + 456,789.
3. 900,123 – 456,789.
4. 7096 × 98,307.
5. 25 × 427,386.
6. $1780.02 ÷ 99.
7. $42,346.20 ÷ 7.
8. 52,720 ÷ 500.

II. 1. MCCXCIV = (?).
2. 427,625 + 49,836.
3. 302,123 – 56,877.
4. 4009 × 92,638.
5. 75 × 326,004.
6. 500 × 24,876.
7. $180.18 ÷ 91.
8. 46,500 ÷ 250.

III. 1. CLXXXIX = (?).
2. 697,439 + 89,876.
3. 711,234 – 68,098.
4. 386 × $562.75.
5. 50 × 486,368.
6. 75 × 544,664.
7. $446.22 ÷ 67.
8. 52,320 ÷ 125.

IV. 1. LXXXVIII = (?).
2. 587,629 + 89,987.
3. 834,213 – 96,789.
4. 7237 × 41,826.
5. 936 × $700.99.
6. 25 × 336,404.
7. 500 × 27,460.
8. $176.30 ÷ 43.
9. 56,140 ÷ 70.
10. 57,235 ÷ 130.

V. 1. MCCCIV = (?).
2. 829,968 + 73,875.
3. 501,230 – 66,463.
4. 5009 × 80,008.
5. 672 × $899.50.
6. 75 × 42,764.
7. 250 × 6744.
8. $55.86 ÷ 57.
9. 12,012 ÷ 77.
10. 57,620 ÷ 400.

REVIEW AND DRILL

47. Nature and Purpose of Review and Drill. At the end of each chapter in this book there occurs a section like this one. The work is called Review and Drill, and it is optional with the teacher whether or not it shall be used and how much of it shall be taken. None of these sections is merely a review of the chapter immediately preceding it, for review work is already provided in the chapter itself; on the contrary, the review is general and covers all the important processes in arithmetic that have been studied. This renders possible a continual return to fundamental and first principles, a constant brushing up in the essentials of arithmetic. This constant brushing up, drilling, and reviewing of the essentials is necessary for the attainment of good results. It is easily made interesting; it contributes to efficiency in computation; and it will, if carried out in the right spirit, help to remove the charge that most pupils are not quick and accurate in their calculations.

When the length of the school year permits, the study of these Review and Drill sections is strongly recommended; but the work being supplementary and optional, it doubtless will often be desirable for the teachers to select from these exercises only what is specially adapted to their immediate needs, omitting the rest. A certain flexibility is thus offered that will make the textbook more adaptable to varying local requirements than has heretofore been the case.

Each section consists of three parts: first, a drill in operations, emphasizing the *mechanical* side of arithmetic; second, an exercise in problems without numbers, emphasizing the *analytic* side; and finally, live industrial problems that photograph real situations, emphasizing the *practical* side.

It will at once occur to every teacher that in these Review and Drill sections, with their threefold, inclusive emphasis, there is ready at hand a series of *efficiency tests* which will make it easy at frequent intervals to check up the work of the school as well as the work of the individual pupils.

I. DRILL SECTION

ORAL EXERCISE

1. State the sum and also the difference of:

98	73	66	80	88	35	45
11	21	22	30	30	28	36

2. Multiply, in turn, by 2, 3, 4, and so on to 9:

20 30 40 70 60 21 31

3. Divide, in turn, by 2, 3, 4, and so on to 9:

10 20 50 70 30 25 48

WRITTEN EXERCISE

Add the following:

1. 273 + 421 + 68 + 9026 + 34,379 + 75 + 286.

2. 3972 + 8298 + 4000 + 17,625 + 31 + 9 + 407.

Subtract the following:

3. 876,481 – 298,327.

4. 173,209 – 106,492.

5. 298,008 – 43,661.

6. 381,200 – 179,699.

Multiply the following:

7. 628 × 325.

8. 872 × 896.

9. 1268 × 3472.

10. 2081 × 6235.

11. 469 × 832.

12. 4287 × 3009.

Divide the following:

13. 65,664 ÷ 456.

14. 221,336 ÷ 102.

15. 7,605,920 ÷ 112.

16. 77,112 ÷ 189.

17. 328,320 ÷ 360.

18. 7,913,576 ÷ 209.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. How do you add a number of two figures to another number of two figures when the sum of the units is more than ten? Give your reasons.

2. How do you subtract a number of two figures from a greater number when the units' figure to be subtracted is greater in value than the units' figure of the minuend?

3. How do you multiply by a number of three figures, the middle figure being zero?

4. How do you check your work in addition? How do you check your work in subtraction?

5. How do you check your work in division? How do you check your work in multiplication?

6. If you know the cost of a certain number of acres of land, how do you find the average cost per acre?

7. If you know the weight of each member of your class, how do you find the average weight?

8. If you have the product of two numbers and one of the two numbers given, how do you find the other number?

9. If you have the dividend, remainder, and quotient given, how do you find the divisor?

10. If you have the multiplicand and product given, how do you find the multiplier?

11. How do you multiply by a power of ten? How do you multiply by a multiple of a power of ten?

12. How do you divide by a power of ten? How do you divide by a multiple of a power of ten?

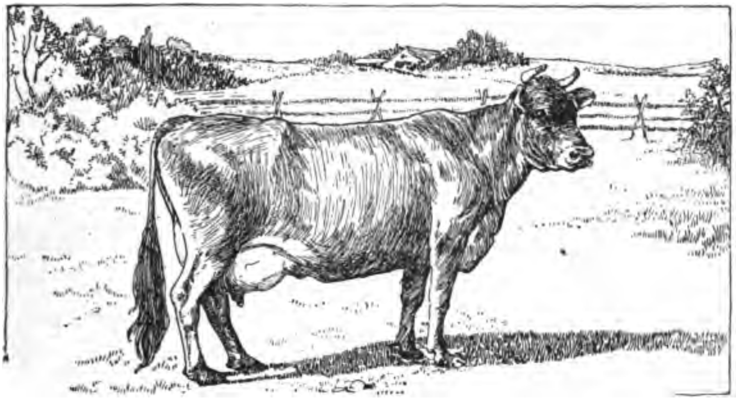
III. INDUSTRIAL PROBLEMS

THE COST OF WASTEFULNESS

1. If a farm wagon that cost $60 is left out in the yard instead of being kept in the shed, it will last about 6 yr., but if kept under cover, it will last about twice as long. How much does a farmer pay for his carelessness per year if he leaves the wagon out?

2. Each of two neighbors bought for his farm work a two-horse cultivator, and each paid $25. One of them took good care of his cultivator, and at the end of 15 yr. it was still giving good service. The other left his out during all kinds of weather and had to buy a new one at the end of every 5 yr. How much did the careless one lose in 15 yr. by his neglect?

3. A farmer after threshing his wheat had 16 tons of straw left. A ton of this straw contains 12 lb. of nitrogen, worth 20¢ a pound, and 10 lb. of potash and 2 lb. of phosphoric acid, each worth 5¢ a pound. If the farmer wastes the straw instead of using it on the soil as fertilizer, how much money does he waste?



DOES BLOOD TELL IN THE DAIRY?

1. Ralph's father explained to him what was meant by grading the cows, keeping their records for milk, and disposing of those with poor records. He showed him the records of twelve farms, as follows:

With Systematic Grading			*Without Systematic Grading*		
Herd	Annual cost of food per cow	Annual profit per cow	Herd	Annual cost of food per cow	Annual profit per cow
a	$34.28	$34.02	*g*	$31.65	$13.34
b	33.37	36.19	*h*	40.58	8.13
c	47.11	25.83	*i*	38.30	22.22
d	36.72	38.27	*j*	37.40	22.33
e	31.19	48.86	*k*	33.78	38.93
f	31.57	42.81	*l*	32.88	14.91

Ralph and his father then figured out the average annual cost of food and profit per cow, in each class of farms. They did this by dividing each of the four totals by six. What were the results?

2. They then found the differences between the two cost averages and the two profit averages. What are these two differences?

RICHARD'S VEGETABLE GARDEN

1. Richard's father has rented him a vegetable garden. Richard is to have the profits for a camping trip. On April 16 he plants some early potatoes. He counts on these potatoes being ready to dig in 79 da. When must he be home to begin to dig them?

2. If Richard allows 18 da. after the date found in Ex. 1 in which to let all the potatoes mature, and to sell them, what is the earliest day on which he can go camping?

3. On April 24 Richard sets out some early cauliflower, and he has to allow 72 da. for this to be ready to sell. When must he be on hand to sell the cauliflower?

4. Richard raised $4\frac{1}{2}$ bu. of early potatoes and sold them at $1 a bushel. How much did he receive for the potatoes?

5. Richard sold 25 heads of early cauliflower at 10¢ a head. How much did he receive for the cauliflower?

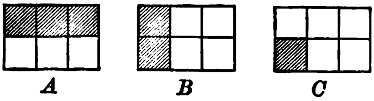
6. Richard pays his father 75¢ rent for his garden and for his materials. Make out a statement showing Richard's income and expenses, and the amount that he has left for his camping trip.

CHAPTER II

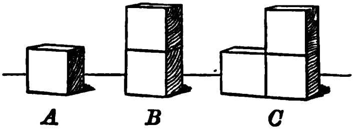
COMMON FRACTIONS

ORAL EXERCISE

1. When anything is divided into 3 equal parts what is each part called? What is each part called when the object is divided into 5 equal parts? into 12 equal parts?

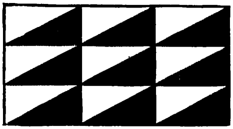


2. In the above figure, what part of *A* is shaded? What part of *B* is shaded? What part of *C* is shaded? From *A* we see that $\frac{1}{2}$ equals how many sixths? From *B* we see that $\frac{1}{3}$ equals how many sixths?



3. In this figure of the cubes, *A* is what part as large as *B*? as *C*? *B* is what part as large as *C*?

4. In the above figure of the cubes, if *C* is called 1, what are *A* and *B* called? If *B* is called 1, what are *A* and *C* called? If *A* is called 1, what are *B* and *C* called?



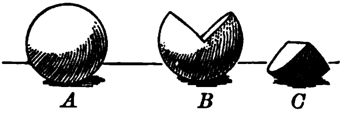
5. Each of these small rectangles, or oblongs, is what part of the whole rectangle? Each shaded triangle is what part of the whole rectangle? What does this tell you about $\frac{1}{2}$ of $\frac{1}{9}$? What does it tell you about $\frac{1}{9}$ of $\frac{1}{2}$?

48. Fraction. A number which shows what number of equal parts of a unit is taken is called a *fraction.*

Thus, $\frac{1}{2}$ shows that one of the two equal parts of a unit is taken; $\frac{3}{4}$ shows that three of the four equal parts of a unit are taken.

49. Denominator. The number which shows into how many equal parts a unit has been divided to make a fraction is called the *denominator* of the fraction.

50. Numerator. The number which shows how many equal parts of a unit have been taken to make a fraction is called the *numerator* of the fraction.



For example, in this figure, *B* contains three of the four equal parts of a sphere; that is, *B* is $\frac{3}{4}$ of a sphere, 4 is the denominator of the fraction $\frac{3}{4}$, and 3 is the numerator.

51. Terms. The numerator and the denominator together are called the *terms* of the fraction.

52. Common Fraction. A fraction which has both terms expressed is called a *common fraction.*

53. Proper Fraction. A fraction whose numerator is less than the denominator is called a *proper fraction.*

54. Unit Fraction. A fraction whose numerator is 1 is called a *unit fraction.*

55. Improper Fraction. A fraction whose numerator equals or exceeds the denominator is called an *improper fraction.*

In the above figure, *A* is $\frac{4}{4}$ of a sphere, and $A + C$ is $\frac{5}{4}$ of a sphere.

56. Integer. A whole number, or a number expressed without fractions, is called an *integer.*

57. Mixed Number. A number composed of an integer and a fraction taken together is called a *mixed number.*

For example, if the above sphere *A* is 1, $A + B$ is $1\frac{3}{4}$ and $A + C$ is $1\frac{1}{4}$.

ORAL EXERCISE

Read aloud the following:

1. $\frac{3}{8}$ $\frac{7}{8}$ $\frac{1}{16}$ $\frac{3}{5}$ $\frac{7}{16}$ $\frac{9}{16}$
2. $\frac{5}{8}$ $\frac{2}{3}$ $\frac{3}{16}$ $\frac{15}{16}$ $\frac{11}{16}$ $\frac{7}{32}$
3. $2\frac{4}{5}$ $3\frac{7}{8}$ $9\frac{2}{3}$ $8\frac{3}{16}$ $2\frac{5}{32}$ $25\frac{2}{3}$
4. $12\frac{1}{2}$ $33\frac{1}{3}$ $15\frac{4}{25}$ $9\frac{3}{32}$ $6\frac{7}{64}$ $157\frac{7}{8}$
5. An inch is $\frac{1}{12}$ ft., $\frac{1}{36}$ yd., or $\frac{1}{198}$ rd.

Fractions having denominators larger than 8 are rarely used in computation, but in measuring we use fractions with larger denominators, usually not exceeding 64.

WRITTEN EXERCISE

Write the following fractions:

1. Seven sixteenths.
2. Nine sixteenths.
3. Nineteen thirty-seconds.
4. Eight ninths.
5. Seven thirty-seconds.
6. Seventeen eighteenths.
7. Nine sixty-fourths.
8. Five thirty-seconds.

Write in words the following:

9. $\frac{1}{2}$ $\frac{1}{8}$ $\frac{5}{8}$ $\frac{7}{8}$ $\frac{5}{16}$ $\frac{5}{24}$ $\frac{15}{16}$ $\frac{17}{32}$
10. $\frac{1}{5}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{3}{8}$ $\frac{7}{16}$ $\frac{23}{24}$ $\frac{19}{32}$ $\frac{21}{64}$
11. $\frac{1}{3}$ $\frac{2}{3}$ $1\frac{2}{3}$ $5\frac{4}{5}$ $62\frac{7}{8}$ $27\frac{1}{2}$ $6\frac{2}{3}$ $15\frac{5}{8}$

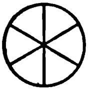
In practical business $\frac{1}{2}$¢ or over, in the final result, is called 1¢; less than $\frac{1}{2}$¢ is neglected in many cases, although in making out bills any fraction of a cent usually counts as a cent.

Write the following in words, and also write the number of dollars and cents that would be taken in making out bills:

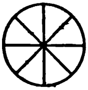
12. \$$125.12\frac{1}{2}$ \$$6.66\frac{2}{3}$ \$$333.33\frac{1}{3}$ \$$275.16\frac{2}{3}$
13. \$$135.87\frac{1}{2}$ \$$14.37\frac{1}{2}$ \$$112.16\frac{7}{8}$ \$$235.45\frac{1}{3}$

ORAL EXERCISE

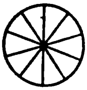
1. Look at figure *A* and state how many sixths you see in one half.



A



B



C

2. Look at figure *B* and state how many eighths you see in one half; in three quarters.

3. Look at figure *C* and state how many tenths you see in one half; in one fifth; in four fifths.

4. How many fifths do you see in two tenths? in four tenths? in six tenths? in eight tenths?

58. Reducing Fractions. We see that $\frac{2}{4}$ may be obtained from $\frac{1}{2}$ by multiplying both terms of the fraction by 2, and that $\frac{1}{2}$ may be obtained from $\frac{5}{10}$ by dividing both terms by 5. That is,

Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.

Both terms of a fraction may be divided by the same number without changing the value of the fraction.

When we change the terms without changing the value of a fraction we *reduce* the fraction.

When both terms cannot be divided by the same number, the fraction is said to be *in lowest terms.*

To reduce a fraction to lowest terms, divide by the largest number that will divide both terms without a remainder.

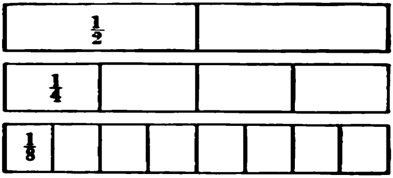
$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}, \text{ lowest terms.}$$

$$\boxed{\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}} = \frac{2}{3}}$$

In this example the common factor 4 is said to be *canceled* from both terms when the work is written as here shown.

ORAL EXERCISE

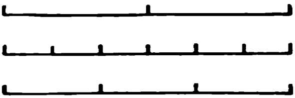
1. Calling the large rectangle *one*, how many halves do you see in 1?



2. How many fourths do you see in $\frac{1}{2}$? in 1?

3. How many eighths do you see in $\frac{1}{4}$? in $\frac{1}{2}$? in 1?

4. Calling the line *one*, how many sixths do you see in $\frac{1}{2}$? in 1?



5. $1\frac{1}{2}$ = how many sixths?

6. How many sixths do you see in $\frac{1}{3}$? in $\frac{2}{3}$? in $\frac{3}{3}$? in 1?

WRITTEN EXERCISE

1. Reduce to halves:

$\frac{2}{4}$ $\frac{4}{8}$ $\frac{8}{8}$ $\frac{12}{8}$ $\frac{16}{8}$ $\frac{24}{8}$ $\frac{32}{8}$ $\frac{40}{8}$

2. Reduce to fourths:

$\frac{1}{2}$ $\frac{2}{8}$ $\frac{4}{8}$ $\frac{16}{8}$ $\frac{10}{8}$ $\frac{20}{8}$ $\frac{18}{8}$ $\frac{32}{8}$

3. Reduce to eighths:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{6}{4}$ $\frac{7}{2}$ $\frac{2}{16}$ $\frac{4}{16}$

4. Reduce to twelfths:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{2}{24}$

5. Reduce to lowest terms:

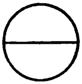
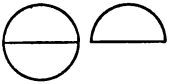
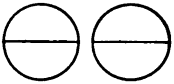
$\frac{2}{4}$ $\frac{4}{8}$ $\frac{10}{8}$ $\frac{8}{10}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{6}{8}$ $\frac{5}{10}$

6. Express $\frac{1}{3}$ in. and $\frac{1}{2}$ in. as sixths of an inch, and tell which is the greater.

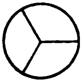
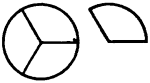
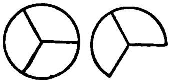
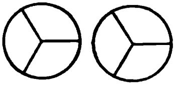
7. Express $\frac{1}{3}$ in. and $\frac{1}{4}$ in. as twelfths of an inch, and tell which is the greater.

ORAL EXERCISE

1. How many halves do you see in 1? in $1\frac{1}{2}$? in 2? Express $\frac{3}{2}$ as a mixed number. Express $\frac{4}{2}$ as a whole number.

2. How many thirds do you see in 1? in $1\frac{1}{3}$? in $1\frac{2}{3}$? in 2? Express $\frac{4}{3}$ and $\frac{5}{3}$ as mixed numbers; $\frac{6}{3}$ as a whole number.

WRITTEN EXERCISE

1. How many fourths are there in 1? in $1\frac{1}{4}$? in $1\frac{3}{4}$? in 2? Write the results in this way: $1 = \frac{4}{4}$, $1\frac{1}{4} = \frac{5}{4}$, and so on.

2. How many fifths are there in 1? in $1\frac{1}{5}$? in $1\frac{2}{5}$? in $1\frac{3}{5}$? in $1\frac{4}{5}$? in $1\frac{5}{5}$, or 2?

3. How many sixths are there in 1? in $1\frac{1}{6}$? in $1\frac{5}{6}$? in 2? in $2\frac{1}{6}$? in $2\frac{5}{6}$? in $2\frac{6}{6}$, or 3?

4. How many sevenths are there in 1? in $1\frac{1}{7}$? in $1\frac{2}{7}$? in $1\frac{3}{7}$? in $1\frac{4}{7}$? in $1\frac{5}{7}$? in $1\frac{6}{7}$? in 2?

5. How many eighths are there in 1? in 2? in $2\frac{1}{8}$? in $2\frac{3}{8}$? in $2\frac{5}{8}$? in $2\frac{7}{8}$? in 3?

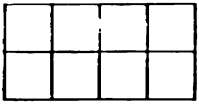
6. How many ninths are there in 1? in 2? in 3? in 4? in $1\frac{1}{9}$? in $2\frac{2}{9}$? in $2\frac{8}{9}$?

7. How many tenths are there in 1? in 2? in 3? in $3\frac{1}{10}$? in $3\frac{3}{10}$? in $3\frac{7}{10}$? in $3\frac{10}{10}$?

8. How many twelfths are there in 1? in 2? in 3? in 4? in $2\frac{1}{12}$? in $3\frac{5}{12}$? in $4\frac{7}{12}$? in $4\frac{11}{12}$?

ORAL EXERCISE

1. From the figure, $\frac{1}{2}$ equals how many eighths? how many fourths?



2. From the figure, $\frac{6}{8}$ equals how many fourths?

3. In what cases, in Exs. 1 and 2, did we reduce to lower terms, and in what cases to higher terms?

Reduce as indicated:

4. $\frac{1}{2}$ to sixteenths $\frac{1}{2}$ to sixths $\frac{4}{5}$ to tenths
5. $\frac{1}{4}$ to twelfths $\frac{1}{4}$ to eighths $\frac{3}{4}$ to twelfths
6. $\frac{3}{8}$ to sixteenths $\frac{2}{8}$ to fourths $\frac{6}{16}$ to eighths

WRITTEN EXERCISE

Reduce to any lower terms:

1. $\frac{4}{8}$ $\frac{6}{16}$ $\frac{12}{16}$ $\frac{8}{10}$ $\frac{8}{12}$ $\frac{14}{32}$ $\frac{16}{32}$ $\frac{24}{32}$

Reduce to sixteenths:

2. $\frac{3}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ $\frac{24}{32}$ $\frac{30}{32}$ $\frac{16}{64}$ $\frac{20}{64}$ $\frac{40}{64}$

Reduce to thirty-seconds:

3. $\frac{1}{4}$ $\frac{3}{4}$ $\frac{5}{8}$ $\frac{7}{8}$ $\frac{15}{16}$ $\frac{18}{64}$ $\frac{32}{64}$ $\frac{24}{64}$

Reduce to sixty-fourths:

4. $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{16}$ $\frac{9}{16}$ $\frac{9}{32}$ $\frac{15}{32}$ $\frac{16}{32}$ $\frac{12}{128}$

Reduce to twelfths:

5. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{6}$ $\frac{12}{24}$ $\frac{10}{24}$

Reduce to twenty-fourths:

6. $\frac{3}{8}$ $\frac{1}{2}$ $\frac{7}{12}$ $\frac{1}{3}$ $\frac{2}{48}$ $\frac{6}{48}$ $\frac{5}{12}$ $\frac{11}{12}$

ORAL EXERCISE

1. How many half spheres in 1 sphere? in $1\frac{1}{2}$ spheres?

2. How many halves in 2? in $2\frac{1}{2}$? in 3? in 5? in $5\frac{1}{2}$?

3. How many fourths in 1? in $1\frac{1}{4}$? in $1\frac{1}{2}$? in $1\frac{3}{4}$? in $5\frac{3}{4}$?

4. Express the following as thirds:

2 4 5 8 $1\frac{1}{3}$ $2\frac{1}{3}$ $3\frac{2}{3}$ $10\frac{1}{3}$

Express the following as fourths:

5. 6 8 $2\frac{1}{4}$ 5 $3\frac{1}{4}$ $3\frac{3}{4}$ $8\frac{1}{4}$ 12

6. 7 9 $1\frac{3}{4}$ $2\frac{3}{4}$ $4\frac{1}{4}$ $4\frac{3}{4}$ $5\frac{1}{4}$ $5\frac{3}{4}$

7. Express the following as eighths:

10 7 $6\frac{1}{8}$ $6\frac{3}{8}$ $7\frac{1}{8}$ $7\frac{5}{8}$ $8\frac{1}{8}$ $8\frac{7}{8}$

Reduce $4\frac{5}{8}$ to eighths.

Since $1=\frac{8}{8}$, we see that $4=\frac{4\times 8}{8}=\frac{32}{8}$.

Therefore, $4\frac{5}{8}=\frac{32}{8}+\frac{5}{8}=\frac{37}{8}$.

WRITTEN EXERCISE

Reduce as indicated:

1. 6 to eighths.
2. 9 to thirds.
3. $8\frac{1}{2}$ to halves.
4. $7\frac{4}{5}$ to fifths.
5. $8\frac{2}{3}$ to thirds.
6. $6\frac{5}{6}$ to sixths.
7. $7\frac{1}{9}$ to ninths.
8. $8\frac{3}{5}$ to fifths.
9. $7\frac{1}{3}$ to thirds.
10. $8\frac{1}{4}$ to fourths.
11. $5\frac{4}{9}$ to ninths.
12. $9\frac{3}{5}$ to fifths.
13. $9\frac{7}{8}$ to eighths.
14. $8\frac{5}{9}$ to ninths.
15. $6\frac{2}{9}$ to ninths.
16. 41 to tenths.
17. $7\frac{2}{5}$ to fifths.
18. $4\frac{2}{3}$ to thirds.
19. $9\frac{3}{4}$ to fourths.
20. $7\frac{1}{8}$ to eighths.
21. $3\frac{3}{8}$ to eighths.

59. Reduction to an Integer or a Mixed Number. Reduce $\frac{124}{5}$ to an integer or a mixed number.

Since $\frac{1}{5}$ means that 1 has been divided into 5 equal parts, $\frac{124}{5}$ means that 124×1 has been divided into 5 equal parts.

Therefore $$\frac{124}{5} = 124 \div 5 = 24\frac{4}{5}.$$

A fraction is an indicated division.

To reduce an improper fraction to an integer or a mixed number, divide the numerator by the denominator.

ORAL EXERCISE

1. Express $2\frac{1}{2}$ as halves; $\frac{5}{2}$ as a mixed number.

2. In $5 there are how many half dollars? Express 5 as halves. Express $\frac{10}{2}$ as an integer. Express $\frac{15}{5}$, $\frac{20}{4}$, $\frac{30}{3}$, and $\frac{40}{10}$ as integers.

3. Express the following as integers:

$\frac{12}{6}$ $\frac{15}{3}$ $\frac{16}{2}$ $\frac{81}{9}$ $\frac{25}{5}$ $\frac{125}{25}$ $\frac{64}{4}$ $\frac{48}{12}$

4. Express the following as mixed numbers:

$\frac{15}{7}$ $\frac{17}{8}$ $\frac{20}{3}$ $\frac{32}{10}$ $\frac{37}{7}$ $\frac{73}{12}$ $\frac{41}{5}$ $\frac{63}{8}$

WRITTEN EXERCISE

Reduce the following to integers or mixed numbers:

1. $\frac{33}{3}$ $\frac{125}{24}$ $\frac{632}{32}$ $\frac{127}{4}$ $\frac{81}{8}$ $\frac{107}{8}$
2. $\frac{47}{6}$ $\frac{298}{16}$ $\frac{190}{12}$ $\frac{87}{12}$ $\frac{92}{12}$ $\frac{116}{12}$
3. $\frac{42}{6}$ $\frac{236}{32}$ $\frac{425}{32}$ $\frac{93}{32}$ $\frac{76}{32}$ $\frac{160}{32}$
4. $\frac{78}{5}$ $\frac{349}{32}$ $\frac{276}{16}$ $\frac{198}{16}$ $\frac{725}{32}$ $\frac{525}{32}$
5. $\frac{65}{8}$ $\frac{427}{12}$ $\frac{629}{24}$ $\frac{332}{16}$ $\frac{342}{12}$ $\frac{353}{12}$
6. $\frac{69}{8}$ $\frac{625}{12}$ $\frac{221}{12}$ $\frac{625}{24}$ $\frac{896}{16}$ $\frac{257}{16}$
7. $\frac{29}{3}$ $\frac{175}{32}$ $\frac{113}{8}$ $\frac{125}{8}$ $\frac{441}{3}$ $\frac{549}{3}$

60. Similar Fractions. If several fractions have the same denominator, they are said to be *similar fractions* and to have a *common denominator.*

61. Least Common Denominator. If the common denominator of several fractions is the smallest possible, the fractions are said to have the *least common denominator* (L.C.D.).

For example, $\frac{4}{8}$ and $\frac{2}{8}$ have a common denominator, but they may be reduced to $\frac{2}{4}$ and $\frac{1}{4}$, and 4 is the least common denominator.

62. Reduction to L.C.D. In making some fine panels for a door a workman had to glue together three strips of wood. The first was $\frac{1}{4}$ in. thick, the second $\frac{3}{8}$ in., and the third $\frac{5}{12}$ in. How thick were the three together?

We shall find that to add $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{12}$ we must reduce them to fractions having the least common denominator, and we must learn to do this before we can answer such a question as the one we have just read.

$$\boxed{\begin{aligned} \frac{1}{4} &= \frac{6 \times 1}{6 \times 4} = \frac{6}{24} \\ \frac{3}{8} &= \frac{3 \times 3}{3 \times 8} = \frac{9}{24} \\ \frac{5}{12} &= \frac{2 \times 5}{2 \times 12} = \frac{10}{24} \end{aligned}}$$

Since the required denominator must contain 4, it must contain all the factors of 4; and since it must contain 8 and 12, it must also contain all their factors.

We see that 4, 8, and 12 each contains 4, that 8 also contains 2, and that 12 contains 3. Therefore the least common denominator must contain the factors 4, 2, and 3, and is $4 \times 2 \times 3$, or 24.

Since $24 \div 4 = 6$, the terms of $\frac{1}{4}$ must be multiplied by 6, to reduce $\frac{1}{4}$ to 24ths, and we have $\frac{1}{4} = \frac{6}{24}$.

Since $24 \div 8 = 3$, the terms of $\frac{3}{8}$ must be multiplied by 3 to reduce $\frac{3}{8}$ to 24ths, and we have $\frac{3}{8} = \frac{9}{24}$.

Since $24 \div 12 = 2$, the terms of $\frac{5}{12}$ must be multiplied by 2 to reduce $\frac{5}{12}$ to 24ths, and we have $\frac{5}{12} = \frac{10}{24}$.

ORAL EXERCISE

Reduce the following to similar fractions:

1. $\frac{2}{3}, \frac{1}{2}$.
2. $\frac{3}{5}, \frac{2}{3}$.
3. $\frac{4}{16}, \frac{2}{8}$.
4. $\frac{2}{5}, \frac{7}{10}$.
5. $\frac{5}{15}, \frac{2}{30}$.
6. $\frac{1}{7}, \frac{2}{14}$.
7. $\frac{3}{5}, \frac{4}{10}$.
8. $\frac{3}{8}, \frac{3}{4}$.
9. $\frac{10}{20}, \frac{15}{30}$.
10. $\frac{6}{8}, \frac{9}{10}$.
11. $\frac{2}{3}, \frac{3}{30}$.
12. $\frac{1}{4}, \frac{3}{10}$.

Reduce to fractions having the least common denominator:

13. $\frac{1}{2}, \frac{2}{3}$.
14. $\frac{3}{5}, \frac{1}{2}$.
15. $\frac{8}{10}, \frac{3}{5}$.
16. $\frac{1}{4}, \frac{2}{3}$.
17. $\frac{7}{8}, \frac{1}{16}$.
18. $\frac{3}{16}, \frac{5}{32}$.
19. $\frac{7}{12}, \frac{1}{4}$.
20. $\frac{1}{2}, \frac{4}{5}$.
21. $\frac{2}{3}, \frac{3}{4}, \frac{1}{6}$.
22. $\frac{2}{9}, \frac{2}{3}, \frac{5}{6}$.
23. $\frac{3}{8}, \frac{3}{5}, \frac{3}{10}$.
24. $\frac{7}{8}, \frac{1}{5}, \frac{1}{40}$.

WRITTEN EXERCISE

Reduce to fractions having the least common denominator:

1. $\frac{1}{2}, \frac{5}{6}$.
2. $\frac{1}{4}, \frac{5}{6}$.
3. $\frac{3}{4}, \frac{1}{6}$.
4. $\frac{1}{8}, \frac{5}{6}$.
5. $\frac{5}{8}, \frac{3}{4}$.
6. $\frac{7}{8}, \frac{1}{3}$.
7. $\frac{5}{6}, \frac{2}{3}$.
8. $\frac{7}{8}, \frac{2}{3}$.
9. $\frac{3}{5}, \frac{2}{3}$.
10. $\frac{4}{5}, \frac{3}{8}$.
11. $\frac{5}{12}, \frac{7}{24}$.
12. $\frac{3}{16}, \frac{5}{24}$.
13. $\frac{7}{8}, \frac{5}{12}$.
14. $\frac{5}{8}, \frac{11}{24}$.
15. $\frac{7}{8}, \frac{13}{24}$.
16. $\frac{7}{8}, \frac{5}{32}$.
17. $\frac{5}{8}, \frac{11}{32}$.
18. $\frac{7}{16}, \frac{5}{32}$.
19. $\frac{15}{16}, \frac{31}{32}$.
20. $\frac{13}{16}, \frac{29}{32}$.
21. $\frac{5}{8}, \frac{7}{24}$.
22. $\frac{7}{8}, \frac{21}{24}$.
23. $\frac{5}{12}, \frac{3}{32}$.
24. $\frac{7}{12}, \frac{5}{32}$.
25. $\frac{11}{12}, \frac{15}{32}$.
26. $\frac{1}{24}, \frac{1}{32}$.
27. $\frac{5}{24}, \frac{3}{32}$.
28. $\frac{7}{24}, \frac{9}{32}$.
29. $\frac{7}{24}, \frac{9}{48}$.
30. $\frac{11}{24}, \frac{13}{48}$.
31. $\frac{7}{8}, \frac{9}{64}$.
32. $\frac{5}{16}, \frac{11}{64}$.
33. $\frac{11}{32}, \frac{15}{64}$.
34. $\frac{1}{12}, \frac{3}{64}$.
35. $\frac{5}{12}, \frac{7}{64}$.
36. $\frac{1}{24}, \frac{1}{64}$.
37. $\frac{7}{24}, \frac{25}{64}$.
38. $\frac{3}{8}, \frac{5}{16}, \frac{7}{24}$.
39. $\frac{7}{8}, \frac{5}{12}, \frac{11}{24}$.
40. $\frac{5}{8}, \frac{7}{12}, \frac{15}{32}$.

Reduce the fractional parts of the following mixed numbers to fractions having the least common denominator:

41. $4\frac{7}{12}, 6\frac{9}{16}$.
42. $5\frac{3}{16}, 7\frac{5}{24}$.
43. $9\frac{11}{24}, 7\frac{15}{32}$.

63. Adding Numbers having the Same Name. If we ask for the sum of 3 boys and 2 girls, the answer cannot be boys alone or girls alone. But we can say,

$$\begin{aligned} 3 \text{ boys} &= 3 \text{ children} \\ 2 \text{ girls} &= \underline{2 \text{ children}} \\ & \ 5 \text{ children, all together} \end{aligned}$$

In adding, we think of things as having the same name.

In the same way, if we wish to find the sum of $\frac{3}{4}$ and $\frac{1}{2}$, we must think of these as $\frac{3}{4}$ and $\frac{2}{4}$, the answer being $\frac{5}{4}$, or $1\frac{1}{4}$.

In adding fractions we think of them as having the same name, that is, as similar fractions.

Numerous illustrations of fractions have been given on pages 47–53. The same idea should be carried throughout, but it acquires more force if, from now on, the objective work is done by the pupil. Addition and subtraction of fractions should be taught exactly as addition and subtraction of integers were taught in the second grade — objectively, as suggested in the oral work on this page. Paper cutting, and diagrams of circles, rectangles, and lines should precede the use of figures. This work is best done by the teacher and pupils working together, the textbook furnishing the examples for practice.

ORAL EXERCISE

1. Express $\frac{1}{4}$ as eighths. To the result add $\frac{5}{8}$.

2. If you fasten a strip of molding $\frac{1}{4}$ in. thick to a strip of wood $\frac{5}{8}$ in. thick, how thick are the two together?

3. Express $\frac{4}{5}$ as tenths. To the result add $\frac{3}{10}$.

4. If you lay an arithmetic that is $\frac{4}{5}$ in. thick on a notebook that is $\frac{3}{10}$ in. thick, how thick are the two together?

5. Express $\frac{2}{3}$ as sixths. To the result add $\frac{1}{6}$.

6. If you sew a piece of cloth $\frac{2}{3}$ yd. wide to a strip $\frac{1}{6}$ yd. wide, how wide will the new piece be?

64. Addition of Fractions. As in § 63, we see that if we add 3 marbles and 5 pencils, we have as a sum neither marbles alone nor pencils alone, but just 8 things, of two different kinds. So if we add $\frac{3}{4}$ and $\frac{5}{6}$ we shall have as a sum neither fourths nor sixths, but just 8 parts of a unit, of two different sizes.

Therefore, to add fractions that are not similar and obtain a definite sum, we must first reduce them to similar fractions.

Suppose that we wish to find the sum of $\frac{3}{5}$, $\frac{9}{10}$, and $\frac{7}{12}$.

The L.C.D. is evidently $5 \times 2 \times 2 \times 3 = 60$. Dividing 60 by the three denominators, we have

$$60 \div 5 = 12, \text{ and } \frac{3}{5} = \frac{12 \times 3}{12 \times 5} = \frac{36}{60};$$

$$60 \div 10 = 6, \text{ and } \frac{9}{10} = \frac{6 \times 9}{6 \times 10} = \frac{54}{60};$$

and

$$60 \div 12 = 5, \text{ and } \frac{7}{12} = \frac{5 \times 7}{5 \times 12} = \frac{35}{60}.$$

Furthermore, $\frac{36}{60} + \frac{54}{60} + \frac{35}{60} = \frac{125}{60} = 2\frac{5}{60} = 2\frac{1}{12}$.

Therefore the sum of $\frac{3}{5}$, $\frac{9}{10}$, and $\frac{7}{12}$ is $2\frac{1}{12}$.

To add fractions, reduce them to fractions having the least common denominator, and add the numerators for the numerator of the sum, writing this numerator over the least common denominator. Then reduce to lowest terms.

ORAL EXERCISE

1. How many twelfths are $\frac{7}{12} + \frac{1}{12}$? How many thirds?

State the sum of each of the following pairs of fractions:

2. $\frac{1}{2} + \frac{1}{4}$ $\frac{1}{2} + \frac{3}{4}$ $\frac{1}{2} + \frac{1}{8}$ $\frac{1}{2} + \frac{7}{8}$ $\frac{1}{12} + \frac{5}{12}$
3. $\frac{1}{4} + \frac{3}{4}$ $\frac{3}{8} + \frac{5}{8}$ $\frac{2}{3} + \frac{5}{6}$ $\frac{1}{8} + \frac{3}{8}$ $\frac{1}{12} + \frac{11}{12}$
4. $\frac{1}{8} + \frac{5}{8}$ $\frac{1}{5} + \frac{4}{5}$ $\frac{1}{5} + \frac{2}{5}$ $\frac{1}{4} + \frac{1}{8}$ $\frac{5}{12} + \frac{7}{12}$

WRITTEN EXERCISE

Add the following fractions:

1. $\frac{1}{2} + \frac{1}{3}$.
2. $\frac{1}{2} + \frac{2}{3}$.
3. $\frac{1}{2} + \frac{1}{5}$.
4. $\frac{1}{2} + \frac{2}{5}$.
5. $\frac{1}{2} + \frac{3}{5}$.
6. $\frac{1}{2} + \frac{4}{5}$.
7. $\frac{1}{2} + \frac{3}{4}$.
8. $\frac{1}{2} + \frac{3}{8}$.
9. $\frac{1}{2} + \frac{7}{8}$.
10. $\frac{3}{4} + \frac{5}{8}$.
11. $\frac{3}{4} + \frac{7}{8}$.
12. $\frac{4}{5} + \frac{1}{8}$.
13. $\frac{3}{5} + \frac{3}{8}$.
14. $\frac{2}{5} + \frac{7}{8}$.
15. $\frac{4}{5} + \frac{7}{8}$.
16. $\frac{1}{2} + \frac{5}{6}$.
17. $\frac{3}{4} + \frac{5}{6}$.
18. $\frac{1}{6} + \frac{1}{8}$.
19. $\frac{1}{6} + \frac{3}{8}$.
20. $\frac{5}{6} + \frac{7}{8}$.

21. A piece of cardboard $\frac{1}{64}$ in. thick is laid on a book $\frac{5}{8}$ in. thick. How thick are the two together?

Add the following fractions:

22. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
23. $\frac{1}{2} + \frac{3}{4} + \frac{1}{8}$.
24. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$.
25. $\frac{1}{3} + \frac{1}{4} + \frac{1}{12}$.
26. $\frac{1}{3} + \frac{3}{4} + \frac{1}{12}$.
27. $\frac{2}{3} + \frac{3}{4} + \frac{7}{12}$.
28. $\frac{1}{3} + \frac{1}{8} + \frac{1}{12}$.
29. $\frac{2}{3} + \frac{1}{8} + \frac{1}{12}$.
30. $\frac{2}{3} + \frac{5}{8} + \frac{7}{12}$.

31. Add $\frac{1}{2}$ in., $\frac{1}{3}$ in., $\frac{1}{4}$ in., $\frac{1}{6}$ in., and $\frac{1}{12}$ in.

32. If you lay a notebook $\frac{1}{2}$ in. thick on a book $\frac{7}{8}$ in. thick, how thick are the two together?

33. Some plaster $\frac{3}{8}$ in. thick is coated with a finer plaster $\frac{3}{16}$ in. thick. How thick is the plaster then?

34. A plate of brass $\frac{1}{32}$ in. thick is laid on a plate of iron $\frac{1}{8}$ in. thick. What is the total thickness?

35. An iron rod $\frac{7}{8}$ in. in diameter is covered with a brass plating $\frac{1}{32}$ in. thick. What is the diameter of the rod then?

Add the following fractions:

36. $\frac{1}{4}, \frac{5}{24}$.
37. $\frac{3}{4}, \frac{11}{24}$.
38. $\frac{3}{8}, \frac{7}{24}$.
39. $\frac{5}{8}, \frac{11}{24}$.
40. $\frac{3}{16}, \frac{1}{24}$.
41. $\frac{5}{16}, \frac{5}{24}$.
42. $\frac{7}{16}, \frac{11}{24}$.
43. $\frac{9}{16}, \frac{13}{24}$.
44. $\frac{1}{2}, \frac{1}{48}$.
45. $\frac{3}{4}, \frac{1}{48}$.
46. $\frac{3}{4}, \frac{15}{28}$.
47. $\frac{3}{8}, \frac{13}{48}$.
48. $\frac{5}{8}, \frac{17}{48}$.
49. $\frac{7}{8}, \frac{23}{48}$.
50. $\frac{1}{16}, \frac{17}{48}$.
51. $\frac{7}{16}, \frac{31}{48}$.

65. Addition of Mixed Numbers. If Rachel needs $4\frac{2}{3}$ yd. of cloth for one piece of work, and $2\frac{2}{3}$ yd. of the same kind of cloth for another piece, how many yards should she buy?

We evidently have to add $4\frac{2}{3}$ yd. and $2\frac{2}{3}$ yd.

First adding the thirds we have

$$\frac{2}{3}+\frac{2}{3}=\frac{4}{3}=1\frac{1}{3}.$$

We write the $\frac{1}{3}$ in the fractions' column and add the 1 to the next column.

$$\boxed{\begin{array}{r} 4\frac{2}{3} \\ \underline{2\frac{2}{3}} \\ 7\frac{1}{3} \end{array}}$$

We then have $1+2+4=7$, and we write the 7 in the units' column.

The sum is $7\frac{1}{3}$, and so Rachel should buy $7\frac{1}{3}$ yd. of cloth.

WRITTEN EXERCISE

1. Robert is putting some picture molding around a room. The four sides of the room are $14\frac{1}{4}$ ft., 12 ft., $14\frac{1}{4}$ ft., and 12 ft. How much molding does he need?

2. One of Mary's books weighs $1\frac{1}{4}$ lb. and another weighs $\frac{3}{4}$ lb. How much do the two together weigh?

Add the following:

3.	4.	5.	6.	7.	8.
$12\frac{1}{2}$	$24\frac{1}{3}$	$12\frac{2}{3}$	$16\frac{2}{3}$	$28\frac{1}{3}$	$17\frac{2}{3}$
$14\frac{1}{2}$	$16\frac{1}{3}$	$3\frac{2}{3}$	$14\frac{2}{3}$	$14\frac{2}{3}$	$4\frac{2}{3}$
$16\frac{1}{2}$	$17\frac{2}{3}$	$4\frac{2}{3}$	$15\frac{2}{3}$	$15\frac{2}{3}$	$8\frac{2}{3}$

9.	10.	11.	12.	13.	14.
$41\frac{1}{2}$	$62\frac{1}{2}$	$39\frac{1}{2}$	$37\frac{1}{3}$	$57\frac{1}{3}$	$15\frac{2}{3}$
27	$17\frac{1}{2}$	$12\frac{1}{2}$	$29\frac{1}{3}$	$22\frac{2}{3}$	$4\frac{1}{3}$
$33\frac{1}{2}$	12	$33\frac{1}{2}$	16	$33\frac{2}{3}$	$17\frac{1}{3}$
42	$15\frac{1}{2}$	$40\frac{1}{2}$	$4\frac{1}{3}$	$4\frac{1}{3}$	$9\frac{2}{3}$

66. Addition of Mixed Numbers Continued. Sarah, Helen, and Emily are making dolls' dresses. Sarah uses $2\frac{1}{2}$ yd. of cloth, Helen $4\frac{2}{3}$ yd., and Emily $3\frac{1}{10}$ yd. How much do they use in all?

We see that we must add $2\frac{1}{2}$, $4\frac{2}{3}$, and $3\frac{1}{10}$.

The L.C.D. must contain the factor 2 because of the fraction $\frac{1}{2}$, the factor 3 because of the fraction $\frac{2}{3}$, and the factors 2 and 5 because of the fraction $\frac{1}{10}$. The L.C.D. must therefore contain the factors 2, 3, and 5, and so the L.C.D. is 30.

Reducing each fraction to a fraction with denominator 30, we have

$2\frac{15}{30} + 4\frac{20}{30} + 3\frac{3}{30} = 9\frac{38}{30} = 10\frac{8}{30} = 10\frac{4}{15}$,

and so the girls use $10\frac{4}{15}$ yd. of cloth.

$$\boxed{\begin{array}{r} 2\frac{1}{2} = 2\frac{15}{30} \\ 4\frac{2}{3} = 4\frac{20}{30} \\ 3\frac{1}{10} = \underline{3\frac{3}{30}} \\ 9\frac{38}{30} = 10\frac{4}{15} \end{array}}$$

To add mixed numbers, add separately the integers and the fractions and find the sum of the results. Then reduce the fraction in this sum to lowest terms.

WRITTEN EXERCISE

1. A teacher's desk is $4\frac{1}{3}$ ft. long and $3\frac{1}{12}$ ft. wide. In planning some molding around the desk we wish to find the perimeter. What is the perimeter?

2. Some boys built a hut $7\frac{3}{4}$ ft. long and $5\frac{3}{8}$ ft. wide, and wish to find the perimeter. What is the perimeter?

3. The three sides of a triangular flower bed are $3\frac{2}{3}$ ft., $4\frac{1}{2}$ ft., and $4\frac{5}{16}$ ft. What is the perimeter?

4. If a boy weighs $56\frac{7}{8}$ lb., his books $1\frac{3}{4}$ lb., and his dog $20\frac{1}{2}$ lb., how much do all together weigh?

Add the following:

5. $1\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
6. $2\frac{1}{3} + \frac{1}{6} + \frac{1}{2}$.
7. $3\frac{1}{2} + 2\frac{3}{4} + 1\frac{5}{8}$.
8. $1\frac{2}{3} + 2\frac{5}{6} + \frac{1}{2}$.
9. $3\frac{1}{4} + 4\frac{1}{8} + 3\frac{1}{16}$.
10. $2\frac{3}{4} + 1\frac{7}{8} + 4\frac{1}{16}$.

67. Subtraction of Fractions. Maud has $\frac{1}{2}$ yd. of lace, and in trimming a doll's waist she uses $\frac{3}{10}$ yd. How much lace has she left?

We see that we must subtract $\frac{3}{10}$ from $\frac{1}{2}$. The two denominators are 2 and 10, and 2 is a factor of 10. Therefore 10 is the L.C.D. of the fractions. Then we have

$$\frac{1}{2} = \frac{5}{10},$$

and $$\frac{5}{10} - \frac{3}{10} = \frac{2}{10} = \frac{1}{5}.$$

$$\begin{array}{l} \frac{1}{2} = \frac{5}{10} \\ \frac{3}{10} = \underline{\frac{3}{10}} \\ \qquad \frac{2}{10} = \frac{1}{5} \end{array}$$

Hence the difference is $\frac{1}{5}$, and so Maud has $\frac{1}{5}$ yd. of lace left.

To subtract fractions, reduce them to fractions having the least common denominator, and subtract the numerators for the numerator of the difference. Then reduce to lowest terms.

In practical work we rarely have to add or subtract fractions which have very large denominators, and the examples given in this book are no more difficult than the hardest one that we are likely to meet.

WRITTEN EXERCISE

1. If Kate has $\frac{7}{8}$ yd. of ribbon and uses $\frac{1}{2}$ yd. in trimming some sleeves, how much has she left?

2. If you live $\frac{7}{8}$ mi. from school, how far have you still to go when you have gone $\frac{3}{4}$ mi. of the distance?

Subtract the following:

3. $\frac{1}{2} - \frac{5}{12}$.
4. $\frac{7}{8} - \frac{1}{16}$.
5. $\frac{3}{8} - \frac{1}{4}$.
6. $\frac{2}{3} - \frac{5}{12}$.
7. $\frac{3}{4} - \frac{3}{16}$.
8. $\frac{5}{8} - \frac{1}{2}$.
9. $\frac{7}{8} - \frac{5}{24}$.
10. $\frac{3}{4} - \frac{7}{12}$.
11. $\frac{5}{12} - \frac{1}{4}$.
12. $\frac{1}{4} - \frac{1}{12}$.
13. $\frac{7}{16} - \frac{1}{8}$.
14. $\frac{3}{4} - \frac{1}{3}$.
15. $\frac{3}{4} - \frac{2}{3}$.
16. $\frac{1}{5} - \frac{1}{6}$.
17. $\frac{3}{5} - \frac{1}{2}$.
18. $\frac{5}{12} - \frac{3}{8}$.
19. $\frac{9}{10} - \frac{3}{5}$.
20. $\frac{5}{8} - \frac{1}{12}$.
21. $\frac{7}{8} - \frac{1}{4}$.
22. $\frac{7}{8} - \frac{2}{3}$.
23. $\frac{2}{3} - \frac{1}{2}$.
24. $\frac{7}{8} - \frac{1}{2}$.
25. $\frac{5}{8} - \frac{3}{32}$.
26. $\frac{9}{10} - \frac{1}{2}$.

68. Subtracting a Mixed Number. Albert has a board 12 in. wide from which he saws a strip $3\frac{2}{3}$ in. wide. How wide is the part of the board that is left?

We see that we must subtract $3\frac{2}{3}$ from 12.

We see that we cannot take $\frac{2}{3}$ from nothing, although we can evidently take $3\frac{2}{3}$ from 12.

We therefore think of 12 as $11\frac{3}{3}$, which we can do because $\frac{3}{3} = 1$. We then have

$$\frac{3}{3} - \frac{2}{3} = \frac{1}{3},$$

and

$$11 - 3 = 8.$$

$$\boxed{\begin{array}{r} 12 \\ 3\frac{2}{3} \\ \hline 8\frac{1}{3} \end{array}}$$

Therefore the difference between 12 and $3\frac{2}{3}$ is $8\frac{1}{3}$, and so the width of the board that Albert has left is $8\frac{1}{3}$ in.

We check the work by seeing that $8\frac{1}{3} + 3\frac{2}{3} = 12$.

WRITTEN EXERCISE

1. Fred is making a bird house. He has a piece of wood 18 in. long, and from this he saws a piece $6\frac{3}{4}$ in. long. How long is the piece which remains?

2. Alice has a piece of ribbon 22 in. long. She uses $14\frac{1}{2}$ in. in trimming a bag. How much ribbon has she left?

3. If you start out on an automobile trip of 38 mi., and stop to call on a friend who lives $7\frac{1}{4}$ mi. along the road, how much farther have you to go after leaving your friend?

Subtract the following:

4. $8 - 2\frac{2}{3}$.
5. $6 - 4\frac{3}{8}$.
6. $8 - 3\frac{2}{5}$.
7. $7 - 6\frac{7}{8}$.
8. $6 - 2\frac{3}{5}$.
9. $27 - 15\frac{3}{8}$.
10. $14 - 12\frac{3}{5}$.
11. $16 - 4\frac{5}{16}$.
12. $18 - 3\frac{7}{16}$.
13. $20 - 5\frac{9}{16}$.
14. 5 in. − $3\frac{3}{4}$ in.
15. 7 in. − $6\frac{3}{32}$ in.
16. 6 in. − $1\frac{7}{32}$ in.
17. 8 in. − $2\frac{11}{32}$ in.
18. 9 in. − $3\frac{1}{64}$ in.

69. Subtracting from a Mixed Number. From a mixed number we may have to subtract a whole number, a fraction, or a mixed number.

1. From $17\frac{1}{3}$ to subtract 8.

We have $\frac{1}{3} - 0 = \frac{1}{3}$, and $17 - 8 = 9$. Therefore the result is $9\frac{1}{3}$.

$$\boxed{\begin{array}{r} 17\frac{1}{3} \\ 8 \\ \hline 9\frac{1}{3} \end{array}}$$

2. From $17\frac{1}{3}$ to subtract $\frac{5}{6}$.

We may think of $17\frac{2}{6}$ as $16\frac{8}{6}$, and $16\frac{8}{6} - \frac{5}{6} = 16\frac{3}{6} = 16\frac{1}{2}$.

$$\boxed{\begin{array}{r} 17\frac{1}{3} = 17\frac{2}{6} \\ \frac{5}{6} \\ \hline 16\frac{1}{2} \end{array}}$$

3. From $17\frac{1}{3}$ to subtract $8\frac{5}{6}$.

$$17\frac{1}{3} - 8\frac{5}{6} = 17\frac{2}{6} - 8\frac{5}{6}.$$

We may now think of $17\frac{2}{6}$ as $16\frac{8}{6}$, and $16\frac{8}{6} - 8\frac{5}{6} = 8\frac{3}{6} = 8\frac{1}{2}$.

WRITTEN EXERCISE

Subtract the following:

1. $91\frac{7}{8} - 19$.
2. $3\frac{1}{8} - \frac{3}{4}$.
3. $9\frac{1}{2} - 6\frac{2}{3}$.
4. $83\frac{5}{8} - 27$.
5. $5\frac{3}{8} - \frac{1}{2}$.
6. $8\frac{1}{4} - 5\frac{7}{8}$.
7. $120\frac{3}{4} - 69$.
8. $7\frac{1}{12} - \frac{3}{8}$.
9. $15\frac{2}{5} - 9\frac{3}{8}$.
10. $103\frac{5}{6} - 78$.
11. $8\frac{5}{12} - \frac{7}{8}$.
12. $16\frac{1}{3} - 7\frac{3}{4}$.
13. $200\frac{7}{12} - 96$.
14. $6\frac{2}{3} - \frac{4}{5}$.
15. $25\frac{1}{5} - 16\frac{7}{8}$.

16. If one book is $7\frac{1}{4}$ in. long and another is $6\frac{15}{16}$ in. long, what is the difference in their lengths?

17. If we have a piece of board $17\frac{1}{8}$ in. long and we need a piece only $15\frac{3}{4}$ in. long, how much shall we cut off?

18. If a lady has $14\frac{1}{4}$ yd. of cloth and uses $8\frac{3}{4}$ yd., how much cloth has she left?

19. How much must you add to $26\frac{7}{8}$ in. to make a yard?

20. How much must you take from 1 yd. to leave $16\frac{3}{8}$ in.?

WRITTEN EXERCISE

Add or subtract as indicated:

1. $2\frac{1}{2} + 3\frac{1}{4} + 4\frac{3}{4}$.

2. $2\frac{1}{4} + 9\frac{3}{4} - \frac{1}{8}$.

3. $3\frac{1}{2} + 5\frac{1}{4} + 5\frac{3}{4}$.

4. $3\frac{1}{4} + 7\frac{1}{8} - 2\frac{1}{3}$.

5. $5\frac{1}{2} + 6\frac{3}{4} + 4\frac{3}{4}$.

6. $5\frac{1}{4} + 6\frac{1}{8} - 3\frac{2}{3}$.

7. $8\frac{1}{2} + 3\frac{3}{8} + 9\frac{3}{4}$.

8. $8\frac{7}{8} + 5\frac{3}{8} - 2\frac{3}{4}$.

9. $4\frac{1}{2} + 4\frac{7}{8} + 8\frac{3}{4}$.

10. $7\frac{7}{8} + 3\frac{5}{8} - 4\frac{3}{4}$.

11. $5\frac{1}{2} + 7\frac{5}{8} + 11\frac{1}{3}$.

12. $8\frac{7}{8} + 5\frac{2}{3} - 7\frac{1}{4}$.

13. $7\frac{2}{3} + 8\frac{1}{2} + 12\frac{1}{4}$.

14. $15\frac{1}{2} - 2\frac{1}{4} - 1\frac{2}{3}$.

15. $5\frac{5}{6} + 12\frac{2}{3} + 21\frac{1}{4}$.

16. $22\frac{3}{4} - 3\frac{3}{8} - 2\frac{1}{2}$.

17. $7\frac{1}{6} + 18\frac{4}{5} + 23\frac{3}{8}$.

18. $17\frac{5}{6} - 2\frac{1}{12} - 3\frac{2}{3}$.

19. $8\frac{5}{6} + 21\frac{11}{12} + 20\frac{3}{4}$.

20. $23\frac{5}{6} - 3\frac{5}{12} - 2\frac{2}{3}$.

21. $7\frac{1}{6} + 18\frac{11}{12} + 22\frac{3}{4}$.

22. $33\frac{1}{6} - 2\frac{7}{12} - 3\frac{3}{4}$.

23. $9\frac{5}{6} + 15\frac{1}{12} + 13\frac{1}{3}$.

24. $41\frac{1}{6} - 3\frac{11}{12} - 2\frac{1}{3}$.

25. How much greater is $57\frac{1}{4}$ mi. than $29\frac{7}{8}$ mi.?

26. How much less is $36\frac{7}{8}$ mi. than $90\frac{1}{4}$ mi.?

27. If a room is $16\frac{1}{8}$ ft. long and $14\frac{1}{2}$ ft. wide, what is the perimeter of the room?

28. From a board 16 ft. long a man saws off a piece $2\frac{1}{4}$ ft. long and another piece $3\frac{1}{2}$ ft. long. How long is the remaining part?

29. A boy puts in his basket $3\frac{1}{2}$ lb. of meat, 2 lb. of sugar, and $1\frac{1}{4}$ lb. of figs. The basket weighs $2\frac{1}{8}$ lb. What is the weight of the basket and contents?

30. If a piece of ribbon $3\frac{3}{4}$ yd. long has another piece sewed to it so as to increase the total length by $3\frac{1}{2}$ yd., and then $5\frac{1}{4}$ yd. is cut off and used for trimming a hat, how many yards of ribbon are left?

WRITTEN EXERCISE

1. Frank is $4\frac{1}{8}$ ft. tall, and Rob lacks 4 in. of being as tall as Frank. How tall is Rob?

2. John is $11\frac{2}{3}$ yr. old, Mary is $\frac{1}{2}$ yr. older, and Tom is $1\frac{1}{4}$ yr. younger than Mary. How old is Tom?

3. If my finger nail is $\frac{7}{16}$ in. long, and the rest of my finger is $2\frac{3}{4}$ in. long, what is the total length of my finger?

4. If you weigh $56\frac{3}{4}$ lb., and your dog weighs $18\frac{1}{8}$ lb., and your books weigh $3\frac{1}{2}$ lb., what is the total weight?

5. Harry had a kite string $198\frac{2}{3}$ yd. long. His kite was up in a shower and the string shrank $1\frac{7}{8}$ yd. How long was it then?

6. A plate of glass $23\frac{1}{4}$ in. by $37\frac{1}{8}$ in. was set in a picture frame that covered it $\frac{3}{8}$ in. from each edge. What are the dimensions of the glass not covered by the frame?

7. In making a dress ruffle $3\frac{7}{8}$ in. wide, enough cloth must be allowed to turn in $\frac{7}{8}$ in. on one side and $1\frac{1}{4}$ in. on the other. What is the total width of cloth needed?

8. In running a gas pipe into a schoolroom the gas fitter found he needed three pieces of pipe of these lengths: 6 ft. $2\frac{1}{2}$ in., 5 ft. $3\frac{7}{8}$ in., and 10 ft. 2 in. He also needed a fourth piece half as long as the third one. What was the total length required?

9. A tenpenny nail is $2\frac{3}{4}$ in. long, and a fourpenny nail is $1\frac{3}{8}$ in. shorter; how long is a fourpenny nail? A sixpenny nail is $\frac{5}{8}$ in. longer than a fourpenny nail; how long is a sixpenny nail?

10. Five cans of sirup were measured very carefully and the first was found to contain $2\frac{1}{4}$ qt., the second $2\frac{3}{8}$ qt., the third $2\frac{7}{16}$ qt., the fourth $2\frac{5}{32}$ qt., and the fifth $2\frac{2}{3}$ qt. How much sirup was there in all?

From time to time teachers should read the note at the bottom of page 26 and remind their pupils of its importance.

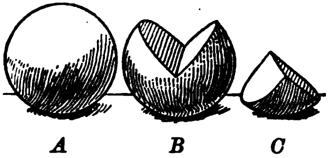
ORAL EXERCISE

1. How much is $\frac{1}{2}$ of \$4? $\frac{1}{2}$ of 4 fifths? $\frac{1}{2}$ of $\frac{4}{7}$?

2. How much is $\frac{1}{3}$ of 6 bu.? $\frac{1}{3}$ of 6 sevenths? $\frac{1}{3}$ of $\frac{6}{11}$?

3. How much is $\frac{1}{4}$ of $\frac{16}{17}$? $\frac{1}{4}$ of $\frac{8}{15}$? $\frac{1}{4}$ of $\frac{24}{25}$? $\frac{1}{3}$ of $\frac{6}{5}$?

4. In each of these cases in which you found a fraction of a fraction, how did you proceed?



5. In the above figure, *A* represents a whole sphere. What represents $\frac{3}{4}$ of a sphere? What represents $\frac{1}{3}$ of $\frac{3}{4}$ of a sphere?

State the values of the following:

6. $\frac{1}{2}$ of $\frac{2}{3}$ $\frac{1}{5}$ of $\frac{15}{16}$ $\frac{1}{6}$ of $\frac{12}{13}$ $\frac{1}{2}$ of $\frac{8}{9}$

7. $\frac{1}{8}$ of $\frac{16}{21}$ $\frac{1}{9}$ of $\frac{18}{25}$ $\frac{1}{10}$ of $\frac{20}{21}$ $\frac{1}{12}$ of $\frac{24}{25}$

8. Draw on the blackboard a rectangle $2\frac{1}{2}$ ft. long and 1 ft. high; mark it off into fifths; mark off each fifth into halves. Then how much is $\frac{1}{2}$ of $\frac{1}{5}$? How much is $\frac{1}{5}$ of $\frac{1}{2}$?

9. Draw another rectangle as in Ex. 8, marking off each fifth into thirds. How much is $\frac{1}{3}$ of $\frac{1}{5}$?

State the values of the following:

10. $\frac{1}{3}$ of $\frac{1}{5}$ $\frac{1}{5}$ of $\frac{1}{3}$ $\frac{1}{5}$ of $\frac{1}{4}$ $\frac{1}{4}$ of $\frac{1}{5}$

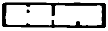
11. $\frac{1}{2}$ of $\frac{1}{5}$ $\frac{1}{2}$ of $\frac{1}{10}$ $\frac{1}{3}$ of $\frac{1}{10}$ $\frac{1}{2}$ of $\frac{1}{3}$

12. In Exs. 10 and 11 what did you do to the two denominators?

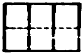
Pages 68 and 69 contain development work for the multiplication of fractions, which begins on page 70.

ORAL EXERCISE

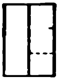
Draw these figures on the blackboard, and then answer the following questions:



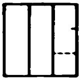
A



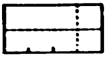
B



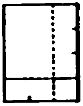
C



D



E



F

1. What part of A is $\frac{1}{2}$ of $\frac{1}{2}$ of A?

2. What part of B is $\frac{1}{2}$ of $\frac{1}{3}$ of B?

3. Show that $\frac{1}{2}$ of $\frac{1}{3}$ of B is the same as $\frac{1}{3}$ of $\frac{1}{2}$ of B.

4. What part of C is $\frac{1}{3}$ of $\frac{1}{2}$ of C?

5. What part of D is $\frac{1}{3}$ of $\frac{1}{3}$ of D?

6. Point to $\frac{1}{2}$ of E; to $\frac{1}{4}$ of $\frac{1}{2}$ of E. Also point to $\frac{1}{2}$ of $\frac{1}{4}$ of E. What part of E is $\frac{1}{2}$ of $\frac{1}{4}$ of E, or $\frac{1}{4}$ of $\frac{1}{2}$ of E?

7. Point to $\frac{1}{2}$ of $\frac{3}{4}$ of E; to $\frac{3}{4}$ of $\frac{1}{2}$ of E. What part of E is $\frac{3}{4}$ of $\frac{1}{2}$ of E?

8. Point to $\frac{1}{3}$ of F; to $\frac{1}{4}$ of $\frac{1}{3}$ of F. Also point to $\frac{1}{4}$ of F, and to $\frac{1}{3}$ of $\frac{1}{4}$ of F. Each result is what part of F?

WRITTEN EXERCISE

Draw rectangles to show the following:

1. $\frac{1}{5}$ of $\frac{1}{3}$, and $\frac{1}{3}$ of $\frac{1}{5}$.
2. $\frac{2}{3}$ of $\frac{1}{5}$, and $\frac{1}{5}$ of $\frac{2}{3}$.
3. $\frac{3}{5}$ of $\frac{2}{3}$, and $\frac{2}{3}$ of $\frac{3}{5}$.
4. $\frac{1}{8}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{8}$.
5. $\frac{5}{8}$ of $\frac{1}{5}$, and $\frac{1}{5}$ of $\frac{5}{8}$.
6. $\frac{1}{10}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{10}$.
7. $\frac{1}{2}$ of $\frac{1}{6}$, and $\frac{1}{6}$ of $\frac{1}{2}$.
8. $\frac{1}{2}$ of $\frac{5}{6}$, and $\frac{5}{6}$ of $\frac{1}{2}$.
9. $\frac{4}{5}$ of $\frac{1}{6}$, and $\frac{1}{6}$ of $\frac{4}{5}$.
10. $\frac{3}{8}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{3}{8}$.
11. $\frac{3}{5}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{3}{5}$.
12. $\frac{3}{10}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{3}{10}$.

ORAL EXERCISE

1. How much is 2 times 3 in.? 2 times $\frac{3}{7}$? 5 times $\frac{2}{11}$? How did you multiply these fractions by an integer?

2. How much is 4 times $\frac{1}{4}$? 4 times $\frac{3}{4}$? 2 times $\frac{3}{4}$? 5 times $\frac{7}{40}$? 7 times $\frac{4}{49}$? How did you multiply these fractions by an integer?

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70. Multiplication of a Fraction by a Whole Number. From the answers to the above questions we see that the following is true:

To multiply a fraction by a whole number, multiply the numerator by the multiplier.

Before actually multiplying, indicate the multiplication and cancel common factors if possible.

For example, to multiply $\frac{11}{63}$ by 18. Since we have 11 sixty-thirds, if we have 18 times as much we shall have

$$\frac{\overset{2}{\cancel{18}} \times 11}{\underset{7}{\cancel{63}}},\ \text{or}\ \frac{22}{7},\ \text{or}\ 3\tfrac{1}{7}.$$

Multiply the following, using cancellation whenever possible:

3. $\frac{5}{9}$ by 9.
4. $\frac{3}{4}$ by 2.
5. $\frac{3}{8}$ by 16.
6. $\frac{5}{8}$ by 40.
7. $\frac{5}{8}$ by 80.
8. $\frac{3}{8}$ by 8.
9. $\frac{4}{5}$ by 10.
10. $\frac{7}{8}$ by 24.
11. $\frac{4}{5}$ by 5.
12. $\frac{7}{8}$ by 4.
13. $\frac{5}{8}$ by 24.
14. $\frac{5}{8}$ by 48.
15. $\frac{9}{10}$ by 5.
16. $\frac{3}{10}$ by 5.
17. $\frac{3}{16}$ by 48.
18. $\frac{7}{12}$ by 24.

WRITTEN EXERCISE

Multiply the following, using cancellation whenever possible:

1. $\frac{4}{25}$ by 35.
2. $\frac{5}{48}$ by 84.
3. $\frac{7}{72}$ by 16.
4. $\frac{7}{16}$ by 24.
5. $\frac{7}{32}$ by 28.
6. $\frac{5}{12}$ by 132.
7. $\frac{11}{64}$ by 36.
8. $\frac{7}{64}$ by 96.
9. $\frac{7}{36}$ by 27.
10. $\frac{5}{48}$ by 64.
11. $\frac{9}{32}$ by 48.
12. $\frac{11}{64}$ by 192.

ORAL EXERCISE

1. How much is $\frac{1}{2}$ of 6? $\frac{1}{2}$ of 60? $\frac{1}{2}$ of 600?

2. How much is $\frac{1}{3}$ of 6? $\frac{2}{3}$ of 6? $\frac{1}{3}$ of 60? $\frac{2}{3}$ of 60?

3. How much is $\frac{1}{4}$ of 8? $\frac{3}{4}$ of 8? $\frac{1}{4}$ of 80? $\frac{3}{4}$ of 80?

4. How much is $\frac{1}{5}$ of 25? $\frac{2}{5}$ of 25? $\frac{3}{5}$ of 25? $\frac{4}{5}$ of 25?

Find the following parts of numbers:

5. $\frac{1}{8}$ of 24 $\frac{3}{8}$ of 24 $\frac{5}{8}$ of 24 $\frac{7}{8}$ of 24
6. $\frac{1}{6}$ of 30 $\frac{5}{6}$ of 30 $\frac{1}{6}$ of 48 $\frac{5}{6}$ of 48
7. $\frac{1}{9}$ of 72 $\frac{2}{9}$ of 72 $\frac{4}{9}$ of 72 $\frac{5}{9}$ of 72

8. How do you find a fractional part of a number?

WRITTEN EXERCISE

1. Find $\frac{1}{8}$ of 736. Then find $\frac{3}{8}$ of 736.

2. Find $\frac{1}{12}$ of 1728. Then find $\frac{5}{12}$ of 1728.

Find the following parts of numbers:

3. $\frac{3}{8}$ of 1768 $\frac{5}{8}$ of 1768 $\frac{5}{8}$ of 2576
4. $\frac{7}{8}$ of 2576 $\frac{1}{6}$ of 1740 $\frac{5}{6}$ of 1752
5. $\frac{5}{6}$ of 1764 $\frac{5}{6}$ of 1776 $\frac{1}{16}$ of 3376
6. $\frac{3}{8}$ of 3376 $\frac{5}{8}$ of 3392 $\frac{7}{16}$ of 3408
7. $\frac{5}{8}$ of 6752 $\frac{5}{8}$ of 6784 $\frac{5}{32}$ of 6816

8. A man has to travel 117 mi. After traveling $\frac{5}{9}$ of the distance, how many miles has he still to travel?

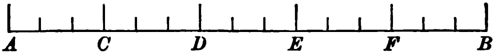
9. A man owns 176 head of cattle. If he sells $\frac{3}{8}$ of them, how many does he sell? How many has he left?

10. A man owns 336 acres of land. If he sells $\frac{3}{8}$ of it, how many acres does he sell? How many acres has he left?

71. Multiplication by a Fraction. To take a fractional part of a number is called *multiplying by a fraction.*

We write "$\frac{1}{2}$ of 4" or "$\frac{1}{2} \times 4$," the × being read "of."

72. Multiplication of a Fraction by a Fraction. Suppose that we have to multiply $\frac{4}{5}$ by $\frac{2}{3}$.



In this figure let the line *AB* be divided into five equal parts, and let each of the five parts be divided into three equal parts. Then each part evidently is $\frac{1}{15}$ of *AB*.

That is, $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ of the whole line.
Then $\frac{1}{3}$ of $\frac{4}{5}$ must be 4 times as much, or $\frac{4}{15}$,
and $\frac{2}{3}$ of $\frac{4}{5}$ must be twice $\frac{4}{15}$, or $\frac{8}{15}$.

Therefore, *to multiply a fraction by a fraction, find the product of the numerators for the required numerator and the product of the denominators for the required denominator.*

Always cancel if possible. For example,

$$\frac{11}{12} \text{ of } \frac{15}{22} = \frac{\cancel{11} \times \overset{5}{\cancel{15}}}{\underset{4}{\cancel{12}} \times \underset{2}{\cancel{22}}} = \frac{5}{8}.$$

WRITTEN EXERCISE

Multiply the following:

1. $\frac{3}{4} \times \frac{8}{9}$.
2. $\frac{2}{3} \times \frac{9}{10}$.
3. $\frac{2}{3} \times \frac{15}{16}$.
4. $\frac{2}{3} \times \frac{11}{24}$.
5. $\frac{2}{3} \times \frac{25}{32}$.
6. $\frac{3}{4} \times \frac{5}{24}$.
7. $\frac{1}{8} \times \frac{4}{5}$.
8. $\frac{3}{8} \times \frac{7}{8}$.
9. $\frac{7}{8} \times \frac{4}{7}$.
10. $\frac{5}{8} \times \frac{3}{10}$.
11. $\frac{3}{16} \times \frac{8}{9}$.
12. $\frac{7}{16} \times \frac{8}{21}$.
13. $\frac{1}{32} \times \frac{4}{5}$.
14. $\frac{1}{48} \times \frac{24}{25}$.
15. $\frac{7}{48} \times \frac{16}{21}$.
16. $\frac{1}{2} \times \frac{4}{5}$.
17. $\frac{1}{5} \times \frac{5}{8}$.
18. $\frac{2}{5} \times \frac{5}{8}$.
19. $\frac{4}{5} \times \frac{5}{8}$.
20. $\frac{7}{32} \times \frac{16}{21}$.

Find the products of:

21. $\frac{3}{5} \times \frac{5}{48}$.
22. $\frac{4}{5} \times \frac{25}{48}$.
23. $\frac{5}{9} \times \frac{81}{100}$.
24. $\frac{5}{8} \times \frac{24}{25}$.
25. $\frac{3}{8} \times \frac{16}{21}$.
26. $\frac{7}{8} \times \frac{32}{35}$.
27. $\frac{9}{16} \times \frac{4}{9}$.
28. $\frac{7}{16} \times \frac{5}{7}$.
29. $\frac{5}{16} \times \frac{32}{45}$.
30. $\frac{13}{16} \times \frac{2}{3}$.
31. $\frac{15}{16} \times \frac{2}{5}$.
32. $\frac{7}{12} \times \frac{4}{9}$.
33. $\frac{5}{12} \times \frac{3}{5}$.
34. $\frac{11}{12} \times \frac{4}{5}$.
35. $\frac{11}{16} \times \frac{4}{11}$.
36. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.
37. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$.
38. $\frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}$.
39. $\frac{3}{5} \times \frac{3}{4} \times \frac{1}{8}$.
40. $\frac{3}{4} \times \frac{3}{8} \times \frac{4}{5}$.
41. $\frac{5}{8} \times \frac{7}{8} \times \frac{3}{8}$.

A few examples with denominators larger than 10 are given only for drill.

42. If an express train runs 1 mi. in $\frac{3}{4}$ min., how long will it take the train to run $\frac{7}{8}$ mi. at the same rate?

43. If a train runs $\frac{1}{3}$ of a certain distance in 1 hr., what part of the distance can it run in $\frac{3}{4}$ hr.?

44. If the distance to a certain house is $\frac{3}{8}$ mi., what fraction of a mile has a boy gone when he has covered $\frac{5}{16}$ of the distance?

45. If it takes a train $\frac{3}{4}$ hr. to reach a certain station, what fraction of an hour will it take to cover $\frac{2}{5}$ of the distance?

46. If you live $\frac{4}{5}$ mi. from school, and Jack lives $\frac{1}{8}$ as far, how far does he live from school?

47. If $\frac{3}{4}$ yd. of lace is needed to trim two neckbands, how much lace will be needed to trim one neckband?

48. If an automobile travels a mile in $\frac{49}{60}$ min. in a certain race, how long would it take to travel $\frac{5}{8}$ mi. at the same rate?

49. If Kate lives $\frac{7}{8}$ mi. from school, and Mary lives $\frac{1}{3}$ as far, how far from school does Mary live? If Sue lives $\frac{1}{2}$ as far as Mary, how far from school does Sue live?

50. A milliner used $\frac{7}{8}$ yd. of velvet for the upper brim and the crown of a hat, and $\frac{2}{5}$ as much for the lower brim. How much velvet did she use for the hat?

ORAL EXERCISE

1. How much is 4 times 5? 4 times $\frac{2}{9}$? 4 times $5\frac{2}{9}$?
2. How much is 2×8? $\frac{3}{4}$ of 8? $2\frac{3}{4}$ times 8?
3. How much is $\frac{10}{3}$ of $\frac{6}{5}$? Then how much is $3\frac{1}{3} \times 1\frac{1}{5}$?
4. How much is $\frac{15}{4}$ of $\frac{4}{3}$? Then how much is $3\frac{3}{4} \times 1\frac{1}{3}$?

73. Multiplication of Mixed Numbers. From these answers we see the truth of the following statements:

To multiply a mixed number by an integer, multiply the whole number and the fractional part separately and add the products.

For example, $6 \times 3\frac{2}{5} = 18\frac{12}{5} = 20\frac{2}{5}$.

To multiply an integer by a mixed number, multiply by the whole number and the fractional part separately and add the products.

For example, $3\frac{2}{5} \times 6 = 3 \times 6 + \frac{2}{5}$ of $6 = 18\frac{12}{5} = 20\frac{2}{5}$.

To multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply the results.

For example, $5\frac{2}{5} \times 3\frac{3}{4} = \frac{27}{\cancel{5}} \text{ of } \frac{\overset{3}{\cancel{15}}}{4} = \frac{81}{4} = 20\frac{1}{4}$.

WRITTEN EXERCISE

Multiply the following:

1. $17 \times 2\frac{19}{34}$.
2. $27 \times 41\frac{5}{9}$.
3. $49 \times 81\frac{6}{7}$.
4. $81 \times 62\frac{5}{9}$.
5. $48 \times 37\frac{3}{10}$.
6. $26\frac{3}{4} \times 32$.
7. $37\frac{4}{5} \times 45$.
8. $29\frac{4}{9} \times 36$.
9. $49\frac{7}{8} \times 68$.
10. $25\frac{2}{3} \times 18$.
11. $47\frac{1}{10} \times 65$.
12. $26\frac{1}{2} \times 63$.
13. $35\frac{7}{8} \times 36$.
14. $6\frac{7}{8} \times 7\frac{7}{8}$.
15. $9\frac{1}{3} \times 15\frac{7}{8}$.
16. $16\frac{3}{4} \times 14\frac{2}{3}$.
17. $15\frac{5}{9} \times 14\frac{3}{7}$.
18. $14\frac{3}{4} \times 16\frac{4}{5}$.
19. $18\frac{4}{7} \times 23\frac{7}{8}$.
20. $18\frac{1}{3} \times 27\frac{3}{5}$.
21. $15\frac{3}{4} \times 28\frac{4}{9}$.

ORAL EXERCISE

1. If you have $\frac{3}{8}$ yd. of cloth and need $\frac{1}{3}$ of it for a piece of work, what part of a yard do you need?

2. If the class has $\frac{7}{8}$ gal. of milk and $\frac{4}{7}$ of it is used in one lesson, what part of a gallon is used?

Think of $\frac{4}{7} \times \frac{7}{8}$, and cancel 4 and 7 mentally.

Usually $\frac{1}{3}$ of $\frac{3}{8}$ is written $\frac{1}{3} \times \frac{3}{8}$, × being read "of" after a fraction, but "times" after a whole number or a mixed number.

3. If the distance to a certain house is $\frac{5}{16}$ mi., how far has a boy gone when he has covered $\frac{4}{5}$ of this distance?

Find the values of the following:

4. $\frac{1}{2}$ of $\frac{1}{2}$. 6. $\frac{1}{4}$ of $\frac{1}{2}$. 8. $\frac{1}{2}$ of $\frac{2}{3}$. 10. $\frac{1}{3}$ of $\frac{1}{4}$.
5. $\frac{1}{2}$ of $\frac{1}{4}$. 7. $\frac{1}{2}$ of $\frac{1}{3}$. 9. $\frac{2}{3}$ of $\frac{1}{2}$. 11. $\frac{1}{3}$ of $\frac{3}{4}$.

WRITTEN EXERCISE

Add the following:

1. $4\frac{1}{3}$, $3\frac{2}{3}$, $5\frac{1}{3}$. 3. $4\frac{7}{8}$, $3\frac{2}{3}$, $5\frac{11}{24}$. 5. $2\frac{1}{10}$, $16\frac{3}{20}$, $27\frac{7}{30}$.
2. $2\frac{1}{8}$, $1\frac{1}{8}$, $6\frac{3}{8}$. 4. $3\frac{1}{16}$, $5\frac{3}{32}$, $7\frac{11}{64}$. 6. $8\frac{7}{16}$, $15\frac{15}{32}$, $73\frac{51}{64}$.

7. The sides of a triangle are $3\frac{3}{4}$ in., $2\frac{7}{8}$ in., and $3\frac{1}{2}$ in., respectively. What is the perimeter?

Subtract the following:

8. $28\frac{5}{8} - 7\frac{25}{32}$. 10. $12\frac{1}{3} - 8\frac{1}{4}$. 12. $31\frac{5}{6} - 7\frac{7}{12}$.
9. $32\frac{7}{8} - 9\frac{31}{32}$. 11. $15\frac{1}{4} - 7\frac{1}{3}$. 13. $42\frac{5}{6} - 8\frac{5}{12}$.

14. From the sum of $16\frac{1}{2}$ ft. and $4\frac{1}{2}$ ft. subtract $12\frac{3}{4}$ ft.

Find the values of the following:

15. $\frac{1}{2}$ of $\frac{3}{4}$. 17. $\frac{5}{8}$ of $\frac{4}{5}$. 19. $\frac{13}{16}$ of $\frac{2}{3}$. 21. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.
16. $\frac{3}{4}$ of $\frac{1}{3}$. 18. $\frac{3}{8}$ of $\frac{8}{21}$. 20. $\frac{15}{16}$ of $\frac{2}{5}$. 22. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.

23. A piece of tin $\frac{1}{40}$ in. thick is folded once. How thick are both parts together?

24. A piece of paper $\frac{1}{60}$ in. thick is folded once, and then folded again, making four leaves. How thick are the four leaves together?

25. If the height of the upright piece in a sundial is $\frac{5}{32}$ of the diameter of the dial and the diameter of the dial is 8 in., what is the height of the upright piece?

26. If the length of a paddle of a water wheel is $\frac{3}{7}$ of the diameter and the diameter is 21 in., what is the length of the paddle?

27. A certain recipe calls for 8 oz. of sugar. To make $2\frac{1}{2}$ times the recipe, how many ounces of sugar must we use?

28. A certain recipe calls for 2 cups of cream. To make $3\frac{1}{2}$ times the recipe, how much cream must we use?

29. If an automobile goes a mile in 59 sec. (or $\frac{59}{60}$ min.) in a certain race, how long will it take to go $\frac{3}{4}$ mi. at the same rate?

30. A man has a city lot that contains $\frac{12}{25}$ A. If he sells $\frac{5}{8}$ of it, how much does he sell? How much has he left?

31. If it takes $2\frac{7}{8}$ yd. of cloth to make a boy's suit, how many yards will it take to make a dozen suits?

32. A tin cup is found to hold $\frac{15}{16}$ pt. When it is $\frac{2}{3}$ full the cup contains what part of a pint?

33. If a glass jar holds $\frac{15}{16}$ qt., what part of a quart will it contain when $\frac{4}{5}$ full? What part of a quart will it contain when $\frac{2}{5}$ full? when $\frac{3}{5}$ full?

34. In making the plans for a house an architect wishes to draw a line that shall be $\frac{2}{5}$ as long as one that is $\frac{15}{16}$ in. long. How long shall he draw it?

74. Cash Checks. In large stores the clerks are required to fill out cash checks like the following:

Sold by No. 127		Amount rec'd, $15.00		
½ yd.	Silk	.57		29
4⅜ "	Summer silk	.38	1	67
11½ "	Silk	.99	11	39
	Total		13	35
	Change due		1	65
			15	00

WRITTEN EXERCISE

Make out cash checks for the following sales:

1. 14½ yd. gingham @ 25¢, 9 yd. cotton @ 12½¢, 10½ yd. madras @ 35¢. Amount received, $10.

2. 6½ yd. cheviot @ $1.10, 9½ yd. velveteen @ 98¢, 5⅜ yd. silk @ $1.25. Amount received, $25.

3. 11½ yd. India linen @ 35¢, 6⅝ yd. dimity @ 25¢, 12⅛ yd. linen suiting @ 75¢. Amount received, $20.

4. 9½ yd. India linen @ 38¢, 5⅛ yd. cheviot @ $1.25, 15 yd. cotton @ 12½¢. Amount received, $15.

5. 7 yd. muslin @ 25¢, 12½ yd. lining @ 11¢, 6⅝ yd. India linen @ 45¢, ½ yd. velvet @ $3. Amount received, $10.

6. 14½ yd. muslin @ 24¢, 4⅛ yd. velvet @ $2.40, 16¾ yd. lining @ 12¢, 7⅜ yd. silk @ $1.60, 5¾ yd. suiting @ 80¢, 6½ yd. ribbon @ 30¢. Amount received, $50.

The teacher should ask the pupils to make problems similar to those given above, using the local prices of common materials.

75. Cancellation. We have seen, on page 50, that we may divide both terms of a fraction by the same number without changing the value of the fraction; that is, we may *cancel* any factor that is found in both terms.

Teachers should make it clear that this is a case of *division*; that we cannot *subtract* the same number from both terms of the fraction.

In order to divide both terms of a fraction by the same number we must know more about different divisors.

76. Common Divisor. A factor that is common to two or more numbers is called a *common divisor.*

For example, 5 is a common divisor of 15 and 25. It might be called a common factor, but common divisor is the usual name.

77. Greatest Common Divisor. The greatest factor common to two or more numbers is called their *greatest common divisor* (G.C.D.). For example, although 3 is a common divisor of 12, 18, and 48, the number 6 is the *greatest* common divisor.

78. Finding the G.C.D. The greatest common divisor of such numbers as we have met or shall meet in fractions is easily found. For example, consider 12 and 30. We see that

$$12 = 2 \times 3 \times 2, \qquad 30 = 2 \times 3 \times 5,$$

and that 2 and 3 are the only common factors. Therefore 2 × 3, or 6, is the greatest common divisor.

79. Prime Number. A number that has no factors except itself and 1 is called a *prime number.* For example, 7, 11, and 17 are prime numbers.

80. Composite Number. A number not prime is called *composite.* For example, 25, 39, and 51 are composite numbers.

We use the terms "prime" and "composite" only in connection with whole numbers.

81. Even Number. A number that contains the factor 2 is called an *even number.* For example, 20, 38, and 1112 are even numbers.

82. Odd Number. A number that is not even is called an *odd number.* For example, 21, 39, and 127 are odd numbers.

83. Prime Factor. A factor that is a prime number is called a *prime factor.* For example, the prime factors of 68 are 2, 2, and 17, and the prime factors of 25 are 5 and 5.

84. Mutually Prime Numbers. Numbers that have no common factors are said to be *prime to each other.* For example, 12 and 35.

ORAL EXERCISE

1. What do you mean by the factors of a number?

2. Name two factors of 35; of 77; of 26; of 34.

3. Name three factors of 30; of 70; of 66; of 110.

4. Name a factor that is common to 35 and 45.

5. Name two factors common to 30 and 48.

6. Find a common divisor of 20 and 22; of 16 and 36.

7. Find the greatest common divisor of 75 and 100.

8. Which of the following numbers have no factors except 1 and themselves: 4, 5, 6, 7, 10, 11, 12, 13?

9. In reducing $\frac{12}{16}$ to lowest terms is it sufficient to cancel the common divisor 2? What kind of common divisor must be canceled in reducing a fraction to lowest terms?

10. Find the odd numbers and the prime numbers from 1 to 30.

11. Find the prime factors of 20; of 48; of 60; of 77; of 50.

12. Find the greatest common divisor of 8, 12, 20, and 24.

13. Find the greatest common divisor of 9, 15, 21, and 60.

85. Divisible Numbers. When we speak of one number being *divisible* by another we mean that the quotient is a whole number.

86. Divisibility by 2. *A number is divisible by 2 if the units are so divisible.* For example, 64, or 60 + 4, must be divisible by 2 if 4, the number of units, is divisible by 2, because any multiple of 10 is divisible by 2.

87. Divisibility by 5. *A number is divisible by 5 if it ends in 0 or 5.* For example, 180 and 275 are divisible by 5.

88. Divisibility by 3. *A number is divisible by 3 if the sum of its digits is so divisible.* For example, the sum of the digits of 411 is 6, and 6 is divisible by 3. Hence 411 is also divisible by 3.

ORAL EXERCISE

1. Is 2 a factor of 14? of 27? of 50? of 45? of 600? How do you tell whether 2 is a factor of a number?

2. Is 2 a factor of 10? of any number of 10's? Then is 2 a factor of any number if it is a factor of the units?

3. Is 5 a factor of 10? of any number of 10's? Then is 5 a factor of any number if it is a factor of the units?

WRITTEN EXERCISE

1. From this list select the numbers divisible by 2:

660 4907 6255 3027 1356 37,268

2. From this list select the numbers divisible by 3:

4867 7075 9636 8124 3672 12,475

3. From this list select the numbers divisible by 5:

1235 5778 9100 8328 3445 31,035

89. Other Tests of Divisibility. There are various tests of divisibility besides those on page 80. They are easily illustrated or explained, and may be given or not as the teacher prefers. The more important are the following:

A number is divisible by 4 if the number represented by its two right-hand figures is so divisible.

A number is divisible by 6 if it is even and if the sum of its digits is divisible by 3.

A number is divisible by 8 if the number represented by its three right-hand figures is so divisible.

A number is divisible by 9 if the sum of its digits is so divisible.

A number is divisible by 11 if the difference between the sums of the digits in its even and odd places is so divisible.

For example, 430,507 is divisible by 11, for $7+5+3=15$, $0+0+4=4$, and $15-4=11$.

ORAL EXERCISE

1. State the prime factors of 15, 18, 27, 35, 42, and 75.
2. State one factor of 395, 123, 777, 692, 1275, and 1263.
3. Is 5 a factor of 20? of 78? of 45? of 72? of 800?
4. State all the odd numbers from 10 to 30.
5. State all the prime numbers from 1 to 40.
6. Select the numbers divisible by 2, 3, and 5 respectively:
   4263 7065 2022 3708 5445 13,005
7. Select the numbers divisible by 4, 6, and 8 respectively:
   3104 3204 3304 7275 3116 54,012
8. Select the numbers divisible by 9 and 11 respectively:
   5004 1001 9009 6264 2244 12,342

ORAL EXERCISE

State the answers to the following:

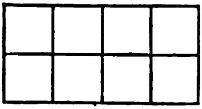
1. 15 ft. ÷ 5 $\frac{15}{16} \div 5$ $\frac{15}{32} \div 5$ $\frac{15}{64} \div 5$

2. 25 pt. ÷ 5 $\frac{25}{24} \div 5$ $\frac{25}{8} \div 5$ $\frac{25}{32} \div 5$

3. 14 in. ÷ 7 $\frac{14}{15} \div 7$ $\frac{14}{9} \div 7$ $\frac{14}{3} \div 7$

4. Dividing the numerator of a fraction does what to the fraction itself? Give an example.

5. If you divide $\frac{1}{2}$ of this rectangle into two equal parts, each is what part of the rectangle? How much is $\frac{1}{2} \div 2$?



6. If you divide $\frac{1}{4}$ of the rectangle into two equal parts, each is what part of the rectangle? How much is $\frac{1}{4} \div 2$? What have you done to the denominator?

7. If you divide $\frac{3}{4}$ of the rectangle into two equal parts, how many eighths of the rectangle do you then have? How much is $\frac{3}{4} \div 2$? What have you done to the denominator?

8. Multiplying the denominator of a fraction does what to the fraction itself?

Divide the following fractions by dividing the numerators:

9. $\frac{4}{5} \div 2$ $\frac{15}{16} \div 3$ $\frac{15}{32} \div 5$ $\frac{25}{64} \div 5$ $\frac{21}{32} \div 7$

10. $\frac{6}{7} \div 3$ $\frac{12}{25} \div 6$ $\frac{16}{25} \div 8$ $\frac{27}{32} \div 9$ $\frac{33}{64} \div 11$

Divide the following fractions by multiplying the denominators:

11. $\frac{1}{3} \div 5$ $\frac{2}{3} \div 7$ $\frac{3}{5} \div 4$ $\frac{3}{8} \div 7$ $\frac{3}{4} \div 5$

12. $\frac{4}{5} \div 9$ $\frac{7}{8} \div 3$ $\frac{3}{8} \div 3$ $\frac{5}{8} \div 9$ $\frac{3}{5} \div 7$

Divide the following:

13. $\frac{4}{15} \div 2$ $\frac{3}{5} \div 2$ $\frac{7}{8} \div 4$ $\frac{5}{8} \div 2$ $\frac{7}{8} \div 2$

14. $\frac{15}{16} \div 10$ $\frac{25}{32} \div 5$ $\frac{5}{12} \div 4$ $\frac{15}{8} \div 5$ $\frac{7}{8} \div 5$

90. How to Divide a Fraction by a Whole Number. From the answers to the questions on page 82 we have the following:

Dividing the numerator by a whole number divides the fraction by that number.

Multiplying the denominator by a whole number divides the fraction by that number.

If we have a mixed number to be divided, we may reduce the mixed number to an improper fraction.

Thus $2\frac{1}{3} \div 3 = \frac{7}{3} \div 3 = \frac{7}{9}$.

Sometimes, however, it is easier to divide the mixed numbers. For example, to divide a number like $244\frac{1}{2}$ by 3 it is easier not to reduce to an improper fraction.

Thus $244\frac{1}{2} \div 3 = 81 + \frac{1}{3} \text{ of } 1\frac{1}{2} = 81 + \frac{1}{2} = 81\frac{1}{2}$.

WRITTEN EXERCISE

Divide the following:

1. $\frac{1}{2} \div 2$ $\frac{1}{3} \div 2$ $\frac{1}{4} \div 2$ $\frac{3}{4} \div 2$ $\frac{3}{8} \div 2$
2. $\frac{1}{2} \div 3$ $\frac{2}{3} \div 3$ $\frac{1}{4} \div 3$ $\frac{3}{4} \div 3$ $\frac{3}{8} \div 3$
3. $\frac{2}{5} \div 3$ $\frac{4}{5} \div 3$ $\frac{1}{8} \div 2$ $\frac{6}{5} \div 3$ $\frac{8}{5} \div 4$
4. $\frac{5}{8} \div 5$ $\frac{5}{8} \div 2$ $\frac{5}{8} \div 3$ $\frac{5}{8} \div 4$ $\frac{7}{8} \div 7$
5. $\frac{6}{7} \div 2$ $\frac{6}{7} \div 3$ $\frac{6}{7} \div 5$ $\frac{6}{7} \div 6$ $\frac{5}{8} \div 6$
6. $\frac{5}{8} \div 5$ $3\frac{3}{4} \div 5$ $7\frac{1}{2} \div 3$ $4\frac{1}{5} \div 7$ $4\frac{2}{3} \div 7$

Divide the whole part and the fractional part separately, and add the results:

7. $12\frac{3}{4} \div 3$ $21\frac{1}{2} \div 3$ $36\frac{3}{5} \div 3$ $24\frac{4}{5} \div 4$ $35\frac{15}{16} \div 5$
8. $23\frac{1}{3} \div 7$ $25\frac{1}{2} \div 8$ $28\frac{1}{8} \div 9$ $28\frac{3}{4} \div 5$ $134\frac{1}{2} \div 5$

ORAL EXERCISE

1. How many quarters of a dollar in $1?

2. How many thirds of an inch in 1 in.? Then $1 \div \frac{1}{3}$ is how many? What can you say about the values of $1 \div \frac{1}{3}$ and 3×1?

3. How many thirds in 1? in 2? What can you say about the values of $2 \div \frac{1}{3}$ and 3×2?

4. How many eighths of an inch in 1 in.? What can you say about the values of $1 \div \frac{1}{8}$ and 8×1?

5. How many fifths of an inch in 1 in.? Then $1 \div \frac{1}{5}$ is how many? What can you say about the values of $1 \div \frac{1}{5}$ and 5×1?

6. Tell the values of

$1 \div \frac{1}{8}$ $\quad 1 \div \frac{1}{9}$ $\quad 1 \div \frac{1}{7}$ $\quad 1 \div \frac{1}{10}$ $\quad 1 \div \frac{1}{20}$

7. How do you divide by a unit fraction?

8. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets pressed together will have a thickness of $\frac{3}{16}$ in.?

9. If a sheet of felt is $\frac{1}{16}$ in. thick, how many sheets are there in a pile $1\frac{1}{16}$ in. thick?

10. If a bicycle is traveling at the rate of a mile in $\frac{1}{8}$ hr., how many miles will it travel in $3\frac{1}{8}$ hr.?

11. How many weights of $\frac{1}{16}$ lb. (or 1 oz.) each will it take to weigh $2\frac{1}{16}$ lb.?

12. How many pints ($\frac{1}{8}$ gal.) are there in 3 gal.? How many pints are there in $2\frac{1}{2}$ gal.?

13. How many inches ($\frac{1}{12}$ ft.) are there in 3 ft.? How many inches are there in 10 ft.?

14. If each book is $\frac{1}{2}$ in. thick, how many books are there in a pile that is $3\frac{1}{2}$ in. high?

91. Dividing by a Unit Fraction. From the work given on page 84 it is clear that division by a unit fraction is simple.

To divide a number by a unit fraction, multiply the number by the denominator of the fraction.

Thus $1 \div \frac{1}{8} = 1 \times 8 = 8,$

$3 \div \frac{1}{8} = 3 \times 8 = 24,$

and $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \times 8 = 6.$

WRITTEN EXERCISE

Divide the following:

1. $25 \div \frac{1}{3}$.
2. $32 \div \frac{1}{4}$.
3. $37 \div \frac{1}{5}$.
4. $43 \div \frac{1}{6}$.
5. $51 \div \frac{1}{2}$.
6. $63 \div \frac{1}{8}$.
7. $27 \div \frac{1}{2}$.
8. $36 \div \frac{1}{12}$.
9. $51 \div \frac{1}{16}$.
10. $81 \div \frac{1}{8}$.
11. $16 \div \frac{1}{4}$.
12. $23 \div \frac{1}{4}$.
13. $32 \div \frac{1}{6}$.
14. $54 \div \frac{1}{4}$.
15. $29 \div \frac{1}{3}$.
16. $48 \div \frac{1}{5}$.
17. $63 \div \frac{1}{8}$.
18. $27 \div \frac{1}{6}$.
19. $17 \div \frac{1}{12}$.
20. $97 \div \frac{1}{12}$.
21. $39 \div \frac{1}{20}$.
22. $3\frac{1}{2} \div \frac{1}{16}$.
23. $7\frac{3}{8} \div \frac{1}{32}$.
24. $9\frac{7}{16} \div \frac{1}{64}$.
25. $\frac{2}{3} \div \frac{1}{2}$.
26. $\frac{4}{5} \div \frac{1}{10}$.
27. $\frac{3}{8} \div \frac{1}{32}$.
28. $\frac{3}{4} \div \frac{1}{8}$.
29. $\frac{5}{6} \div \frac{1}{12}$.
30. $2\frac{1}{2} \div \frac{1}{2}$.

31. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets pressed together will have a thickness of $1\frac{1}{8}$ in.?

32. If a sheet of blotting paper is $\frac{1}{32}$ in. thick, how many sheets are there in a pile that is $15\frac{5}{8}$ in. thick?

33. How many weights of $\frac{1}{16}$ lb. (or 1 oz.) each will it take to weigh $17\frac{3}{4}$ lb.?

34. If a wagon wheel makes $\frac{1}{12}$ of a revolution in going 1 foot, how many feet will it go in making $17\frac{3}{4}$ revolutions?

ORAL EXERCISE

1. I have a room 18 ft. wide. I wish to find how many strips of carpet $\frac{3}{4}$ yd. wide will be needed to run lengthwise of the room. First I must find how many yards there are in 18 ft.

2. If I have to divide 6 yd. by $\frac{3}{4}$ yd., I may think of the 6 yd. as fourths. Express 6 as fourths.

3. If now I have to divide $\frac{24}{4}$ by $\frac{3}{4}$, I may think of the results in the cases of 24¢ ÷ 3¢, 24 ft. ÷ 3 ft., and 24 in. ÷ 3 in. What are the results?

4. Then how much is $\frac{24}{4} \div \frac{3}{4}$? How many strips of carpet shall I need to carpet the room mentioned in Ex. 1?

5. Divide $\frac{6}{5}$ by $\frac{2}{5}$. Multiply $\frac{6}{5}$ by $\frac{5}{2}$. Compare the results.

6. Show that $\frac{4}{5} \div \frac{2}{5}$ is the same as $\frac{5}{2} \times \frac{4}{5}$.

7. Study the following:

$$15 \div \frac{3}{5} = \frac{5}{\cancel{3}} \times \overset{5}{\cancel{15}} = 25.$$

8. Study the following:

$$\frac{3}{8} \div \frac{5}{16} = \frac{\overset{2}{\cancel{16}}}{5} \times \frac{3}{\cancel{8}} = \frac{6}{5} = 1\tfrac{1}{5}.$$

WRITTEN EXERCISE

1. How many paper pads, each $\frac{5}{16}$ in. thick, will it take to make a package 5 in. thick?

2. How many thicknesses of board, each $\frac{3}{4}$ in. thick, will it take to make a pile $9\frac{3}{4}$ in. high?

Divide the following:

3. $12 \div \frac{2}{3}$.	**5.** $9 \div \frac{3}{4}$.	**7.** $4 \div \frac{4}{5}$.	**9.** $\frac{7}{8} \div \frac{5}{8}$.
4. $14 \div \frac{2}{3}$.	**6.** $5 \div \frac{3}{4}$.	**8.** $\frac{5}{8} \div \frac{4}{5}$.	**10.** $5\frac{1}{3} \div \frac{1}{2}$.

92. Dividing by a Fraction. An inverted fraction is called the *reciprocal* of the fraction. That is, the reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$.

From page 86 we have found how to divide by a fraction. We shall now look a little more carefully at the reason. We found on page 85 that when we have to divide by $\frac{1}{3}$ we may multiply by its reciprocal, 3. We also know that when we multiply the divisor by 2 we divide the quotient by 2.

That is, $6 \div \frac{1}{3} = 3 \times 6,$

and $6 \div \frac{2}{3} = \text{half as much} = \frac{3 \times 6}{2} = 9.$

That is, $6 \div \frac{2}{3} = \frac{3}{2} \text{ of } 6.$

Therefore, *to divide by a common fraction, multiply by the reciprocal of that fraction.*

That is, $$15 \div \frac{3}{5} = \frac{5}{\cancel{3}} \times \overset{5}{\cancel{15}} = 25.$$

Similarly, $$\frac{3}{8} \div \frac{5}{6} = \frac{\overset{3}{\cancel{6}}}{5} \times \frac{3}{\underset{4}{\cancel{8}}} = \frac{9}{20},$$

and $$4\tfrac{2}{3} \div 2\tfrac{4}{5} = \frac{14}{3} \div \frac{14}{5} = \frac{5}{\cancel{14}} \times \frac{\cancel{14}}{3} = \frac{5}{3} = 1\tfrac{2}{3}.$$

WRITTEN EXERCISE

Divide the following:

1. $\frac{7}{8} \div \frac{2}{3}$.
2. $\frac{5}{6} \div \frac{3}{4}$.
3. $\frac{9}{10} \div \frac{4}{5}$.
4. $\frac{5}{12} \div \frac{7}{8}$.
5. $\frac{3}{8} \div \frac{5}{12}$.
6. $\frac{7}{16} \div \frac{3}{32}$.
7. $\frac{5}{64} \div \frac{7}{32}$.
8. $\frac{9}{24} \div \frac{5}{16}$.
9. $\frac{3}{40} \div \frac{6}{25}$.
10. $\frac{7}{48} \div \frac{7}{12}$.
11. $\frac{3}{4} \div 1\frac{1}{16}$.
12. $2\frac{7}{16} \div 3\frac{1}{8}$.
13. $9\frac{7}{12} \div 5\frac{3}{4}$.
14. $5\frac{1}{4} \div 18\frac{3}{8}$.
15. $3\frac{9}{20} \div 2\frac{3}{10}$.
16. $4\frac{5}{9} \div 13\frac{2}{3}$.
17. $6\frac{3}{10} \div 1\frac{4}{5}$.
18. $4\frac{9}{10} \div 12\frac{5}{6}$.

ORAL EXERCISE

Divide the following:

1. $\frac{1}{2} \div 2$ $\frac{1}{2} \div \frac{1}{2}$ $\frac{1}{2} \div \frac{1}{4}$ $\frac{1}{2} \div \frac{1}{8}$ $\frac{1}{2} \div \frac{1}{3}$
2. $\frac{1}{2} \div \frac{3}{4}$ $\frac{1}{2} \div \frac{3}{8}$ $\frac{1}{2} \div \frac{5}{8}$ $\frac{1}{2} \div \frac{7}{8}$ $\frac{1}{2} \div \frac{1}{16}$
3. $\frac{1}{4} \div \frac{1}{8}$ $\frac{3}{4} \div \frac{1}{8}$ $\frac{3}{4} \div \frac{5}{8}$ $\frac{3}{4} \div \frac{7}{8}$ $\frac{3}{4} \div \frac{9}{8}$

4. Divide $\frac{1}{3}$ by 2; 2 by $\frac{1}{3}$; 2 by 3; 3 by 2.

5. Divide $\frac{3}{4}$ by 3; 3 by $\frac{3}{4}$; 3 by $\frac{3}{5}$; 3 by $\frac{3}{8}$.

6. Divide $\frac{4}{5}$ by 2; 2 by $\frac{4}{5}$; 2 by $\frac{2}{3}$; $\frac{2}{3}$ by 2.

7. Divide $\frac{3}{8}$ by $\frac{5}{8}$; $\frac{5}{8}$ by $\frac{3}{8}$; $\frac{3}{4}$ by $\frac{5}{4}$; $\frac{5}{4}$ by $\frac{3}{4}$.

8. How many half-yard pieces can be made by cutting 5 yd. of ribbon? by cutting 7 yd. of ribbon?

9. If your recipe for making cake calls for $\frac{1}{2}$ lb. of sugar, 6 lb. of sugar is enough for how many times as much cake?

WRITTEN EXERCISE

Divide the following:

1. $\frac{2}{3} \div \frac{3}{4}$.
2. $\frac{3}{4} \div \frac{2}{3}$.
3. $\frac{2}{3} \div \frac{4}{5}$.
4. $\frac{4}{5} \div \frac{2}{3}$.
5. $\frac{3}{8} \div \frac{3}{4}$.
6. $\frac{3}{4} \div \frac{3}{8}$.
7. $\frac{5}{8} \div \frac{4}{5}$.
8. $\frac{4}{5} \div \frac{5}{8}$.
9. $\frac{7}{8} \div \frac{3}{4}$.
10. $\frac{3}{4} \div \frac{7}{8}$.
11. $\frac{4}{5} \div \frac{7}{8}$.
12. $\frac{7}{8} \div \frac{4}{5}$.
13. $\frac{5}{9} \div \frac{2}{3}$.
14. $\frac{2}{3} \div \frac{5}{9}$.
15. $\frac{5}{9} \div \frac{4}{5}$.
16. $\frac{4}{5} \div \frac{5}{9}$.
17. $\frac{3}{5} \div \frac{2}{3}$.
18. $\frac{2}{3} \div \frac{3}{5}$.
19. $\frac{5}{12} \div \frac{2}{3}$.
20. $\frac{2}{3} \div \frac{5}{12}$.
21. $5 \div \frac{5}{8}$.
22. $5\frac{1}{4} \div \frac{5}{8}$.
23. $3\frac{3}{4} \div \frac{7}{8}$.
24. $8\frac{1}{3} \div \frac{5}{6}$.
25. $3\frac{5}{8} \div \frac{3}{4}$.
26. $3\frac{5}{8} \div \frac{3}{5}$.
27. $4\frac{7}{8} \div \frac{3}{8}$.
28. $5\frac{4}{9} \div 2\frac{1}{3}$.

29. How many mats, each $\frac{3}{4}$ yd. long, can be made from a strip of carpet 9 yd. long, making no allowance for turning in or hemming?

30. How many towels, each $\frac{5}{8}$ yd. long, can be made from a strip of toweling $11\frac{1}{4}$ yd. long, making no allowance for hemming?

WRITTEN EXERCISE

Divide the following:

1. $21 \div 5\frac{1}{4}$.
2. $77 \div 9\frac{1}{3}$.
3. $69 \div 5\frac{3}{4}$.
4. $35 \div 5\frac{5}{8}$.
5. $21 \div 4\frac{2}{3}$.
6. $14 \div 2\frac{5}{8}$.
7. $61\frac{1}{4} \div 8\frac{3}{4}$.
8. $71 \div 8\frac{7}{8}$.
9. $17\frac{7}{8} \div 4\frac{7}{8}$.
10. $33 \div 7\frac{4}{5}$.
11. $15\frac{2}{3} \div 6\frac{1}{3}$.
12. $17\frac{1}{8} \div 2\frac{1}{2}$.
13. $12\frac{1}{2} \div 3\frac{1}{2}$.
14. $15\frac{1}{2} \div 3\frac{1}{8}$.
15. $78 \div 8\frac{17}{32}$.
16. $16\frac{4}{5} \div 4\frac{1}{5}$.
17. $79\frac{4}{5} \div 8\frac{1}{7}$.
18. $324 \div 7\frac{1}{5}$.
19. $25\frac{3}{5} \div 5\frac{1}{3}$.
20. $52\frac{3}{5} \div 8\frac{3}{16}$.
21. $61\frac{7}{8} \div 4\frac{13}{16}$.
22. $77 \div 6\frac{5}{12}$.
23. $27\frac{1}{2} \div 33\frac{1}{3}$.
24. $40\frac{3}{4} \div 6\frac{1}{3}$.
25. $28\frac{11}{16} \div 2\frac{1}{8}$.
26. $29\frac{1}{5} \div 58\frac{2}{5}$.
27. $20\frac{1}{8} \div 15\frac{1}{3}$.
28. $31\frac{5}{8} \div 15\frac{13}{16}$.
29. $13\frac{9}{10} \div 4\frac{2}{5}$.
30. $151\frac{7}{8} \div 15\frac{3}{16}$.
31. $876\frac{1}{2} \div 129\frac{5}{12}$.
32. $202\frac{1}{8} \div 14\frac{7}{16}$.
33. $171\frac{7}{8} \div 17\frac{3}{16}$.

34. How many strips of cloth, each $5\frac{3}{8}$ yd. long, can we get from a strip $37\frac{5}{8}$ yd. long?

35. How many city lots, each $31\frac{3}{4}$ ft. front, can a man get from a piece of land having $222\frac{1}{4}$ ft. front?

36. How many yards of cloth at $33\frac{1}{3}$¢ a yard can be bought for \$7? for \$19? for \$27?

37. How many books, each $\frac{7}{8}$ in. thick, will it take to make a pile $21\frac{7}{8}$ in. high?

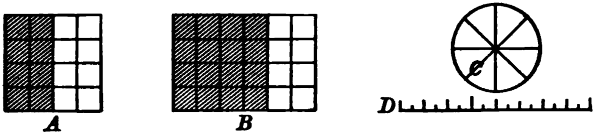
38. How many strips of wood, each $\frac{9}{16}$ in. thick, will it take to make a pile $19\frac{1}{8}$ in. high?

39. How many sheets of bookbinding board, each $\frac{3}{32}$ in. thick, will it take to make a pile $9\frac{3}{16}$ in. high?

40. If the product of two numbers is 46, and one of the numbers is $2\frac{7}{8}$, what is the other?

ORAL EXERCISE

1. In line *D*, point to $\frac{3}{16}$ of the line; also to $\frac{6}{16}$. We see that $\frac{6}{16}$ is how many times $\frac{3}{16}$? How does multiplying the numerator by 2 change the value of the fraction? How does dividing the numerator by 2 change the value of the fraction?



These figures can be used more easily if drawn on the board.

2. Point to $\frac{1}{2}$ of square *A*; also to $\frac{1}{4}$. We see that $\frac{1}{2}$ is how many times $\frac{1}{4}$? How does multiplying the denominator by 2 change the value of the fraction? How does dividing the denominator by 2 change the value of the fraction?

3. In rectangle *B*, show that $\frac{2}{3} = \frac{4}{6}$. Then does it change the value of a fraction to multiply or divide both terms by the same number? Show this with the circle *C*.

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93. Review of Important Principles. From the above exercise we again see the truth of these important principles:

Multiplying the numerator of a fraction by any number multiplies the fraction by that number.

Dividing the numerator of a fraction by any number divides the fraction by that number.

Multiplying the denominator of a fraction by any number divides the fraction by that number.

Dividing the denominator of a fraction by any number multiplies the fraction by that number.

Dividing both terms of a fraction by the same number does not change the value of the fraction.

ORAL EXERCISE

1. How many cents in \$⅛? Then 16 times \$⅛ is the same as 16 times how many cents?

2. How many cents in \$⅓? Then 9 times \$⅓ is the same as 9 times how many cents?

3. How many cents in \$⅙? Then 24 times \$⅙ is the same as 24 times how many cents?

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94. Use of Aliquot Parts in Multiplication. It is easier to multiply \$⅛, \$⅙, and \$⅓ than it is to multiply 12½¢, 16⅔¢, and 33⅓¢. Such parts of a dollar or of any other unit are often called, as we found in § 28, aliquot parts.

At 33⅓¢ each, 9 neckties cost 9 times \$⅓, or \$3.

At 16⅔¢ each, 12 bottles of ink cost 12 times \$⅙, or \$2.

At 12½¢ each, 9 balls cost 9 times \$⅛, or \$1⅛, or \$1.12½.

While goods are seldom marked 16⅔¢ or \$⅙, they are often marked 6 for \$1, which is the same thing.

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4. At 12½¢ a yard, how much will 16 yd. of cloth cost?

5. At 16⅔¢ a yard, how much will 18 yd. of cloth cost?

6. At 33⅓¢ a yard, how much will 21 yd. of cloth cost?

7. At 8 doz. marbles for \$1, how much will 6 doz. cost?

8. At 16⅔¢ each, find the cost of 9 water pitchers.

9. At the rate of 3 for \$1, how much must the school pay for 15 arithmetics?

10. At 12½¢ each, find the cost of 10 glasses; of 4 lamp chimneys; of 20 cans of soup; of 32 yd. of calico.

11. How many jackets can be cut from 17½ yd. of cloth, allowing 2½ yd. to a jacket?

ORAL EXERCISE

1. How much is $8 \times \$\frac{1}{8}$? $8 \times \$1\frac{1}{8}$? $8 \times \$1.12\frac{1}{2}$?

2. How much is $6 \times \$\frac{1}{6}$? $6 \times \$1\frac{1}{6}$? $6 \times \$1.16\frac{2}{3}$?

3. How much is $3 \times \$\frac{1}{3}$? $3 \times \$1\frac{1}{3}$? $3 \times \$1.33\frac{1}{3}$?

4. At $\$1.33\frac{1}{3}$ a yard, how much will 15 yd. of lace cost?

5. At $\$1.16\frac{2}{3}$ a yard, how much will 9 yd. of silk cost?

6. At $\$1.12\frac{1}{2}$ a yard, how much will 6 yd. of velvet cost?

7. Find the cost of 12 books at $\$1.12\frac{1}{2}$ each; at $\$1.16\frac{2}{3}$ each.

8. At $\$1.33\frac{1}{3}$ each, find the cost of 3 chairs; of 18 chairs.

9. At $\$1.16\frac{2}{3}$ each, find the cost of 6 maps; of 12 maps; of 30 maps; of 54 maps.

10. At $\$1.12\frac{1}{2}$ each, find the cost of 8 books; of 40 books; of 32 books; of 20 books.

At $\$1.12\frac{1}{2}$ each, 7 books cost $\$7 + \$\frac{7}{8}$, or $\$7.87\frac{1}{2}$ ($7.88).
At $\$1.16\frac{2}{3}$ each, 5 books cost $\$5 + \$\frac{5}{6}$, or $\$5.83\frac{1}{3}$ ($5.83).
At $\$1.33\frac{1}{3}$ each, 8 lamps cost $\$8 + \$\frac{8}{3}$, or $\$10\frac{2}{3}$ ($10.67).

WRITTEN EXERCISE

1. At $12\frac{1}{2}$¢ each, find the cost of 147 balls.

2. At $16\frac{2}{3}$¢ each, find the cost of 113 notebooks.

3. At $33\frac{1}{3}$¢ each, find the cost of 101 handkerchiefs.

4. At $\$1.33\frac{1}{3}$ a volume, find the cost of 165 books.

5. At $\$1.16\frac{2}{3}$ a yard, find the cost of 142 yd. of lace.

6. At $\$1.12\frac{1}{2}$ each, find the cost of 112 yd. of velvet.

7. At $\$2.16\frac{2}{3}$ each, find the cost of 72 umbrellas.

8. What is the difference in cost of 144 books at $\$1.16\frac{2}{3}$ a volume, and the same number at $\$1.12\frac{1}{2}$ a volume?

WRITTEN REVIEW DRILL

Add the following:

1. $\frac{1}{2}+\frac{1}{3}$ $47\frac{1}{2}+93\frac{1}{3}$ $72\frac{1}{4}+83\frac{1}{6}$ $29\frac{3}{4}+47\frac{1}{5}$
2. $\frac{1}{5}+\frac{1}{3}$ $62\frac{1}{5}+35\frac{1}{3}$ $85\frac{4}{5}+16\frac{2}{3}$ $87\frac{5}{6}+19\frac{2}{3}$
3. $\frac{1}{2}+\frac{1}{6}$ $67\frac{1}{2}+29\frac{1}{6}$ $89\frac{3}{8}+23\frac{1}{2}$ $67\frac{4}{5}+49\frac{1}{2}$
4. $\frac{4}{5}+\frac{3}{4}$ $39\frac{4}{5}+28\frac{3}{4}$ $87\frac{3}{5}+62\frac{7}{10}$ $37\frac{4}{5}+18\frac{1}{10}$

Subtract the following:

5. $\frac{4}{5}-\frac{1}{2}$ $79\frac{4}{5}-22\frac{1}{2}$ $86\frac{2}{3}-37\frac{1}{2}$ $48\frac{3}{4}-23\frac{7}{8}$
6. $\frac{3}{4}-\frac{2}{5}$ $92\frac{3}{4}-37\frac{2}{5}$ $30\frac{2}{3}-19\frac{7}{8}$ $82\frac{1}{2}-68\frac{7}{8}$
7. $\frac{2}{5}-\frac{1}{8}$ $72\frac{2}{5}-56\frac{1}{8}$ $41\frac{3}{8}-16\frac{5}{6}$ $272-46\frac{2}{3}$

Multiply the following:

8. $27\times 14\frac{2}{3}$ $2\frac{1}{2}\times 92\frac{1}{2}$ $2\frac{1}{2}\times 3\frac{1}{4}$ $74\frac{2}{5}\times 65$
9. $18\times 96\frac{3}{4}$ $5\frac{1}{2}\times 68$ $4\frac{7}{8}\times 2\frac{1}{2}$ $40\frac{1}{3}\times 96$
10. $85\times 32\frac{2}{5}$ $1\frac{3}{4}\times 96$ $6\frac{3}{8}\times 4\frac{1}{2}$ $21\frac{7}{8}\times 35\frac{1}{2}$
11. $10\times 12\frac{1}{2}$ $6\frac{1}{3}\times 7\frac{1}{5}$ $5\frac{1}{3}\times 3\frac{1}{2}$ $27\frac{1}{5}\times 4\frac{1}{4}$
12. $65\times 9\frac{4}{5}$ $29\times 35\frac{1}{4}$ $5\frac{1}{4}\times 6\frac{3}{8}$ $42\frac{1}{2}\times 408\frac{2}{3}$

Divide the following:

13. $\frac{2}{3}\div\frac{3}{8}$ $\frac{3}{8}\div\frac{2}{3}$ $\frac{7}{8}\div\frac{4}{5}$ $2\frac{4}{5}\div\frac{7}{8}$
14. $\frac{5}{8}\div\frac{3}{4}$ $\frac{3}{4}\div\frac{5}{8}$ $\frac{3}{4}\div\frac{7}{8}$ $3\frac{3}{4}\div 5\frac{1}{2}$
15. $\frac{5}{8}\div 2\frac{1}{5}$ $\frac{5}{8}\div\frac{2}{3}$ $\frac{3}{8}\div 4\frac{3}{8}$ $7\frac{3}{8}\div 1\frac{3}{4}$
16. $\frac{2}{3}\div 3\frac{1}{3}$ $\frac{1}{3}\div\frac{2}{3}$ $\frac{4}{5}\div 3\frac{1}{2}$ $5\frac{5}{8}\div 1\frac{1}{8}$
17. $\frac{1}{5}\div\frac{1}{3}$ $\frac{1}{3}\div 1\frac{1}{5}$ $\frac{9}{10}\div 2\frac{1}{8}$ $2\frac{4}{5}\div 3\frac{1}{2}$

Find the value of:

18. $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 1 cu. ft. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 1 cu. in.
19. $\frac{1}{2}$ of 5×8 sq. ft. $\frac{2}{5}$ of 15×48 sq. ft.
20. $\frac{7}{8}$ of 4×16 sq. in. $\frac{3}{7}$ of 13×14 sq. in.

ORAL EXERCISE

1. What part of 4 is 2? What part of 10 is 8? How do you find what part one number is of another?

2. What part of 7 is 5? of 9 is 4? of 7 is 6? of 16 is 12? of 25 is $12\frac{1}{2}$? of 100 is $33\frac{1}{3}$? of 100 is $16\frac{2}{3}$?

95. Fractional Parts. From these answers we have:

To find what part the second of two numbers is of the first, divide the second by the first.

For example, what part of 51 is 17? 17 is $\frac{17}{51}$, or $\frac{1}{3}$, of 51.

WRITTEN EXERCISE

Find what part the second of these numbers is of the first:

1. 14, 8.
2. 18, 14.
3. $\frac{7}{8}$, $\frac{2}{3}$.
4. $\frac{22}{7}$, $\frac{6}{11}$.
5. $3\frac{1}{3}$, $1\frac{2}{3}$.
6. 17, $8\frac{1}{2}$.
7. 82, $9\frac{1}{9}$.
8. $41\frac{3}{5}$, $5\frac{1}{5}$.
9. $66\frac{2}{3}$, $16\frac{2}{3}$.
10. 243, 27.
11. 100, $37\frac{1}{2}$.
12. $45\frac{3}{5}$, $7\frac{3}{5}$.
13. $16\frac{5}{8}$, $2\frac{3}{8}$.
14. $42\frac{1}{2}$, $14\frac{1}{6}$.
15. $62\frac{1}{8}$, $7\frac{7}{8}$.
16. $66\frac{2}{3}$, $33\frac{1}{3}$.
17. 225, $37\frac{1}{2}$.
18. $22\frac{1}{2}$, $16\frac{2}{3}$.
19. $40\frac{1}{8}$, $21\frac{2}{5}$.
20. $37\frac{1}{2}$, $33\frac{1}{3}$.
21. 421, $60\frac{1}{7}$.
22. $33\frac{1}{3}$, $16\frac{2}{3}$.
23. 427, $47\frac{4}{9}$.
24. 325, $29\frac{6}{11}$.

25. Frank is 12 yr. old and his brother is 18 yr. old. Frank's age is what part of his brother's?

26. The age of Frank's brother, who is 18 yr. old, is what part of his uncle's, who is 54 yr. old?

27. If Frank is 12 yr. old and his brother is 18 yr. old, Frank's age was what part of his brother's 6 yr. ago? It will be what part of his brother's age 6 yr. from now?

ORAL EXERCISE

1. 10 is $\frac{1}{2}$ of what number? $\frac{1}{3}$ of what number?

2. 12 is $\frac{1}{3}$ of what number? If 12 is $\frac{2}{3}$ of some number, what is $\frac{1}{3}$ of the number? What is $\frac{3}{3}$ of the number?

3. If 15 is $\frac{3}{7}$ of some number, what is $\frac{1}{7}$ of the number? What is the number?

96. Given a Part of a Number to find the Number. If $33\frac{1}{3}$ is $\frac{2}{3}$ of some number, what is the number?

Because $\frac{2}{3}$ of some number $= 33\frac{1}{3} = \frac{100}{3}$,
therefore $\frac{1}{3}$ of the number $= \frac{1}{2}$ of $\frac{100}{3} = \frac{50}{3}$,
and $\frac{3}{3}$ of the number $= 3$ times $\frac{50}{3} = 50$.

Therefore $33\frac{1}{3}$ is $\frac{2}{3}$ of 50.

WRITTEN EXERCISE

Find the number of which:

1. 23 is $\frac{1}{3}$.
2. 38 is $\frac{2}{3}$.
3. 63 is $\frac{7}{8}$.
4. 75 is $\frac{3}{8}$.
5. 45 is $\frac{9}{10}$.
6. 150 is $\frac{3}{5}$.
7. 222 is $\frac{6}{7}$.
8. 125 is $\frac{25}{32}$.
9. 144 is $\frac{9}{16}$.
10. \$150 is $\frac{3}{5}$.
11. \$2.25 is $\frac{5}{8}$.
12. \$10.25 is $\frac{5}{9}$.

13. If there are 85 girls in a school, and they are $\frac{5}{9}$ of the total number of pupils, how many pupils are there?

14. In my library there are 164 books on history, and these are $\frac{2}{17}$ of all my books. How many books have I?

15. If a man spends \$1687.50 this year, and this is $\frac{3}{4}$ of his income, how much is his income?

16. A man left to his son $\frac{3}{5}$ of his estate, this share amounting to \$3210. How much was the estate worth?

97. Cancellation Extended. Since a fraction is an indicated division, *equal factors may be canceled from both dividend and divisor without changing the quotient.*

1. Divide $2 \times 7 \times 15$ by $21 \times 5 \times 6$.

$$\frac{\cancel{2} \times \cancel{7} \times \overset{\cancel{5}}{\cancel{15}}}{\underset{\cancel{3}}{\cancel{21}} \times \cancel{5} \times \underset{3}{\cancel{6}}} = \frac{1}{3}$$

Writing this as a fraction, 2 is canceled from 2 and 6; 7 is canceled from 7 and 21; 3 is canceled from 3 (left from the 21) and 15; 5 is canceled from 5 (left from the 15) and 5. Therefore the quotient is $\frac{1}{3}$, for evidently 1 is a factor of the numerator.

2. What is the product of $\frac{1}{15}$, $45\frac{1}{2}$, $33\frac{1}{3}$, and $16\frac{7}{8}$?

$$\frac{1 \times 91 \times \overset{25}{\cancel{100}} \times \overset{\overset{3}{\cancel{9}}}{\overset{\cancel{27}}{\cancel{135}}}}{\underset{\cancel{3}}{\cancel{15}} \times 2 \times \cancel{3} \times \underset{2}{\cancel{8}}} = \frac{6825}{4} = 1706\frac{1}{4}$$

Reducing to improper fractions, $45\frac{1}{2} = \frac{91}{2}$, $33\frac{1}{3} = \frac{100}{3}$, and $16\frac{7}{8} = \frac{135}{8}$. Proceeding as before, the product is $1706\frac{1}{4}$.

WRITTEN EXERCISE

Divide, using cancellation:

1. $7 \times 21 \times 77 \times 6$ by $49 \times 33 \times 42$.
2. $20 \times 25 \times 36 \times 48$ by $27 \times 40 \times 15 \times 12$.
3. $32 \times 27 \times 44 \times 39$ by $55 \times 72 \times 26 \times 8 \times 3$.
4. $125 \times 108 \times 51 \times 8$ by $40 \times 45 \times 85 \times 12 \times 3$.

Multiply, using cancellation:

5. $13\frac{1}{3} \times 32\frac{1}{5} \times 16\frac{7}{8}$.
6. $27\frac{1}{5} \times 24\frac{1}{2} \times 37\frac{1}{2}$.
7. $37\frac{1}{2} \times 22\frac{1}{4} \times 12\frac{4}{5}$.
8. $15\frac{3}{5} \times 16\frac{7}{8} \times 33\frac{1}{3}$.
9. $42\frac{2}{3} \times 66\frac{2}{3} \times 74\frac{1}{4}$.
10. $7\frac{3}{5} \times 2\frac{1}{7} \times 5\frac{1}{4} \times 21\frac{1}{3}$.
11. $6\frac{3}{4} \times 7\frac{3}{4} \times 9\frac{1}{3} \times 12\frac{1}{3}$.
12. $9\frac{3}{5} \times 6\frac{2}{3} \times 3\frac{1}{2} \times 15\frac{2}{3}$.

Multiply, using cancellation when possible:

13. $18 \times \frac{3}{4}$.
14. $27 \times \frac{5}{12}$.
15. $32 \times \frac{7}{8}$.
16. $48 \times \frac{3}{8}$.
17. $56 \times \frac{7}{12}$.
18. $\frac{3}{8}$ of 26.
19. $\frac{7}{8}$ of 34.
20. $\frac{3}{5}$ of 35.
21. $\frac{3}{8}$ of 52.
22. $\frac{5}{6}$ of 33.
23. $\frac{3}{8}$ of $\frac{4}{9}$.
24. $\frac{7}{8}$ of $\frac{3}{4}$.
25. $\frac{7}{8}$ of $\frac{4}{21}$.
26. $\frac{5}{6}$ of $\frac{3}{10}$.
27. $\frac{6}{7}$ of $\frac{7}{12}$.

Using cancellation when possible, find:

28. $\frac{2}{3}$ of $\frac{3}{8}$.
29. $\frac{5}{8}$ of $\frac{4}{9}$.
30. $\frac{3}{5}$ of $\frac{15}{16}$.
31. $\frac{4}{5}$ of $\frac{15}{16}$.
32. $\frac{5}{8}$ of $\frac{4}{15}$.
33. $\frac{3}{7}$ of $\frac{14}{9}$.
34. $\frac{6}{7}$ of $\frac{14}{9}$.
35. $\frac{5}{6}$ of $\frac{3}{16}$.
36. $\frac{5}{6}$ of $\frac{9}{20}$.
37. $\frac{5}{8}$ of $\frac{4}{15}$.
38. $\frac{1}{5}$ of $1\frac{2}{3}$.
39. $\frac{3}{5}$ of $6\frac{2}{3}$.
40. $\frac{7}{8}$ of $4\frac{4}{5}$.
41. $\frac{5}{6}$ of $4\frac{3}{8}$.
42. $\frac{5}{8}$ of $9\frac{3}{5}$.

Find the products of:

43. $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{4}{5}$.
44. $\frac{7}{8}$ of $\frac{4}{5}$ of $\frac{15}{14}$.
45. $\frac{5}{8}$ of $\frac{2}{5}$ of $\frac{4}{9}$ of $\frac{3}{5}$.
46. $\frac{3}{8}$ of $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{6}$.
47. $\frac{5}{8}$ of $1\frac{3}{5} \times 2\frac{1}{2} \times \frac{3}{5}$.
48. $\frac{7}{8}$ of $1\frac{1}{7} \times 3\frac{2}{3} \times \frac{3}{22}$.
49. $1\frac{2}{3} \times 3\frac{1}{5} \times 2\frac{1}{2} \times 5\frac{1}{4}$.
50. $1\frac{7}{8} \times 3\frac{1}{2} \times 2\frac{2}{3} \times 3\frac{1}{4}$.
51. $1\frac{2}{3} \times 2\frac{2}{5} \times 3\frac{1}{2} \times 4\frac{3}{5}$.
52. $2\frac{2}{5} \times 5\frac{1}{2} \times 10 \times 3\frac{3}{8}$.

Using cancellation when possible, find:

53. $14 \div \frac{7}{8}$.
54. $25 \div \frac{15}{16}$.
55. $\frac{3}{8} \div \frac{4}{5}$.
56. $\frac{5}{9} \div \frac{15}{16}$.
57. $\frac{3}{4} \div \frac{9}{10}$.
58. $70\frac{2}{3} \div 8$.
59. $\frac{5}{8} \div \frac{3}{16}$.
60. $\frac{4}{5} \div \frac{2}{3}$.
61. $\frac{3}{5} \div \frac{9}{10}$.
62. $\frac{7}{8} \div \frac{5}{16}$.
63. $\frac{15}{16} \div \frac{7}{8}$.
64. $\frac{3}{8} \div \frac{9}{10}$.
65. $\frac{5}{8} \div 25$.
66. $25 \div \frac{5}{8}$.
67. $3\frac{3}{5} \div 9$.
68. $9 \div 7\frac{1}{5}$.
69. $18 \div 4\frac{1}{2}$.
70. $12\frac{1}{2} \div 3\frac{1}{8}$.
71. $16\frac{2}{3} \div 4\frac{1}{6}$.
72. $37\frac{1}{2} \div 12\frac{1}{2}$.
73. $87\frac{1}{2} \div 37\frac{1}{2}$.

LITTLE EXAMINATIONS

I. 1. Reduce $62\frac{1}{2}$ to halves.
2. Reduce $\frac{27}{8}$ to units.
3. $\frac{2}{3} + \frac{1}{2} + \frac{1}{4}$.
4. $325\frac{1}{2} - 246\frac{7}{8}$.
5. $\frac{3}{8}$ of 3536.
6. $\frac{3}{4} \times \frac{4}{5}$.
7. $3\frac{1}{4} \div \frac{3}{4}$.
8. $8 \times \$1.25$.

II. 1. Reduce $7\frac{3}{4}$ to fourths.
2. Reduce $\frac{75}{15}$ to units.
3. $5\frac{1}{2} + \frac{3}{4} + 3\frac{1}{4}$.
4. $526\frac{1}{3} - 243\frac{1}{2}$.
5. $\frac{3}{4}$ of $\$124.48$.
6. $\frac{4}{5}$ of $12\frac{1}{2}$.
7. $2\frac{1}{2} \div \frac{3}{4}$.
8. $9 \times \$1.33\frac{1}{3}$.

III. 1. Reduce $9\frac{7}{8}$ to eighths.
2. Reduce $\frac{125}{25}$ to units.
3. $16\frac{2}{3} + 33\frac{1}{3} + 175$.
4. $426\frac{1}{4} - 342\frac{7}{8}$.
5. $\frac{2}{3}$ of $\$67.29$.
6. $\frac{3}{4}$ of $16\frac{1}{2}$.
7. $5\frac{1}{4} \div 2\frac{1}{2}$.
8. $75 \times 33\frac{1}{3}$.

IV. 1. Reduce $7\frac{2}{3}$ to thirds.
2. Reduce $\frac{19}{4}$ to units.
3. $7\frac{1}{2} + 9\frac{3}{8} + 4\frac{1}{4}$.
4. $3\frac{7}{8} + 6\frac{3}{4} + 3\frac{1}{2}$.
5. $507\frac{1}{3} - 239\frac{3}{4}$.
6. $\frac{5}{8}$ of $\$17.60$.
7. $\frac{4}{5}$ of 1475.
8. $2\frac{3}{4} \times 96$.
9. $8\frac{3}{4} \div 3\frac{1}{2}$.
10. $7 \times \$1.12\frac{1}{2}$.

V. 1. Reduce $8\frac{4}{5}$ to fifths.
2. Reduce $\frac{17}{3}$ to units.
3. $8\frac{3}{4} + 9\frac{3}{4} + 6\frac{7}{8}$.
4. $4\frac{2}{3} + 7\frac{3}{4} - 2\frac{1}{2}$.
5. $721\frac{3}{4} - 249\frac{7}{8}$.
6. $\frac{7}{8}$ of $\$72.96$.
7. $\frac{3}{16}$ of 3360.
8. $3\frac{4}{5} \times 750$.
9. $9\frac{7}{8} \div 2\frac{1}{2}$.
10. $6 \times \$7.33\frac{1}{3}$.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write in words the numbers 1,020,346 and MCDXCII.

2. Write in figures the number seven million, seven.

3. Add $4276 + 39{,}542 + 278 + 9463 + 48{,}278 + 9368$.

4. Add $\$48.72 + \$63.96 + \$8.75 + \$128.50 + \$2.35$.

5. From 401,324 subtract 96,849.

6. Find the difference between $742.25 and $96.79.

7. Multiply 43,298 by 5276.

8. Multiply $175.75 by 46.

9. Divide the sum of 27,960 and 39,290 by 250.

10. Divide 472,932 by 302, and 945,864 by 604.

11. Reduce to lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{4}{32}$, $\frac{8}{32}$, $\frac{16}{32}$, $\frac{32}{32}$.

12. How many fourths are there in 3? in $3\frac{1}{4}$? in $5\frac{3}{4}$?

13. Reduce $15\frac{3}{4}$ to fourths; to eighths.

14. Reduce to integers or mixed numbers: $\frac{25}{2}$, $\frac{37}{4}$, $\frac{31}{8}$.

15. Find the value of $3\frac{1}{2} + \frac{1}{4} + 2\frac{3}{8} + 4\frac{3}{4} + 1\frac{7}{8}$.

16. From $7\frac{1}{4}$ in. take the sum of $1\frac{1}{2}$ in. and $2\frac{7}{8}$ in.

17. Find $\frac{3}{4}$ of $7\frac{1}{2}$ in., and $\frac{5}{8}$ of $16\frac{1}{2}$ ft.

18. How many strips of molding $8\frac{3}{4}$ in. long can be cut from a strip 3 ft. long, and how much will be left over?

ORAL EXERCISE

1. How much will $2\frac{1}{2}$ lb. of coffee cost at 24¢ a pound?

2. How much will $\frac{3}{4}$ lb. of pepper cost at 24¢ a pound?

3. How much will $2\frac{1}{2}$ lb. of butter cost at 30¢ a pound?

4. How much will 5 oz. of cinnamon cost at 32¢ a pound?

5. How much will 20 lb. of loaf sugar cost at $6\frac{1}{4}$¢ a pound?

6. A grocer buys 240 cans of corn and sells half of them. How many cans of corn does he sell?

7. A merchant buys 200 yd. of cloth and sells a quarter of it. How many yards of cloth does he sell?

8. Find the cost of $1\frac{1}{2}$ lb. of cocoa at 22¢ per half-pound can.

WRITTEN EXERCISE

1. At $1.38 a yard, how much will 37 yd. of carpet cost?

2. What is the cost of $6\frac{1}{4}$ lb. of mackerel @ 16¢, $3\frac{1}{2}$ lb. of codfish @ 12¢, and 4 cans of salmon at the rate of 2 for a quarter?

3. I bought $\frac{1}{2}$ gal. of olive oil at $3.30 a gallon, but $\frac{1}{4}$ of what I bought was lost by leaking. What rate per gallon do I pay for what is left?

4. In buying 48 cans of tomatoes, how much is saved by buying two cases of 2 doz. cans each, at $2.89 a case, over buying at the rate of 3 cans for 40¢?

5. I bought 3 gal. of best maple sirup @ 1.12\frac{1}{2}$ a gallon and $2\frac{1}{2}$ gal. New Orleans molasses @ 62¢ a gallon, and gave the grocer a $5 bill. How much change did I get?

Teachers should send the pupils to local stores to ascertain prices, and these prices should be used in the above and similar problems.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. How do you subtract a number of three figures from a greater number when the units' figure to be subtracted is greater in value than the units' figure of the minuend, and the tens' figure to be subtracted is greater than the tens' figure of the minuend? Give your reasons.

2. How do you multiply by a number of three figures, the last two figures being zero? Give your reasons.

3. How do you divide by a number of three figures, the last two figures being zero? Give your reasons.

4. What do you mean by a power of a number? by the square of a number? by the cube of a number? Illustrate.

5. What is cancellation? What are its advantages? On what principle does cancellation depend?

6. If you know the cost of some land and the cost per acre, how do you find the number of acres?

7. If you know the weight of a cubic foot of water and the number of cubic feet of water in a cistern, how do you find the weight of the water in the cistern?

8. If you have the difference of two numbers and one of the two numbers given, how do you find the other number?

9. If you have the quotient of two fractions given and know the dividend, how do you find the divisor?

10. If you have the product of two fractions given and one of the fractions, how do you find the other fraction?

11. If you know the cost of a certain number of acres of land, how do you find the cost of a certain other number of acres?

III. INDUSTRIAL PROBLEMS

BRAINS PAY

1. Joseph's father has told him he could grow some tomatoes on shares, the father furnishing the land and the plants, and Joseph doing the work. Joseph was given the use of a rectangular plot of land 4 rd. by 5 rd. There being 160 sq. rd. to an acre, what part of an acre does Joseph have to cultivate?

2. Joseph reads that it is possible to average a crop worth $140 an acre for plants started in the most scientific way in a hotbed, but he found that this was the result of more nearly perfect conditions than he met in his garden. He got $\frac{4}{7}$ of this figure. How much did that amount to on his plot of ground?

3. Joseph was given half of the money received. How much was his share worth? How much would it have been worth if he had raised tomatoes at the rate of 6 T. to the acre, selling the tomatoes at $10 a ton? How much did he gain by using his brains?

GETTING A ROPE FOR A SWING

1. Do you know how heavy a rope is needed for a swing or a trapeze? Would it be an inch in diameter or a half inch? Well, it depends on whether it is a manila rope or a hemp rope, and on whether or not it is new. But we can find out nearly enough for our purposes by some rules that have been worked out. The weight needed to break a rope is called the breaking strength. For new manila rope we find the breaking strength in pounds by squaring the diameter (that is, by multiplying this number by itself) and then multiplying the result by 7200. For example, the breaking strength of new manila rope $\frac{3}{4}$ in. thick is found thus:

$$\tfrac{3}{4} \times \tfrac{3}{4} \times 7200 \text{ lb.} = 4050 \text{ lb.}$$

Find the breaking strength of $\frac{3}{16}$-inch and $\frac{3}{8}$-inch ropes.

2. But we do not wish to have the swing break with us, and all ropes are not equally well made, so we divide the answer by 6, a number called the factor of safety. If the breaking strength is 4050 lb., the safe load is $\frac{1}{6}$ of 4050 lb., or 675 lb. Find the safe loads for ropes of the diameters given in Ex. 1.

3. If you decide to use a $\frac{1}{2}$-inch new manila rope, what will be the breaking strength and the safe load? Is this sure to be enough for a good swing?

Although a person in a swing is supported by two ropes, he often swings by one rope only, so we must have this rope strong enough to hold if father or uncle want to swing in this way.

4. If we want the swing to be perfectly safe for two grown persons, will it be all right to use a $\frac{5}{8}$-inch new manila rope?

5. Hemp rope is three fourths as strong as manila rope. Find the safe loads for hemp ropes of diameters $\frac{3}{8}$ in. and $\frac{3}{4}$ in.

The figures on this page are, of course, approximate, for ropes differ in quality, but these figures are based on the best scientific investigations.

WHAT DO PEACHES COST?

1. Let us find the cost of a peach. If a peach grower has 15 acres in his orchard, and it rents for $6 an acre each year, how much does this amount to for 15 acres?

2. He has to have the trees pruned. If he has 367 hr. of work done at 20¢ an hour, how much must he pay for pruning?

3. The peach grower pays $1\frac{1}{4}$¢ a gallon for the spray and must use 100 gal. to an acre. How much does this cost him?

4. He must also have all the imperfect peaches picked off while they are young. He pays for 113 hr. of work at 15¢ an hour. How much does this add to his expense account?

5. The peaches have to be picked and packed carefully. Picking and packing require 964 hr. of work at 15¢ an hour. Find this part of the cost.

6. The other expenses amount to $34.50. Add this to the grower's expense account. How much does this make?

7. The peaches average 142 bu. to the acre. Find how many bushels the grower gets, and how much they actually cost him.

CHAPTER III

DENOMINATE NUMBERS

ORAL EXERCISE

1. How many ounces in a pound? An ounce is what part of a pound?

2. State the number of gills in a pint, the number of pints in a quart, and the number of quarts in a gallon.

3. State the number of inches in a foot and the number of feet in a yard. An inch is what part of a foot?

4. State the number of quarts in a peck and the number of pecks in a bushel. A peck is what part of a bushel?

5. How many seconds in a minute? minutes in an hour? hours in a day? days in a week? days in a common year? days in a leap year? What months have 30 days?

Express the following as indicated:

6. 6 pt. as quarts.
7. 14 da. as weeks.
8. $1\frac{1}{2}$ lb. as ounces.
9. 48 hr. as days.
10. 8 qt. as gallons.
11. 8 pk. as bushels.
12. 15 pt. as quarts.
13. 32 oz. as pounds.
14. 3 yd. as feet.
15. 12 ft. as yards.
16. 4 wk. as days.
17. $2\frac{1}{2}$ qt. as pints.

18. How many pints in a quart? in $1\frac{1}{2}$ qt.? in 2 qt.? in a gallon? in 2 gal.? in $2\frac{1}{2}$ gal.?

19. How many quarts in a peck? in 2 pk.? in a bushel? in 2 bu.? in $2\frac{1}{2}$ bu.? in 10 bu.? in 100 bu.?

WRITTEN EXERCISE

1. If John picked 12 qt. of berries to-day and half as many yesterday, how many quarts did he pick yesterday? How many quarts did he pick in the two days?

2. If Charles gathered 21 qt. of chestnuts and Will gathered $\frac{2}{3}$ as many, how many quarts did Will gather? How many quarts did they both gather?

3. If Mary lives $\frac{1}{2}$ mi. from school and Louise lives half as far, how far from school does Louise live?

4. If school opens at 9 A.M. and closes at 30 min. past 3 P.M., and there is $1\frac{1}{4}$ hr. recess at noon, how many hours are we in school to-day?

5. There being $5\frac{1}{2}$ yd. in a rod, how many yards in $17\frac{2}{3}$ rd.? how many yards in 32 rd.?

6. How much will $8\frac{3}{4}$ doz. pencils cost at the rate of 25¢ (\$$\frac{1}{4}$) for a half dozen?

7. At the rate of $3\frac{3}{4}$ lb. a week, how much sugar will a family use in the $4\frac{3}{7}$ weeks of January?

8. At the rate of $14\frac{7}{8}$ lb. of meat a week, how much meat will a family use in the $4\frac{2}{7}$ weeks of April?

9. How far will a commercial salesman travel in 6 da. at an average of $42\frac{1}{2}$ mi. a day?

10. An ocean steamer averaged $16\frac{1}{8}$ mi. an hour for $23\frac{1}{2}$ hr. What was the total distance?

11. Some blotting paper is $\frac{3}{64}$ in. thick. What will be the height of a pile of $2\frac{3}{4}$ doz. sheets?

12. If an arithmetic is $1\frac{1}{8}$ in. thick and a reader is $\frac{15}{16}$ in. thick, what is the height of a pile of 7 arithmetics and 9 readers?

13. At 28¢ a pound, how much will 4½ lb. of steak cost?

In this and the following examples estimate the approximate answer and write it down. After the problem is solved compare the approximate answer with the real one. This is one of the best checks against absurd results. Thus 5 × 28¢ is a reasonable approximation for 4½ × 28¢.

14. At 24¢ a pound, how much will 6¾ lb. of meat cost?

15. At 16¢ a pound, how much will 5¼ lb. of cheese cost?

16. At 20¢ a yard, how much will 18¾ yd. of calico cost?

17. At $6.50 a ton, how much will 5¾ tons of coal cost?

18. If 6 yd. of silk cost $6.75, what is the price per yard?

19. How many bushels of potatoes will 32 acres produce, if an average acre produces 125 bu.?

20. How many bushels of oats will 81 acres produce, if an average acre produces 33⅓ bu.?

21. How much will 250 doz. yards of millinery braid cost at 62½¢ a dozen yards? at 75¢ a dozen yards?

22. There are 44,000 yd. in 25 mi. How many yards are there in one mile?

23. If 25 tons of coal cost $150, what will be the cost of 11 tons? of 15 tons? of 32 tons? of 75 tons?

24. How long will it take an express train to go 362½ mi. at the rate of 50 mi. an hour?

25. A dealer pays $124 for some gingham at 25¢ a yard. How many yards of gingham does he buy?

26. A dealer pays $160 for some percaline at 25¢ a yard. How many yards of percaline does he buy?

27. If you buy 3⅓ doz. oranges at 40¢ a dozen, and 2 doz. bananas at 25¢ a dozen, how much change should you receive from $2?

98. Tables for Reference. While most of these tables have been learned and used in the Primary Book, they are here inserted for reference and for such review as may be necessary.

99. Table of Length. The following is the table of length:

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

The *hand* (4 in.) is used in measuring the height of horses at the shoulder. Sailors use the *fathom* (6 ft.) and the *cable length* (120 fathoms) for measuring depths, and the *knot* (nautical mile = $1\frac{3}{20}$ common, or statute, miles = $6080\frac{1}{4}$ ft.) for distances at sea.

1. Required to find the number of inches in 3 ft.

$$3 \text{ ft.} = 3 \times 12 \text{ in.} = 36 \text{ in.}$$

2. Required to find the number of feet in 27 rd.

$$27 \text{ rd.} = 27 \times 16\tfrac{1}{2} \text{ ft.} = 445\tfrac{1}{2} \text{ ft.}$$

ORAL EXERCISE

Find the number of inches in:

1. 5 ft.
2. 9 ft.
3. 10 ft.
4. $1\frac{1}{2}$ ft.
5. 1 yd.
6. 10 yd.
7. $\frac{1}{2}$ yd.
8. $\frac{1}{4}$ yd.

Find the number of feet in:

9. 2 yd.
10. 4 yd.
11. 7 yd.
12. 10 yd.
13. 1 rd.
14. 10 rd.
15. $5\frac{1}{2}$ yd.
16. $\frac{1}{2}$ yd.

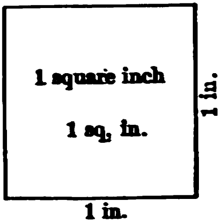
17. If you buy $\frac{1}{4}$ yd. of velvet, how many inches do you buy?

18. If a horse is 16 hands high, what is his height in inches? in feet and inches?

100. Table of Square Measure. The following is the table of square measure:

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Carpenters, architects, and mechanics often write 8″ for 8 in., and 5′ for 5 ft. They also use sq.″ and sq.′ for square inches and square feet. In this book both of these forms are used.

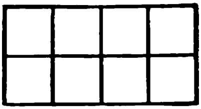


If a square is 1 in. on a side, it is 1 in. square, and its area is 1 sq. in.

If a rectangle is 4 in. long and 2 in. wide, its area is 4 × 2 sq. in., or 8 sq. in.

The pupils may express this in either of these two forms:

4×2 sq. in. = 8 sq. in.,
or 2×4 sq. in. = 8 sq. in.



In practical work we simply multiply 4 by 2, saying: "There are *as many* square inches as 2 × 4, or 8 sq. in."

WRITTEN EXERCISE

1. Find the number of square inches in the top of a desk whose area is 3 sq. ft.

2. The school platform has an area of 3 sq. yd. What is its area in square feet?

3. A flower bed has an area of 2 sq. rd. What is its area in square yards? in square feet?

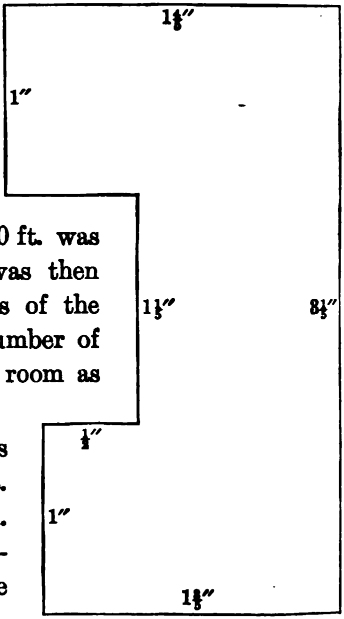
4. The school grounds contain half an acre. What is the area in square rods?

X

WRITTEN EXERCISE

1. A farm is plotted as shown below. The scale is 1 in. to 100 rd. Find the number of square rods and also the number of acres in the farm.

As stated in the Primary Book, to draw to the scale of 1 in. to 100 rd., we represent 100 rd. by 1 in. To draw to the scale $\frac{1}{12}$, we represent 12 in. by 1 in.



2. A rectangular meadow is plotted to the scale of 1 in. to 50 rd. The plot is 2 in. by 3⅕ in. Find the number of acres in the meadow.

3. A plan for a schoolroom drawn to the scale of 1 in. to 50 ft. was 1 in. long and $\frac{7}{10}$ in. wide. It was then decided to reduce the dimensions of the room one fifth. What was the number of square feet of floor space in the room as finally built?

4. A plan for a playhouse is drawn to the scale of 1 in. to 2½ ft. The plan is 6⅖ in. long and 4 in. wide, and the playhouse is rectangular. Compute the area of the playhouse.

5. The class has considered the question of a model kitchen in an apartment house. The pupils drew a plan to the scale $\frac{1}{60}$, representing the kitchen as 3 in. long and 2 in. wide. What was the floor space of the proposed kitchen?

6. In Ex. 5, what would be the floor space of the proposed kitchen if the scale were $\frac{1}{48}$?

WRITTEN EXERCISE

1. Measure this page of the arithmetic. Draw a plan of the page to the scale ¼.

2. Draw a plan of the same page to the scale ½.

3. Measure the inside of the cover of this arithmetic. It is a little larger than the page. Draw a plan to the scale ⅕.

4. Measure the top of your desk. Draw a plan to the scale of 1 in. to 1 ft.

5. Draw a plan of the top of your desk to the scale ⅙.

6. Measure the length and width of the schoolroom. Draw a plan and write below it the scale that you have used.

7. Measure the size of the paper upon which you make your drawings. Draw a plan to the scale ⅛.

8. Measure the size of any other book you are studying. Draw a plan of a page to the scale ⅕.

9. A drawing has been made of a box cover. The cover is 14 in. long and the drawing is 3½ in. long. What scale was used?

10. A drawing of a floor 24 ft. long and 20 ft. wide is 6 in. long and 5 in. wide. What scale was used?

11. A drawing is made of a rectangle. The drawing has an area of 6 sq. in. If a scale twice as large were used, what would be the area of the drawing? Draw a plan of the two rectangles, so as to show that the answer is correct.

12. A drawing is made of a lamp screen 18¾ in. high, and the drawing is 3⅛ in. high. What scale was used?

Teachers should have measurements made of desks, doors, blackboards, and other objects in the schoolroom, and scale drawings made from these measurements. The pupils should gradually come to know what the working drawings of the carpenter, the builder, and the manufacturer mean.

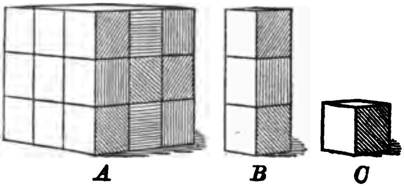
101. Table of Cubic Measure. The following is the table of cubic measure:

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

A *perch* of stone or masonry is usually considered as 1 rd. long, 1 ft. high, and 1½ ft. thick, and it contains 24¾ cu. ft. It varies, however, in different parts of the country. A cubic yard of earth is considered a *load.*

If a block is 1 in. on an edge, and is in the form of a cube, it is a 1-inch cube, and the volume is 1 cu. in., as in *C.*

If a block is 1 in. square at the base and 3 in. high, the volume is 3 cu. in., as in *B.*



If a block is 3 in. square at the base, the base contains 3 × 3 sq. in., or 9 sq. in. If the block is 3 in. high, the volume is 3 × 9 cu. in., or 27 cu. in., as in *A.*

If a block is 4 in. long, 3 in. wide, and 2 in. high, the volume is

2 × 3 × 4 cu. in. = 24 cu. in.

WRITTEN EXERCISE

Find the volumes of boxes of these dimensions:

1. 3 in., 4 in., 5 in.
2. 2 in., 5 in., 9 in.
3. 4 in., 7 in., 12 in.
4. 8 in., 12 in., 16 in.
5. 3½ in., 8 in., 9½ in.
6. 4 ft., 12½ ft., 14 ft.
7. 2½ ft., 3½ ft., 4 ft.
8. 2¾ ft., 3½ ft., 8 ft.
9. 32 in., 25 in., 10 in.
10. 48 in., 27 in., 10 in.
11. How many cubic feet in 8 perches of 24¾ cu. ft. each?

103. Table of Weight. The following is the table of weight:

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

The ton of 2000 lb. is sometimes called the *short ton*, there being a *long ton* of 2240 lb. which is used in some transactions in mining products.

Goldsmiths still use an old table of *troy weight*, and apothecaries and physicians use the table of *apothecaries' weight* in prescriptions of medicine, but neither is of enough importance to justify teaching it.

The avoirdupois pound contains 7000 grains, the troy pound (12 troy ounces) 5760 grains.

A *carat* weight, used in weighing diamonds, is (since 1913) 200 milligrams, or 3.08647 grains. The word "carat" is also used in speaking of the purity of gold and means $\frac{1}{24}$; 16 carats fine means $\frac{16}{24}$ pure gold.

ORAL EXERCISE

Find the number of ounces in:

1. $\frac{1}{2}$ lb. 2. $\frac{1}{4}$ lb. 3. $\frac{3}{4}$ lb. 4. $1\frac{1}{2}$ lb. 5. 2 lb.

Find the number of pounds in:

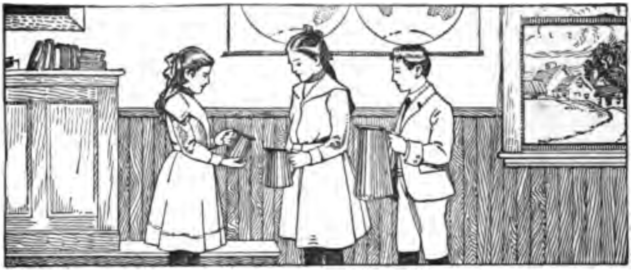
6. $\frac{1}{2}$ T. 7. $\frac{1}{4}$ T. 8. $\frac{3}{4}$ T. 9. $1\frac{1}{2}$ T. 10. 2 T.

WRITTEN EXERCISE

1. How many pounds in $2\frac{1}{2}$ T.? in $1\frac{3}{4}$ T.? in $3\frac{3}{4}$ T.?
2. How many ounces in $2\frac{1}{2}$ lb.? in $2\frac{3}{4}$ lb.? in $3\frac{1}{4}$ lb.?
3. How many ounces in 7 lb.? in $7\frac{1}{4}$ lb.? in $9\frac{3}{4}$ lb.?
4. How many grains are there in $\frac{1}{16}$ lb., or 1 oz.?
5. How many pounds in $2\frac{3}{4}$ T.? in $3\frac{1}{2}$ T.? in $5\frac{1}{4}$ T.?
6. How many pounds in 70 oz.? in 84 oz.? in 112 oz.?
7. How many tons in 2500 lb.? in 3000 lb.? in 4500 lb.?
8. How many ounces in $3\frac{3}{4}$ lb.? in $4\frac{1}{4}$ lb.? in $5\frac{1}{2}$ lb.?

103. Table of Liquid Measure. The following is the table of liquid measure:

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)



A gallon contains 231 cu. in. Barrels vary in size, although in estimating the capacity of tanks and cisterns, 31½ gal. are considered a barrel (bbl.), and 2 bbl. a hogshead.

There is also a table of *apothecaries' liquid measure,* in which 16 fluid ounces make 1 pint.

ORAL EXERCISE

1. If you need ½ pt. of cream for a recipe, how many gills do you need? What part of a quart do you need?

2. If you need 3 pt. of milk for some cooking, how many quarts do you need?

3. A quart is what part of a gallon? Does your answer show you why this measure is called a quart?

4. How many pints are there in a gallon? how many gills?

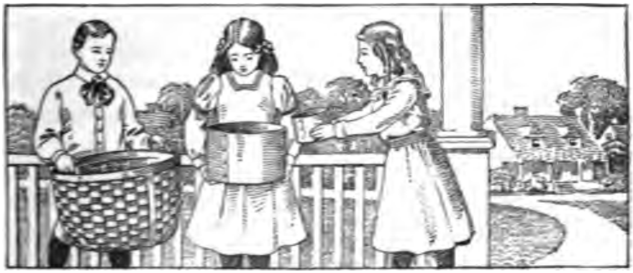
5. Name the various measures shown in the picture.

6. Name something that is sold by the pint; something that is sold by the quart; something that is sold by the gallon.

104. Table of Dry Measure. The following is the table of dry measure:

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A bushel contains nearly 2150$\frac{2}{5}$ cu. in., or about 1$\frac{1}{4}$ cu. ft. The *dry quart* contains 67$\frac{1}{5}$ cu. in. and the *liquid quart* contains 57$\frac{3}{4}$ cu. in.



ORAL EXERCISE

1. In the picture, point to the quart measure, the peck measure, and the bushel measure.

2. If you feed a horse 8 qt. of oats a day, how many days will 1 bu. of oats last?

3. If you have 2 bu. of shelled corn, how many days will this last your chickens if you feed them 1 qt. of corn a day?

4. How many quarts in 2 pk.? in 4 pk.?

5. How many pecks in 2 bu.? in 4 bu.? in 8 bu.?

6. How many quarts in 1 bu.? in 2 bu.? in 10 bu.?

7. How many pints in 3 qt.? in 8 qt.? in 1 pk.? in 2 pk.?

8. Name something that is sold by the bushel; something that is sold by the peck.

105. Table of Time. The following is the table of time:

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)

Thirty days has September,
April, June, and November.

The other months have 31 days, except February, which has 28 days in common years and 29 days in leap years. The *common year* has 365 days, or 52 weeks and 1 day; the *leap year* has 366 days. A *century* is 100 years.

For 45 minutes after 7 o'clock we usually write 7.45, 7 : 45, or 7 45, using A.M. and P.M. respectively to denote morning and afternoon.

Years whose numbers are divisible by 4 but not by 100, and those divisible by 400, are leap years. Thus 1916, 2000, but not 2100, are leap years.

106. Table of Value. The table of value is as follows:

10 mills = 1 cent (ct. or ¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)

The mill is not coined. The $10 gold piece is sometimes called an eagle.

WRITTEN EXERCISE

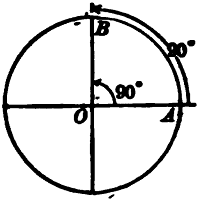
1. If you go to school 9½ mo. a year, and we allow 4 wk. to the month, how many weeks do you go to school each year?

2. If you are in school 5½ hr. a day for 5 da. each week, you are in school how many hours a week?

3. If you are in school 5½ hr. a day, how many minutes a day are you in school?

4. If you are saving 5-cent pieces to buy a 75-cent box of note paper, how many 5-cent pieces will you have to save?

107. Circular Measure. The circumference of a circle is measured in *degrees*.



There are 360 degrees (written 360°) in a circumference and 90° in a right angle. Because the angle at O, the center, increases just as fast as the part of the circumference between its arms increases, an angle is also measured in degrees.

108. Perpendicular Lines. A line that makes a right angle with another line is said to be *perpendicular* to it.

109. Arc. A portion of a circumference is called an *arc*.

110. Arc and Angle Measures. In measuring arcs,

360° = a circumference

That is, we divide the circumference, as stated above, into 360°.

In measuring angles we divide four right angles into 360°. That is,

360° = 4 right angles

In measuring both angles and arcs,

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)

ORAL EXERCISE

1. How many degrees in $\frac{1}{2}$ of a circumference?

2. If one line is perpendicular to another, what kind of angle do the two lines form? How many degrees in the angle?

3. Is an angle acute, right, or obtuse if it contains 100°? 60°? 90°? 120°? 1°?

4. A machinist wishes to put together two pieces of steel making an angle $\frac{1}{3}$ of a right angle. How many degrees is this?

111. Table of Dozens. In some kinds of business this table is used:

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gro.)

Formerly the term "great gross" was used for 12 gross, or 1728, and the word "score" was often used for "twenty."

ORAL EXERCISE

1. How many eggs are 3 doz. eggs? 4 doz. eggs?
2. How many oranges are $2\frac{1}{2}$ doz. oranges? $3\frac{1}{2}$ doz. oranges?
3. How many bananas are $1\frac{1}{4}$ doz. bananas?
4. How many pens in 2 gross? in 6 gross? in 8 gross?
5. How many dozen in $\frac{2}{3}$ of a gross? in $3\frac{1}{2}$ gross?
6. How much will a gross of padlocks cost at $5 a dozen?
7. How much will a gross of shears cost at $6 a dozen?
8. How much will $\frac{1}{2}$ gross of knives cost at $4 a dozen?

WRITTEN EXERCISE

1. How many are 7 gross? $9\frac{2}{3}$ gross? 16 gross?
2. How much will 9 gross of tablets cost at 25¢ a dozen?
3. At $7.20 a gross, how much will a dozen lamp chimneys cost? How much will $2\frac{1}{2}$ doz. cost?
4. A dealer bought 25 gross of blackboard erasers of which $\frac{1}{10}$ were damaged. How many were in good condition?
5. Which is cheaper, to buy a gross of articles for $125 or to buy 12 doz. at $11 a dozen? how much cheaper?
6. How much will $2\frac{1}{2}$ doz. knives cost at $100 a gross?
7. How much will 15 doz. saws cost at $200 a gross?

112. Counting Sheets of Paper. Formerly, sheets of paper were counted thus:

24 sheets = 1 quire

20 quires, or 480 sheets = 1 ream

Now, for convenience in counting, 500 sheets are often called a *ream*, and the word "quire" is used only for folded note paper, other paper being usually sold by the pound.

ORAL EXERCISE

1. How many sheets of fine writing paper in 2 quires? in 5 quires? in $3\frac{1}{2}$ quires? in 10 quires? in $\frac{1}{8}$ of a quire?

2. If writing paper costs 15¢ a quire, how much will 4 quires cost? How many sheets will there be?

Notice that 4 times 24 is the same as 4 times (25 − 1), or 100 − 4.

3. At 20¢ a quire, how much will 7 quires of paper, with $1 worth of envelopes, cost?

4. If a dealer buys paper at 19¢ a quire and sells it at 25¢ a quire, how much will he make on 10 quires?

WRITTEN EXERCISE

1. How many sheets in 17 quires? in $\frac{1}{3}$ of a quire?

2. In a dozen reams of 500 sheets each how many more sheets of paper are there than in a dozen reams of 480 sheets each?

3. At $16\frac{2}{3}$¢ a quire, how much will $7\frac{1}{2}$ quires of paper cost?

Remember the use of aliquot parts.

4. At $12\frac{1}{2}$¢ a quire, how much will $9\frac{1}{2}$ quires of paper cost? $6\frac{1}{2}$ quires? a dozen quires?

5. At 24¢ a quire for paper, and 1¢ each for envelopes, how much will it cost for paper, envelopes, and postage for eight letters from where you live to some other place in this country? Allow 1 sheet of paper to a letter.

113. Denominate Number. A concrete number that expresses measure is called a *denominate number.* For example, 3 ft., $5, and 3 bu. 2 pk. are denominate numbers, but 3 books and 5 chickens are not denominate numbers.

114. Compound Number. A denominate number in which two or more units are expressed is called a *compound number.* For example, 2 ft. 3 in. and 4 mi. 3 rd. 2 ft. are compound numbers.

115. Reduction. We may express 2 ft. 3 in. as 27 in., or as 2¼ ft. This changing of the form of a denominate number to another form without changing the value is called *reduction.*

116. Reduction Descending. Reduction from a higher to a lower denomination is called *reduction descending.* For example, 3 ft. may be reduced to 36 in.

117. Reduction Ascending. Reduction from a lower to a higher denomination is called *reduction ascending.* For example, 24 oz. may be reduced to 1 lb. 8 oz.

Simple cases of reduction, familiar from the work in primary arithmetic, have already been considered in the preceding pages.

Definitions like the above have little value, the subject of compound numbers having lost most of its former importance. We speak of "2 ft. 3 in." instead of "2¼ ft.," but we rarely use the pound and ounce *together*, usually speaking of "2 lb. 4 oz." as "2¼ pounds." But little time is now given to work in denominate numbers, particularly to work involving more than two denominations.

ORAL EXERCISE

1. Express 10 bu. as pecks; as quarts; as pints.
2. Express 64 qt. as pecks; as bushels; as pints.
3. Express 40 pt. as quarts; as gallons; as gills.
4. Express 20 ft. as inches; 21 ft. as yards; 21 yd. as feet.
5. Express 36 in. as yards; 36 ft. as yards; 30 yd. as feet.

118. Examples in Reduction Descending. 1. A strip of carpet is $7\frac{1}{4}$ yd. long. Is it long enough to reach the length of a hall that is 22 ft. long?

Here it is convenient to reduce $7\frac{1}{4}$ yd. to feet, or else to reduce 22 ft. to yards, so as to make the comparison.

$$\begin{array}{r} 7\frac{1}{4}\ \text{(yd.)} \\ 3 \\ \hline 21\frac{3}{4}\ \text{(ft.)} \end{array}$$

Because $\quad 1\text{ yd.} = 3\text{ ft.},$

we see that $\quad 7\frac{1}{4}\text{ yd.} = 7\frac{1}{4} \times 3\text{ ft.} = 21\frac{3}{4}\text{ ft.}$

While we think of multiplying 3 ft. by $7\frac{1}{4}$, we actually multiply $7\frac{1}{4}$ by 3, because this is shorter.

We therefore see that the carpet is too short by $\frac{1}{4}$ ft., or 3 in.

2. Reduce 7 rd. to inches.

$$\begin{array}{r} 16\frac{1}{2}\ \text{(rd.)} \\ 7 \\ \hline 115\frac{1}{2}\ \text{(ft.)} \\ 12 \\ \hline 1386\ \text{(in.)} \end{array}$$

Because $\quad 1\text{ rd.} = 16\frac{1}{2}\text{ ft.},$

we see that $\quad 7\text{ rd.} = 7 \times 16\frac{1}{2}\text{ ft.} = 115\frac{1}{2}\text{ ft.}$

Because $\quad 1\text{ ft.} = 12\text{ in.},$

we see that $\quad 115\frac{1}{2}\text{ ft.} = 115\frac{1}{2} \times 12\text{ in.} = 1386\text{ in.}$

In this case we find it easier to multiply $16\frac{1}{2}$ by 7 than to multiply 7 by $16\frac{1}{2}$.

Such an example is rare. We seldom find a practical case more difficult than Ex. 1. It is very unusual to find a case in which we have to reduce, say, 17 yd. 2 ft. 7 in. to inches.

WRITTEN EXERCISE

Reduce the following:

1. 8 ft. 2 in. to inches.
2. 2 rd. 3 ft. to feet.
3. 5 rd. 9 ft. to feet.
4. 2 lb. 2 oz. to ounces.
5. 2 gal. 3 qt. to quarts.
6. 10 cd. to cubic feet.
7. 3 rd. 5 ft. to feet.
8. 2 A. to square rods.
9. ~~5' 6" to inches.~~
10. 5 yd. 1 ft. to feet.
11. $2\frac{3}{4}$ A. to square rods.
12. $23\frac{1}{3}$ cu. yd. to cubic feet.

13. A carpenter saws a length of 7 ft. 11 in. from a board. How many inches does he saw off?

14. A lady is using 2 yd. 8 in. of cloth. How many inches of cloth is she using?

15. A dealer is putting 7 lb. 12 oz. of candy in boxes. How many ounce boxes can he fill with the candy?

16. A man puts 16½ gal. of gasoline in a tank. How many quarts of gasoline does he put in?

17. A milk dealer sells 2 gal. 2 qt. of cream. How many pints of cream does he sell?

18. A farmer has 16½ bu. of oats in his bin. How many pecks of oats has he? How many quarts?

19. A farmer has built 64 rd. 8 ft. of fence. How many feet of fence has he built?

20. If a pile of wood contains 17 cords, how many cubic feet of wood does it contain?

21. How many square rods are there in 1¼ acres?

22. A rug is 3 ft. 8 in. long and 1 ft. 7 in. wide. How many inches in the perimeter of the rug?

23. A strip of carpet is 19 yd. 2 ft. long. How many feet in the length of the strip?

24. In excavating a cellar 600 cu. yd. of earth are removed. How many cubic feet of earth are removed?

25. A piece of land is 12 rd. long and 7 rd. wide. How many square rods in the piece? How many square feet?

26. A man buys a load of coal weighing 1 T. 750 lb. How many pounds of coal does he buy?

As suggested on page 26, teachers should frequently call the attention of their pupils to the necessity of estimating an answer in advance of the solution.

119. Examples in Reduction Ascending. 1. If the milk dealer sells 200 pt. of cream in a week, how many gallons does he sell?

Here we have to reduce 200 pt. to gallons.

Because 1 pt. = $\frac{1}{2}$ qt.,
we see that 200 pt. = 200 × $\frac{1}{2}$ qt. = 100 qt.
Because 1 qt. = $\frac{1}{4}$ gal.,
we see that 100 qt. = 100 × $\frac{1}{4}$ gal. = 25 gal.

2) 200 (pt.)
4) 100 (qt.)
25 (gal.)

2. Reduce 175 in. to feet and inches.

Because 1 in. = $\frac{1}{12}$ ft.,
we see that 175 in. = 175 × $\frac{1}{12}$ ft.
= $14\frac{7}{12}$ ft. = 14 ft. 7 in.

12) 175 (in.)
14 (ft.)
7 (in.) rem.

It should be understood that these reductions are rather more difficult than those usually found in practical work.

WRITTEN EXERCISE

Reduce the following:

1. 96 in. to feet.
2. 48 oz. to pounds.
3. 176 oz. to pounds.
4. 3600 min. to hours.
5. 27 pk. to bushels.
6. 92 in. to feet.
7. 99 in. to yards.
8. 7200 sec. to hours.
9. 275 ft. to rods and feet.
10. 16,000 lb. to tons.
11. 527 pt. to bushels and pints; to quarts and pints.
12. 500 sq. in. to square feet and square inches.
13. 475 yd. to feet; to a fraction of a mile.
14. $\frac{3}{4}$ in. to a fraction of a foot; to a fraction of a yard.
15. $266\frac{2}{3}$ rd. to yards; to a fraction of a mile.
16. $6\frac{1}{2}$ in. to a fraction of a foot; to a fraction of a yard.
17. A woman uses 81 in. of ribbon. Express this as yards and inches; as yards and a fraction.

18. A carpenter uses 42 in. of molding. Express this as feet and inches; as feet and a fraction; as yards and inches.

19. A milk dealer sells 302 qt. of milk. Express this as gallons and quarts; as gallons and a fraction.

20. A milk dealer sells 73 pt. of cream. Express this as quarts and pints; as quarts and a fraction; as gallons and pints; as gallons and a fraction.

21. A dealer sells 137 qt. of gasoline. Express this as gallons and quarts; as gallons and a fraction.

22. A boy is running a half-mile race. After he has run 660 yd. what fraction of the distance has he run?

23. Express 775 in. as feet; as yards.

24. Express 32 in. as feet; 32 oz. as pounds; 32 ft. as yards; 32 pt. as quarts; 32 qt. as gallons.

25. Express 96 oz. as pounds; 96 in. as yards and a fraction; 96 ft. as yards; 96 min. as hours and minutes.

26. If you need $9\frac{1}{2}$ yd. of cloth for a garment, and have 72 in. already, how many yards will you have to buy?

27. If you cut 86 sq. in. from a piece of cloth containing $2\frac{1}{2}$ sq. ft., how many square feet and square inches are left?

28. If you cut off $\frac{5}{8}$ of a piece of tin containing 3 sq. ft., how many square inches do you cut off?

29. From a sheet of tin 3 ft. long and $2\frac{1}{2}$ ft. wide a piece is cut containing 500 sq. in. How many square feet and square inches are left?

30. From an iron plate 2 ft. 6 in. by 7 ft. 6 in. a piece is cut containing 108 sq. in. How many square feet are left?

31. If from a piece of glass 4 ft. 6 in. long by 3 ft. wide a strip 1 in. wide is cut from each side, how many square inches are left?

120. Addition of Compound Numbers. The general principles of adding compound numbers are similar to those of adding ordinary whole numbers. The only difference is that in compound numbers the number of units of a lower denomination required to make one unit of the next higher varies with the denomination we are using.

In our school shop we wished to make a picture frame 2 ft. 8 in. long and 1 ft. 6 in. wide. How much molding do we need?

There are four sides to the frame, these being 2 ft. 8 in., 2 ft. 8 in., 1 ft. 6 in., and 1 ft. 6 in.

We first add the inches and find the sum to be 28 in., or 2 ft. 4 in. We write the 4 under the inches, and add the 2 with the feet.

Practically we would see that 6 in. + 6 in. = 1 ft., and 8 in. + 8 in. = 1 ft. 4 in.

Adding the feet, we have 8 ft.

Hence the result is 8 ft. 4 in.

ft.	in.
2	8
2	8
1	6
1	6
8	4

In practical life we rarely meet cases more difficult than those given in the following examples.

WRITTEN EXERCISE

Add the following:

1.

ft.	in.
5	8
9	10
22	6

2.

sq. ft.	sq. in.
94	30
82	85
71	63

3.

yd.	in.
11	17
16	20
35	19

4.

lb.	oz.
148	8
862	13
421	9

5.

ft.	in.
28	11
17	9
35	5

6.

sq. ft.	sq. in.
38	48
56	84
93	57

7.

rd.	ft.
26	13
47	9
56	11

8.

lb.	oz.
385	13
421	12
635	9

121. Subtraction of Compound Numbers. The general principles of subtracting compound numbers are quite similar to those of subtracting ordinary whole numbers.

From a board 9 ft. 2 in. long Robert wishes to cut a piece 3 ft. 8 in. long for a table top which he is making. How much will he have left?

ft.	in.
9	2
3	8
5	6

Since we cannot subtract 8 in. from 2 in., we take 1 ft., which equals 12 in., of the 9 ft. and add it to the 2 in. We then have 14 in. − 8 in. = 6 in.

We now have 8 ft. left of the 9 ft.

Then 8 ft. − 3 ft. = 5 ft.

Therefore Robert has left a board 5 ft. 6 in. long.

In practical work we so rarely have to subtract numbers of more than two denominations that in this book, which gives only the essentials of arithmetic, all other cases are excluded.

WRITTEN EXERCISE

Subtract the following:

1.		2.		3.		4.	
ft.	in.	yd.	ft.	yd.	in.	gal.	qt.
62	8	32	2	16	3	32	1
15	10	18	1	9	5	18	3

5.		6.		7.		8.	
ft.	in.	bu.	qt.	rd.	ft.	A.	sq. rd.
91	$4\frac{1}{2}$	31	23	37	2	64	75
38	$8\frac{1}{4}$	17	17	19	15	43	119

9.		10.		11.		12.	
bu.	pk.	qt.	pt.	lb.	oz.	sq. ft.	sq. in.
61	1	71	0	35	8	96	72
37	2	39	1	27	13	38	107

122. Multiplication of Compound Numbers. The general principles of multiplying compound numbers are similar to those of multiplying ordinary whole numbers.

1. A wholesale grocer is packing English walnuts in fancy boxes, 4 lb. 9 oz. to the box. He sends them out in cases of a half dozen boxes. How many pounds of walnuts in six of these boxes?

lb.	oz.
4	9
	6
27	6

Here we have to multiply 4 lb. 9 oz. by 6.

We might multiply as with other numbers and obtain 24 lb. 54 oz., and reduce this to 27 lb. 6 oz. But it is easier to say, "6 × 9 oz. = 54 oz. = 3 lb. 6 oz.; 6 × 4 lb. = 24 lb., which added to 3 lb. = 27 lb."

2. Multiply 243 ft. 7 in. by 27.

ft.	in.
243	7
	27
6561	189
6576	9

Multiplying as with other numbers, the result is 6561 ft. 189 in., which is reduced to 6576 ft. 9 in.

In this case it is not easy to reduce while we are multiplying, as we did in Ex. 1. We therefore multiply as with whole numbers and reduce the product to a simpler form.

The tendency at present is to avoid the multiplication of compound numbers except in very simple cases, chiefly those involving feet and inches. Instead of multiplying 5 lb. 4 oz. by 8, we multiply 5¼ lb. by 8, and similarly in other cases.

ORAL EXERCISE

Multiply the following:

1. 3 ft. 4 in. by 2.
2. 7 ft. 2 in. by 4.
3. 8 ft. 1 in. by 9.
4. 9 yd. 6 in. by 3.
5. 8 yd. 3 in. by 8.
6. 6 yd. 2 in. by 7.
7. 8 lb. 2 oz. by 6.
8. 7 rd. 1 ft. by 8.
9. 11 lb. 3 oz. by 4.
10. 9 gal. 1 qt. by 3.
11. 7 qt. 1 pt. by 8.
12. 15 lb. 4 oz. by 4.
13. 17 lb. 1½ oz. by 10.
14. 20 rd. 6 ft. by 12.

WRITTEN EXERCISE

Multiply the following:

1. 4 ft. 6 in. by 5.
2. 4 yd. 2 in. by 6.
3. 9 ft. 3 in. by 4.
4. 4 lb. 4 oz. by 9.
5. 6 yd. 3 in. by 8.
6. 7 lb. 15 oz. by 4.
7. 8 lb. 13 oz. by 6.
8. 7 lb. 11 oz. by 9.
9. 6 ft. 8 in. by 7.
10. 7 ft. 11 in. by 4.
11. 5 ft. 10½ in. by 2.
12. 4 yd. 27 in. by 3.
13. 6 yd. 33 in. by 9.
14. 27 lb. 3 oz. by 4.
15. 4 mi. 3 ft. by 8.
16. 73 ft. 2 in. by 5.
17. 39 lb. 5 oz. by 7.
18. 30 yd. 1 in. by 8.
19. 3 gal. 2 qt. by 2.
20. 8 gal. 2 qt. by 9.
21. 6 gal. 3 qt. by 7.
22. 4 mi. 113 rd. by 3.
23. 6 mi. 109 rd. by 7.
24. 5 mi. 1250 ft. by 3.

25. If each side of a five-sided figure (pentagon) is 1 ft. 6½ in., what is the perimeter?

26. If each side of a six-sided figure (hexagon) is 1 ft. 7⅔ in., what is the perimeter?

27. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a tank that contains 48 cu. ft.?

28. If the height of each story above the first, or ground, story in a city office building is 11 ft. 9 in., and the height of the first story is 15 ft. 8 in., and there are nine stories in all, what is the height of the building?

29. A man travels in an automobile 6 da. a week for 3 wk. The first week he averages 8 gal. 2 qt. of gasoline a day, the second week 7 gal. 3 qt., and the third week 8 gal. 1 qt. How much gasoline does he use in the three weeks?

123. Division of Compound Numbers. If you have a board 7 ft. 6 in. long, and cut it into two equal parts to make a top for a workshop bench, how long is each part?

Here we have to divide 7 ft. 6 in. by 2.

We see that 7 ft. ÷ 2 = 3 ft. with remainder 1 ft., or 12 in.

Then we have 12 in. + 6 in., or 18 in., still to be divided.

18 in. ÷ 2 = 9 in.

Therefore the result is 3 ft. 9 in.

$$\begin{array}{rrr} & \text{ft.} & \text{in.} \\ 2) & 7 & 6 \\ \cline{2-3} & 3 & 9 \end{array}$$

The teacher will recognize the fact that we seldom have to divide compound numbers, and that when a case arises it usually involves only two denominations.

It is very rare to find a practical case in which we need to divide one compound number by another. If, for example, we have to divide 41 ft. 3 in. by 2 ft. 9 in., we simply reduce both to feet (41¼ ft. ÷ 2¾ ft. = 15) or to inches (495 in. ÷ 33 in. = 15). In this book these unreal cases have no place.

WRITTEN EXERCISE

1. The perimeter of an equilateral (equal-sided) triangle is 11 ft. 4½ in. Find the length of each side.

2. The perimeter of a square is 6 ft. 10 in. Find the length of each side.

3. A steamship changed its longitude 59° 37′ in 7 da. How many degrees of longitude did it sail per day?

4. If you are in school 5 hr. 30 min. a day, and recite half the time, how many hours and minutes do you recite?

Divide as follows:

5. 75 ft. 9 in. ÷ 3.
6. 91 ft. 8 in. ÷ 5.
7. 7 lb. 8 oz. ÷ 10.
8. 6 lb. 3 oz. ÷ 11.
9. 2 mi. 120 rd. ÷ 38.
10. 20 hr. 12 min. ÷ 6.
11. 17 wk. 6 da. ÷ 12½.
12. 36 hr. 59 min. ÷ 7.
13. 143 ft. 4 in. ÷ 8.
14. 39 lb. 12 oz. ÷ 12.

1 **15.** At the rate of a mile in 4 min., how far will an automobile go in 3 hr.? in 3½ hr.?

2 **16.** How far will a horse go in 3 hr., traveling at the rate of a mile in 9 min.?

3 **17.** If a wheel is $10\frac{1}{2}\frac{8}{5}$ ft. in circumference, how many times will it turn in going 25 mi.?

4 **18.** A square plot of ground has a perimeter of 510 ft. 8 in. Find the length of each side.

5 **19.** A cistern capable of holding 464 cu. ft. of water is ⅓ full. Express the amount of water in cubic feet and cubic inches.

6 **20.** At 75 lb. to the acre, how many acres of land can be enriched with 1687½ lb. of fertilizer?

7 **21.** A woman buys 3¼ lb. of coffee for $1.04. How much does she pay per pound?

8 **22.** If a man walks at the rate of a mile in 19½ min., how many miles can he walk in 39 min.?

23. If a cubic foot of gold weighs 1187 lb. 8 oz., and if gold is 19 times as heavy as water, how much does a cubic foot of water weigh?

24. If it takes a man 8 hr. 36 min. to walk a certain distance, how long will it take an automobile to travel the same distance if it goes 12 times as fast?

25. A man can row across a lake 1¾ mi. wide in 28 min. At this rate, how long will it take him to row 1 mi.?

26. A man can row 1 mi. in 20 min. At this rate, how long will it take him to row across a lake 1¾ mi. wide?

27. On a trip of 3 mi. an automobile took 2 min. 34 sec. for the first mile, 2 min. 17 sec. for the second, and 1 min. 56 sec. for the third. What was the average rate per mile?

WRITTEN REVIEW

1. How many feet are there in $\frac{1}{8}$ mi.? in $\frac{1}{4}$ mi.?

2. How many yards are there in $\frac{3}{8}$ mi.? in $\frac{3}{4}$ mi.?

3. How many yards are there in 5 rd.? in $7\frac{1}{2}$ rd.?

4. How many yards are there in 79 ft.? in 113 in.?

5. If a river is $7\frac{1}{2}$ fathoms deep, what is its depth in feet?

6. Calling $1\frac{3}{20}$ mi. a knot, how many miles in 160 knots?

7. How many square inches in $7\frac{1}{2}$ sq. ft.?

8. How many square feet in 2 sq. yd. 7 sq. ft.?

9. How many cubic feet in 49 cu. yd. 7 cu. ft.?

10. A hand is 4 in. If a horse is 15 hands high, what is its height in inches? What is its height in feet?

11. Taking $24\frac{3}{4}$ cu. ft. as a perch, how many perches in 99 cu. ft. of stone?

12. Taking, as is often done, 25 cu. ft. as a perch, how many cubic feet in $17\frac{1}{2}$ perches of stone?

13. A gallon contains 231 cu. in. How many gallons of water will it take to fill an aquarium containing $1\frac{1}{2}$ cu. ft.?

14. Allowing $31\frac{1}{2}$ gal. to the barrel, how many gallons of water will it take to fill a tank that holds $16\frac{1}{2}$ bbl.?

15. Allowing $2150\frac{1}{2}$ cu. in. to the bushel, how many cubic inches in a bin that holds 16 bu.?

16. Allowing $67\frac{1}{5}$ cu. in. to the dry quart, how many cubic inches in a peck? How many cubic inches in a pint?

17. Allowing $2150\frac{1}{2}$ cu. in. to the bushel, how many cubic feet in a bin that holds 3456 bu.?

18. Allowing $67\frac{1}{5}$ cu. in. to the dry quart, how many quarts of oats will it take to fill a box that contains $604\frac{4}{5}$ cu. in.?

19. Allowing $1\frac{1}{4}$ cu. ft. to a bushel, how many bushels of wheat can be put into a bin that will hold 900 cu. ft.?

20. A dry quart contains $67\frac{1}{5}$ cu. in. How many quarts in a measure containing 336 cu. in.?

21. A railway train goes $63\frac{9}{10}$ mi. in an hour and a half. What is its rate per hour?

22. If a man walks at the rate of a mile in $19\frac{1}{2}$ min., how many miles can he walk in 1 hr. $37\frac{1}{2}$ min.?

23. A woman buys $15\frac{1}{2}$ yd. of carpet for $7.44. What does she pay per yard?

24. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a cistern that contains 464 cu. ft.?

25. How many pounds do 5 cu. yd. 18 cu. ft. of earth weigh, if a cubic yard weighs 1 T. 825 lb.?

26. A man can row across a lake, $2\frac{3}{4}$ mi., in 42 min. At this rate, how long will it take him to row 1 mi.?

27. A certain horse is 14 hands high. Express this in inches; in feet and inches; in feet and a fraction.

28. A cubic foot of water weighs nearly 1000 oz. Express this weight in pounds and ounces, and in pounds and a fraction of a pound.

29. On a trip of $2\frac{1}{2}$ mi. an automobile took 2 min. 16 sec. for the first mile, 2 min. 10 sec. for the second, and 1 min. 6 sec. for the last $\frac{1}{2}$ mi. What was the average rate per mile?

30. If a cubic foot of ice weighs 57 lb. 8 oz., what will be the weight in tons of the mass of ice 1 ft. thick that covers a pond whose surface is 100,000 sq. yd.?

31. If you are in school 5 hr. a day, how many minutes are you in school during the 5 da. of the week? how many seconds?

Add the following:

32.		**33.**		**34.**		**35.**	
ft.	in.	ft.	in.	ft.	in.	ft.	in.
127	4	329	7	635	7	246	11
286	3	206	8	82	6	321	9
491	7	129	11	39	10	480	7
302	5	43	5	41	11	39	10

36.		**37.**		**38.**		**39.**	
yd.	in.	gal.	qt.	bu.	pk.	bu.	pk.
278	10	27	1	427	1	171	1
63	13	92	3	63	1	26	3
49	17	167	1	29	3	48	1
326	5	43	2	28	2	39	2

Subtract the following:

40. 9½ lb. – 9 oz.
41. 3⅔ ft. – 8 in.
42. 8⅔ rd. – 5 yd.
43. 7½ bu. – 3 pk.
44. 480 ft. 2 in. – 197 ft. 10 in.
45. 293 lb. 5 oz. – 169 lb. 12 oz.
46. 402 yd. 5 in. – 327 yd. 12 in.
47. 572 bu. 1 pk. – 296 bu. 3 pk.

Multiply the following:

48. 102 lb. 1 oz. by 42.
49. 249 lb. 2 oz. by 32.
50. 243 ft. 2 in. by 75.
51. 63 lb. 2 oz. by 27.
52. 78 lb. 9 oz. by 16; by 32.
53. 329 ft. 7 in. by 96; by 48.

Divide the following:

54. 87 lb. 8 oz. by 4.
55. 648 gal. by 4 gal.
56. 302 lb. 8 oz. by 2.
57. 125 ft. 10 in. by 5.
58. 95 lb. 1 oz. by 3.
59. 981 gal. by 9 gal.
60. 429 lb. 4 oz. by 4; by 8.
61. 425 ft. 5 in. by 25; by 12½.

LITTLE EXAMINATIONS

I. **1.** $72\frac{1}{2}$ ft. = (?) in.
2. 75 in. = (?) ft.
3. $2\frac{1}{2}$ lb. = (?) oz.
4. 36 qt. = (?) pk.
5. $8\frac{1}{4}$ ft. − 9 in.
6. $2\frac{1}{2} \times 3\frac{1}{2}$ ft.
7. 3 × 7 ft. 9 in.
8. $72\frac{1}{2}$ ft. ÷ 3.

II. **1.** $39\frac{3}{4}$ ft. = (?) in.
2. 96 in. = (?) yd.
3. $6\frac{1}{2}$ pk. = (?) qt.
4. 36 sq. ft. = (?) sq. yd.
5. $2\frac{1}{2}$ yd. − 8 in.
6. $3\frac{7}{8} \times 6\frac{3}{4}$ ft.
7. 8 × 4 bu. 2 qt.
8. $43\frac{3}{4}$ ft. ÷ 6.

III. **1.** $16\frac{1}{2}$ ft. = (?) yd.
2. 1280 A. = (?) sq. mi.
3. $3\frac{3}{4}$ T. = (?) lb.
4. 7 sq. ft. = (?) sq. in.
5. $3\frac{1}{2}$ lb. + 9 oz.
6. $5\frac{3}{4} \times 7\frac{1}{2}$ lb.
7. 6 × 8 lb. 6 oz.
8. $7\frac{1}{2}$ yd. ÷ 9.

IV. **1.** 273 ft. = (?) yd.
2. $3\frac{1}{2}$ sq. mi. = (?) A.
3. $62\frac{1}{2}$ gal. = (?) qt.
4. 35 bu. = (?) pk.
5. $2\frac{1}{2}$ cu. ft. = (?) cu. in.
6. $6\frac{1}{2}$ hr. + 15 min.
7. $6\frac{3}{4}$ bu. − 3 pk.
8. $6\frac{3}{8} \times 8\frac{1}{2}$ bu.
9. 9 × 4 yd. 6 in.
10. 6 lb. 3 oz. ÷ 7.

V. **1.** $32\frac{1}{2}$ yd. = (?) in.
2. 256 cu. ft. = (?) cd.
3. $7\frac{1}{2}$ gal. = (?) pt.
4. $2\frac{3}{4}$ hr. = (?) min.
5. 96 gi. = (?) gal.
6. $2\frac{1}{2}$ yd. + 7 ft.
7. 16 ft. − 10 in.
8. $7\frac{3}{4} \times 8\frac{1}{2}$ mi.
9. 7 × 6 gal. 3 qt.
10. 7 rd. 3 ft. ÷ 5.

REVIEW AND DRILL

I. DRILL SECTION

ORAL EXERCISE

Add the following:

1.	2.	3.	4.	5.	6.	7.
42	48	79	58	38	44	78
63	13	32	62	26	29	37

In adding 49 and 65 think of 49 + 60 + 5 = 109 + 5 = 114. In all such cases it is easier and more rapid to begin at the left to add.

Make change from $1, having made purchases as follows:

8. 24¢ 58¢ 74¢ 38¢ 87¢ 49¢ 59¢

9. 35¢ 62¢ 85¢ 76¢ 92¢ 63¢ 82¢

10. Multiply by 25: 24, 48, 56, 16, 64, 32, 84.

11. Multiply by 50: 16¢, 20¢, 34¢, 42¢, 66¢, 28¢, 84¢.

12. Add $\frac{1}{2}$ to 2; to $1\frac{1}{2}$; to $2\frac{1}{2}$; to $1\frac{1}{4}$; to $1\frac{3}{4}$; to $\frac{1}{3}$.

13. Multiply by $2\frac{1}{2}$: 6, 10, 12, 24, 16, 18, 20.

14. Express the following in the next lower denominations:

5 yd. 8 hr. 5 wk. 9 gal. 25 qt. 6 lb. 2 A.

15. Express the following in the next higher denominations:

48 mo. 64 oz. 32 qt. 21 da. 56 wk. 48 pt. 33 ft.

16. Give the statement from the table of denominate numbers that is suggested by each of the following numbers:

160 320 27 128 8 16 9

WRITTEN EXERCISE

Copy, add, and check the following in eight minutes or less:

1.	2.	3.	4.
$359.68	$496.48	$396.04	$982.69
298.76	127.92	72.63	604.05
400.92	600.00	129.67	81.97
640.32	503.52	871.64	631.83
701.24	872.08	280.93	400.00

Multiply the first number by each of the two numbers below it:

5.	6.	7.	8.	9.
28,764	84,216	12,987	18,625	16,284
345	974	317	315	708
678	287	526	475	354

10. In Exs. 5 and 6, divide the first number by each of the two numbers below it.

11. In Exs. 7–9, divide the first number by each of the two numbers below it.

Multiply the following:

12. $27 \times \$42.30$.
13. $40 \times \$78.20$.
14. $6\frac{1}{2} \times \$47.50$.
15. $112 \times \$1.12\frac{1}{2}$.
16. $87 \times 32\frac{2}{3}$.
17. $72 \times \$2.16\frac{2}{3}$.
18. $165 \times \$1.33\frac{1}{3}$.
19. $48 \times \$1.66\frac{2}{3}$.
20. $56 \times \$1.37\frac{1}{2}$.

Perform the following operations:

21. $6\frac{7}{8} \times 6\frac{3}{4} \times 10\frac{2}{3}$.
22. $15\frac{2}{3} \times 33\frac{1}{3} \times 16\frac{7}{8}$.
23. $3\frac{1}{2} + 16\frac{2}{3} + 48\frac{3}{4}$.
24. $92\frac{1}{8} - 37\frac{3}{4} - 2\frac{1}{2}$.

25. Reduce the following to sixteenths:

$\frac{3}{8}$ $\frac{3}{4}$ $\frac{7}{8}$ $\frac{12}{64}$ $\frac{24}{32}$ $\frac{1}{2}$ $\frac{20}{64}$ $\frac{16}{32}$

WRITTEN EXERCISE

1. The title-page of an old book has the date MDCXCIV. In what year was the book printed?

2. Make out a cash account as follows and balance it: Jan. 1, cash on hand, $3.40. Receipts: Jan. 15, shoveling snow, 50¢; Jan. 20, delivering papers for the week, 85¢. Expenses: Jan. 3, repairing skates, 10¢; Jan. 3 to 29, street-car tickets, 45¢. Balanced on Feb. 1.

3. Write the tables of length and of square measure.

4. In a certain city there are eight blocks to a mile. If a boy lives ten blocks from the schoolhouse, how many rods does he go in walking from his home to the school?

5. Without any written computation, write the answer to this question: What will 64 gramophones cost at $25 each?

6. Mt. Whitney lacks 1339 ft. of being 3 mi. high. What is its height in feet? in yards?

7. If each book is 1¼ in. thick, how many books in a pile 30 in. high? in a pile 25 in. high?

8. Find the cost of 112 yd. of silk at $1.12½ a yard.

9. If a train travels a mile in 1¼ min., how far will it travel in 6¼ min.? in 25 min.? in 1 hr. 2½ min.?

10. Measure the cover of your geography and draw a plan of it on the scale of ¼.

11. Which is cheaper, to buy a gross of articles for $250 or to buy 12 doz. at $22 a dozen? How much cheaper?

12. How many feet of fence will be needed to inclose a field 64 rd. 8 ft. long, and 42 rd. wide?

13. When a man is rowing at the rate of 1¾ mi. in 28 min., what is his rate per minute?

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

State the units used in measuring the following:

1. The length of dress goods; the width.

2. The quantity of a liquid.

3. The quantity of a dry substance, like grain.

4. Sugar, in buying it; in cooking.

5. Rice, in buying it; in cooking.

6. Tea, in buying it; in making it for the table.

7. Cocoa, in buying it; in making it for the table.

8. Salt, in buying it; in cooking.

9. Kerosene, in large quantities; in small quantities.

10. Water, in large quantities; in cooking.

11. The length of carpeting; the width.

12. The dimensions of a book cover; of a floor; of a field.

13. The area of a sheet of paper; of a floor; of a field.

14. The capacity of an aquarium.

15. Milk, when sold to a creamery; when sold to a dealer; when sold to a family.

16. Coffee, when sold in large quantities; when sold to families; when used for cooking.

17. Potatoes, when sold in large quantities; when sold in small quantities at a city grocery; when used in cooking.

18. Butter, when sold in large quantities; when sold in small quantities; when used in cooking.

19. Flour, when sold in large quantities; when sold in small quantities; when used in cooking.

III. INDUSTRIAL PROBLEMS

THE BOYS' CORN CLUB

1. A boy rented from his father, at $5 an acre, a field 40 rd. long and 11 rd. wide. How much did he charge to expenses as rent?

2. He estimated his labor at 10¢ an hour and the work of each horse at 25¢ an hour. In one month he put in 42 hr., during 4 hr. of which he had a horse. Find the amount.

3. He needed for seed 16 ears of corn to the acre. How many ears did he need in all? He paid for the seed at the rate of $2\frac{1}{2}$¢ an ear. How much did he pay?

4. He planted the hills 3 ft. 4 in. apart in rows 3 ft. 6 in. apart. How many hills could he put in, making an allowance of 1 ft. 6 in. at each end of the field and 2 ft. 6 in. at each side?

5. It cost him $12.48 per acre to raise the corn, including rental, seed, labor, and fertilizer. How much did it cost him in all? If the corn was worth 40¢ a bushel, and he raised $274\frac{3}{4}$ bu. in all, what was his total profit?

PAVING THE CITY STREETS

1. In preparing a street for paving, John saw that the workmen took out dirt to the average depth of 14 in. The street is 40 ft. wide and $\frac{1}{2}$ mi. long. How many cubic yards of dirt are to be removed, and how much will it cost at 50¢ per cubic yard?

2. John finds that they put in a foundation of 8 in. of concrete costing $1.05 per square yard. How much does this cost?

3. John finds that the concrete is to be covered with brick at $2.35 per square yard, or asphalt at $3.10 per square yard. What is the difference in cost for the street?

4. The curbing for the street costs 78¢ per running foot, and there is a curbing all along both sides, except where there are cross streets. John finds that 480 ft. is to be deducted on each side for intersecting streets. How much will the curbing cost?

5. John's father's lot has a frontage of 66 ft. on the street. If the city requires him to pay half the cost of the paving to the middle of the street, and half the cost of the curbing in front of his lot, how much must he pay if brick is used for the paving?

CHAPTER IV

DECIMAL FRACTIONS

124. Meaning of Decimals. You have learned that $\$3.25$ means 3 dollars + 25 cents, or $\$3 + \$\frac{25}{100}$.

Just as $\$3.25$ means $\$3 + \$\frac{25}{100}$,
so 3.25 ft. means 3 ft. + $\frac{25}{100}$ ft., or $3\frac{1}{4}$ ft.;
4.75 in. means 4 in. + $\frac{75}{100}$ in., or $4\frac{3}{4}$ in.;
2.5 means $2 + \frac{5}{10}$, or $2\frac{1}{2}$;
.5, or 0.5, means $\frac{5}{10}$, or $\frac{50}{100}$, or 0.50, or $\frac{1}{2}$;
.06, or 0.06, means $\frac{6}{100}$ or $\frac{3}{50}$;
and .125, or 0.125, means $\frac{125}{1000}$, or $\frac{1}{8}$.

125. Decimal Fraction. A fraction whose denominator is not written but is some power of 10 is called a *decimal fraction.*

Thus 0.5 (5 tenths) is a decimal fraction. If written in the form $\frac{5}{10}$ it becomes a common fraction.

The expression 0.5 means the same as .5. The 0 may be written or not, as we please. Those who have much computing to do usually find it better to write the 0 before .5, except where the numbers are written in columns, so there shall be no misunderstanding. It is perfectly proper to omit it, however, as is always done when we have columns that are ruled for dollars and cents, and as is often the case in printed numbers. You should accustom yourself to both forms.

Decimal fractions are usually called *decimals.* The period placed at the left of tenths is called the *decimal point.*

126. Reading Decimals. The names of the decimal places most frequently used are given in the following scheme:

Thousands	Hundreds	Tens	Units	(Decimal point)	Tenths	Hundredths	Thousandths	Ten-thousandths
8	3	9	5	.	2	7	6	5

This number is read "eight thousand, three hundred ninety-five and two thousand seven hundred sixty-five ten-thousandths." The orders beyond ten-thousandths are hundred-thousandths, millionths, ten-millionths, and so on. Practically, we do not have much need for decimals beyond thousandths.

Pupils should be given much exercise in reading such numbers from the blackboard, avoiding the use of "and" except between the units and tenths, and the vulgarism of "aught" for "naught."

127. Writing Decimals. Although we write a decimal fraction without its denominator, the denominator is shown by the decimal point and is indicated in reading.

Thus 0.9 has no printed denominator, but when we say "nine tenths" we indicate the fact that the denominator is ten, as well as state that the numerator is nine.

Therefore, *write only the numerator of the decimal fraction, inserting such zeros as may be necessary to indicate the denominator, and place the decimal point before the tenths.*

Thus, to write in figures two hundred one hundred-thousandths, we write 201 and prefix two zeros, preceded by a decimal point, so that the last figure is in hundred-thousandths' place. We then have 0.00201.

ORAL EXERCISE

Read the following numbers:

1. 1.25.
2. 3.78.
3. 2.95.
4. 6.33.
5. 0.125.
6. 48.675.
7. 0.375.
8. 67.005.
9. 32.375.
10. 100.025.
11. 91.075.
12. 300.075.

13. What is the name of the place to the right of tenths? to the right of that? What is the name of the fourth decimal place? the fifth? the sixth?

Since we rarely read the names of numbers beyond three decimal places, even when we write them, the chief drill should be on reading decimal fractions like those in Exs. 1–12.

Read the following numbers:

14. 16.004.
15. 75.137.
16. 82.3745.
17. 12.3147.
18. 91.82734.
19. 16.42398.
20. 68.998232.
21. 14.763458.

WRITTEN EXERCISE

Write the following numbers in figures:

1. Three tenths; five tenths; nine tenths.

2. Four and four tenths; seven and eight tenths.

3. Seventy-five hundredths; thirty-three hundredths.

4. One hundred forty-four thousandths; six hundred seventy-five thousandths; four ten-thousandths.

5. Five and seven thousandths; eight and forty-two thousandths; twenty-five thousandths; one millionth.

6. Twelve hundredths; fifty-five hundredths; nine hundredths; one hundredth; twenty hundredths.

7. One hundred twenty-five thousandths; forty-seven thousandths; ten thousandths; five thousandths.

128. Annexing Zeros to Decimals. Because we know that the fraction $\frac{5}{10} = \frac{50}{100}$, or $0.5 = 0.50$, we see that

Annexing a zero to a decimal fraction does not change its value.

Decimals that have the same number of decimal places are called *similar decimals.*

Therefore, *to change dissimilar decimals to similar decimals, give them the same number of decimal places by annexing or cutting off zeros.*

For example, change 0.25, 2.125, and 3.5 to similar decimals. We see that

$$0.25 = 0.250 \qquad 2.125 = 2.125, \qquad 3.5 = 3.500,$$

and 0.250, 2.125, and 3.500 are similar decimals.

WRITTEN EXERCISE

Change to similar decimals:

1. 0.2, 0.17.
2. 0.8, 0.08.
3. 0.7, 0.002.
4. 0.9, 0.130.
5. 0.6, 0.600.
6. 0.60, 0.600.
7. 0.6, 0.60.
8. 0.6, 0.123.
9. 0.5, 0.700.
10. 0.4, 0.050.
11. 0.1, 0.357.
12. 0.5, 0.50.
13. 0.5, 0.500.
14. 0.50, 0.500.
15. 0.12, 0.103.
16. 0.72, 0.008.
17. 0.69, 0.070.
18. 0.30, 0.700.
19. 0.7, 0.70.
20. 0.7, 0.700.
21. 0.70, 0.700.

22. Express as hundredths: 0.6; 0.7; 0.9; 0.750.

23. Express as tenths: 0.70; 0.90; 0.500; 0.900; 0.800.

24. Express as thousandths: 0.1; 0.3; 0.10; 0.30; 0.7; 0.70.

25. Compare 0.2 and 0.20; 0.2 and 0.200; 0.20 and 0.200.

26. Compare 2.5 and 2.50; $2\frac{1}{2}$ and 2.50; $2\frac{1}{2}$ and 2.500.

129. Changing Decimals to Common Fractions. Because the fraction $0.25 = \frac{25}{100}$, which equals $\frac{1}{4}$, therefore

To change a decimal to a common fraction, write the numerator and the denominator of the decimal and then reduce the common fraction to lowest terms.

In a case like that of $0.33\frac{1}{3}$ we proceed in the same way, thus:

$$0.33\frac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{1}{3}.$$

ORAL EXERCISE

Read these decimals as common fractions in lowest terms:

1. 0.1.
2. 0.3.
3. 0.7.
4. 0.9.
5. 0.11.
6. 0.17.
7. 0.2.
8. 0.4.
9. 0.6.
10. 0.25.
11. 0.5.
12. 0.50.
13. 0.40.
14. 0.80.
15. 0.75.

WRITTEN EXERCISE

Change to common fractions in lowest terms or to mixed numbers:

1. 0.8.
2. 0.60.
3. 0.20.
4. 0.40.
5. 0.35.
6. 0.65.
7. 0.45.
8. 0.55.
9. 0.125.
10. 0.375.
11. 0.625.
12. 0.875.
13. 1.25.
14. 2.75.
15. 5.125.
16. 10.375.
17. Express as hundredths: 0.3, 0.5, 0.1, 0.8, 4, 63.
18. Express as thousandths: 0.5, 0.25, 0.37, 0.09, 31.
19. Express as hundredths: 0.470, 0.680, 0.100, 0.040, 7.8.
20. Express 0.125 as eighths. Find 0.125 of $1600.
21. Express $0.33\frac{1}{3}$ as thirds. Find $0.33\frac{1}{3}$ of 621.
22. Express $0.37\frac{1}{2}$ as eighths. Find $0.37\frac{1}{2}$ of 1768.

130. Changing Common Fractions to Decimals. Since a common fraction indicates division (pages 47, 55), we may divide the numerator by the denominator and thus change the fraction to the decimal form. For example, consider the fraction $\frac{3}{8}$.

We have found that we may annex zeros after the decimal point without changing the value; that is, 3 = 3.000. We may now divide exactly as in United States money, and the result is 0.375.

$$\begin{array}{r} 8\,\underline{)\,3.000} \\ 0.375 \end{array}$$

That this is allowable may be seen from the following: 3.0 means 30 tenths, and 30 tenths ÷ 8 = 3 tenths with remainder 6 tenths. Again 6 tenths = 60 hundredths ($\frac{6}{10} = \frac{60}{100}$), and 60 hundredths ÷ 8 = 7 hundredths with remainder 4 hundredths. But 4 hundredths = 40 thousandths, and 40 thousandths ÷ 8 = 5 thousandths. Therefore $\frac{3}{8} = 0.375$.

Therefore, *to reduce a common fraction to a decimal, place a decimal point after the numerator and divide by the denominator, as in United States money.*

A common fraction cannot always be reduced to an exact decimal; thus $\frac{2}{3} = 0.6\frac{2}{3} = 0.66\frac{2}{3} = 0.666\frac{2}{3}$. It is customary to write a common fraction or the plus sign to indicate that the work is not complete; thus $\frac{2}{3} = 0.666\frac{2}{3}$, 0.666 ⋯, or 0.666 +. This is often written simply 0.666, using as many places as necessary.

WRITTEN EXERCISE

Change to decimals, carrying the reduction only to thousandths in the case of inexact decimals:

1. $\frac{1}{2}$.
2. $\frac{1}{4}$.
3. $\frac{1}{8}$.
4. $\frac{3}{4}$.
5. $\frac{3}{8}$.
6. $\frac{5}{8}$.
7. $\frac{7}{8}$.
8. $\frac{1}{5}$.
9. $\frac{2}{5}$.
10. $\frac{3}{5}$.
11. $\frac{4}{5}$.
12. $\frac{1}{16}$.
13. $\frac{5}{16}$.
14. $\frac{7}{16}$.
15. $\frac{9}{16}$.
16. $\frac{13}{16}$.
17. $\frac{1}{32}$.
18. $\frac{5}{32}$.
19. $\frac{1}{3}$.
20. $\frac{2}{3}$.
21. $\frac{1}{6}$.
22. $\frac{5}{6}$.
23. $\frac{7}{25}$.
24. $\frac{9}{25}$.
25. $2\frac{11}{25}$.
26. $4\frac{9}{75}$.
27. $0.1\frac{5}{8}$.
28. $8.2\frac{5}{8}$.
29. $4.3\frac{3}{5}$.
30. $8.1\frac{3}{4}$.

131. Adding and Subtracting Decimals. We have learned how to add and subtract such numbers as $1.25 and $2.10. We proceed in the same way with decimal fractions.

To add or subtract decimal fractions, write like orders under one another and add or subtract as with whole numbers.

1. Add 3.06, 9.475, and 0.1.

$$\begin{array}{l}3.06\\9.475\\0.1\\\hline 12.635\end{array}$$

If the decimal points are in a column, units will come under units, tenths under tenths, and so on. Adding as with numbers representing dollars and cents or with whole numbers the sum is 12.635.

2. Subtract 0.26 from 4.1.

$$\begin{array}{r}4.10\\0.26\\\hline 3.84\end{array}$$

Arranging the numbers with the decimal points in a column and writing 4.1 as 4.10, the subtraction is performed in the same way as with United States money. The remainder is easily found to be 3.84.

WRITTEN EXERCISE

Add the following:

1.	2.	3.	4.	5.
$2.47	$15.25	28.92	68.23	32.301
3.62	7.42	6.87	.09	41.025
4.81	19.00	9.09	7.40	7.875

Add, and also subtract, the following:

6.	7.	8.	9.	10.
80.25	67.25	49.63	35.70	67.02
40.07	31.50	22.78	12.95	9.80

11.	12.	13.	14.	15.
42.57	48.05	540.00	128.42	142.00
6.64	6.97	.67	37.06	64.92

ORAL EXERCISE

1. A small geography cost $0.60 and a music book $0.35. How much did the two cost together?

2. An arithmetic cost $0.65 and a paint box $0.35. How much did the two cost together?

3. A history cost $0.60. I gave the dealer a 50-cent piece and a quarter of a dollar. How much change did I receive?

4. I bought a large geography for $1.25 and gave the dealer a dollar and a half. How much change did I receive?

5. How much more did the large geography cost than the small geography? How much more than the arithmetic?

WRITTEN EXERCISE

1. The three sides of a triangle are 2.25 in., 3.8 in., and 2.95 in. What is the perimeter?

2. Three pieces of land have areas as follows: 14.85 A., 26.35 A., and 49.8 A. What is the total area?

3. A man has a field of 38.75 A. He takes 1.2 A. for a garden and 9.8 A. for a pasture. How much is left?

Add the following:

4.	**5.**	**6.**	**7.**	**8.**
72.86	22.37	3.725	412.5	7.283
32.96	43.96	4.683	328.7	.796
41.83	77.63	5.928	587.5	.584
82.09	56.04	7.635	671.3	.963
73.57	29.37	4.009	428.9	4.020
61.23	92.08	5.700	571.1	5.000
4.72	70.63	6.302	266.6	.778
16.8	7.92	5.777	733.4	.610

Add, and also subtract:

9.	**10.**	**11.**	**12.**	**13.**
487.3	52.37	4.003	2.803	1.093
209.6	36.99	2.876	.196	.027
14.	**15.**	**16.**	**17.**	**18.**
22.05	39.26	43.01	25.27$\frac{1}{2}$	34.33$\frac{1}{3}$
13.75	13.54	34.17	16.42$\frac{1}{4}$	21.66$\frac{2}{3}$
19.	**20.**	**21.**	**22.**	**23.**
248.7	32.4	5.071	4.638	2.708
178.8	18.05	2.683	.989	1.819
24.	**25.**	**26.**	**27.**	**28.** +
801.3	51.08	9.011	5.117	5.620
628.4	28.72	6.232	2.308	.875
29.	**30.**	**31.**	**32.**	**33.**
912.13	22.38	8.112	6.213	8.112
426.48	19.79	4.348	.487	6.023

34. What number must be added to 7.96 to make 19.8?

35. What number must be taken from 17.5 to leave 6.75?

36. What is the result of adding 4.93 to 17.89?

37. What is the result of subtracting 5.98 from 11.13?

38. What is the result of adding 2.69 to 3.968?

39. The distance from Chicago to Janesville is 91.1 mi., and from Chicago to Fond du Lac 176.7 mi. How far is it, by this route, from Janesville to Fond du Lac?

40. From a barrel of oil containing 31.5 gal. there is drawn off 17.75 gal. How much oil is left?

132. Multiplying Money. We have already studied, in the fourth grade, the multiplication of money by a whole number. For example, if we have to multiply $12.36 by 41 we multiply as if we had 41 × 1236, and then we place the decimal point in the product between the dollars and the dimes.

$$\boxed{\begin{array}{r} \$12.36 \\ 41 \\ \hline 1236 \\ 4944 \\ \hline \$506.76 \end{array}}$$

As an introduction to the multiplication of decimal fractions it is well to review the multiplication of money by a whole number. This is, of course, essentially the multiplication of a decimal fraction by an integer.

WRITTEN EXERCISE

1. If one pair of club skates costs $1.85, how much will 3 pairs cost? How much will 4 pairs cost?

2. If one tennis racket costs $2.50, how much will 5 rackets cost? How much will 3 rackets cost?

3. If one dog collar costs $1.35, how much will 2 dog collars cost? How much will 3 dog collars cost?

4. If 1 doz. tennis balls cost $4.85, how much will 3 doz. tennis balls cost?

5. If one basket ball costs $4.75, how much will 2 basket balls cost?

6. If one football costs $2.25, how much will 3 footballs cost?

7. If one baseman's mitt costs $1.75, how much will 5 such mitts cost?

8. If one baseball mask costs $2.45, how much will 3 such masks cost?

9. At $16.85 each, how much will 17 girls' bicycles cost?

10. At $2.85 each, how much will 33 croquet sets cost?

Multiply the following:

11. 2 × $4.26.
12. 3 × $5.08.
13. 7 × $6.30.
14. 9 × $4.77.
15. 6 × $5.32.
16. 8 × $6.41.
17. 5 × $4.43.
18. 9 × $9.96.
19. 5 × $3.40.
20. 2 × $6.66.
21. 8 × $7.43.
22. 4 × $8.84.
23. 7 × $5.07.
24. 3 × $2.99.
25. 6 × $9.89.
26. 12 × $3.26.
27. 56 × $8.22.
28. 94 × $3.28.
29. 81 × $4.40.
30. 23 × $4.46.
31. 67 × $3.41.
32. 85 × $8.23.
33. 92 × $3.32.
34. 34 × $5.81.
35. 78 × $8.22.
36. 66 × $4.92.
37. 73 × $7.20.
38. 45 × $6.96.
39. 89 × $9.90.
40. 57 × $8.09.
41. 62 × $12.48.
42. 28 × $20.09.
43. 94 × $66.82.
44. 73 × $81.62.
45. 55 × $70.00.
46. 39 × $32.01.
47. 61 × $83.40.
48. 25 × $31.12.
49. 84 × $42.38.
50. 30 × $80.09.
51. $4\frac{1}{2}$ × $36.62.
52. $2\frac{1}{2}$ × $41.14.
53. $6\frac{1}{2}$ × $51.10.
54. $2\frac{1}{4}$ × $88.60.
55. $4\frac{3}{4}$ × $70.08.

56. Rob buys 3 fountain pens for presents. He pays $1.35 for each. How much does he pay for all the pens?

57. A jeweler sells 4 wrist watches at $6.75 each. How much does he receive for all the watches?

58. A book dealer sells 6 sets of Scott's novels at $2.43 a set. How much does he receive for all the sets?

59. A dealer buys 9 rifles at $14.65 each. How much does he pay for the lot?

60. Fred buys 2 minnow nets at $1.55 each. How much does he pay for both nets?

61. A dealer buys 3 pianos at $285.50 each. How much does he pay for all the pianos?

133. Multiplication of a Decimal by a Whole Number. There is a measure called the meter, 39.37 in. long. How many inches are there in 24 meters?

Here we have to multiply 39.37 by 24.

Since 7 hundredths multiplied by 4 is 28 hundredths, we know that this 8 will go in the hundredths' place.

Proceeding with the multiplication in the ordinary way, we finally place the decimal point in the product between the 4 and the 8 so as to have the right-hand figure in the hundredths' place

$$\begin{array}{r} 39.37 \\ 24 \\ \hline 157\,48 \\ 787\,4 \\ \hline 944.88 \end{array}$$

In multiplying a decimal fraction by an integer, there are as many decimal places in the product as there are in the multiplicand.

This is evident from the above explanation. If we multiply tenths by a whole number the product is tenths; if we multiply hundredths the product is hundredths, and so on.

Multiplication by a common fraction or a mixed number is too rarely met to demand special treatment. It is easily understood from the above case.

WRITTEN EXERCISE

Multiply the following:

1. 27.25 by 25.
2. 61.82 by 37.
3. 42.12 by 45.
4. 62.09 by 35.
5. 65.232 by 32.
6. 34.375 by 81.
7. 68.325 by 50.
8. 42.175 by 62.
9. 313.45 by 72.
10. 46.925 by 93.
11. 20.005 by 75.
12. 29.407 by 87.

13. If a twelve-story city building averages 10.62 ft. to a story, how high is the building?

14. If one side of a square is 21.33 in., how long is the perimeter of the square?

15. The measure called the meter, studied later, is 39.37 in. long. How many inches in 62 meters?

134. Multiplying by Tenths. If we multiply 7 by $\frac{1}{2}$ we have $\frac{7}{2}$ as the product, and this we reduce to $3\frac{1}{2}$. In the same way,

$$\tfrac{1}{10} \text{ of } 7 = \tfrac{7}{10},$$

or, in decimal form, $0.1 \times 7 = 0.7.$

In the same way, $\frac{3}{10}$ of $250 = 3 \times \frac{250}{10} = 3 \times 25 = 75,$

or $0.3 \times 250 = 75.0 = 75.$

To multiply a whole number by tenths, multiply as by a whole number, and place a decimal point before the right-hand figure.

ORAL EXERCISE

Multiply the following by 0.1, or .1:

1. 25.
2. 37.
3. 426.
4. 384.
5. 270.
6. 360.
7. 400.
8. 408.
9. 2175.
10. 3286.

Multiply the following by 0.2, or .2:

11. 11.
12. 12.
13. 20.
14. 70.
15. 25.
16. 50.
17. 33.
18. 24.
19. 111.
20. 120.

Multiply the following by 0.3 and 0.5:

21. 10.
22. 20.
23. 30.
24. 40.
25. 11.
26. 12.
27. 100.
28. 200.
29. 111.
30. 101.

WRITTEN EXERCISE

1. How much is 0.7 of 5280 ft.? 0.8 of 1760 yd.?

2. How many feet in 0.6 mi.? in 0.9 mi.?

3. How many yards in 0.7 mi.? in 0.4 mi.? in 0.5 mi.?

4. The distance from Madison to Lancaster is 85.7 mi. This is 85 mi. and how many feet?

ORAL EXERCISE

Multiply the following:

1. $$\begin{array}{r} 2\text{ in.}\\ 3\\ \hline \end{array}\qquad \begin{array}{r} \$2\\ 3\\ \hline \end{array}\qquad \begin{array}{r} \frac{2}{10}\\ 3\\ \hline \end{array}\qquad \begin{array}{r} \frac{2}{5}\\ 3\\ \hline \end{array}\qquad \begin{array}{r} \frac{2}{100}\\ 3\\ \hline \end{array}\qquad \begin{array}{r} 0.2\\ 3\\ \hline \end{array}\qquad \begin{array}{r} 0.02\\ 3\\ \hline \end{array}$$

2. $$\begin{array}{r} 4\text{ ft.}\\ 2\\ \hline \end{array}\qquad \begin{array}{r} \$4\\ 2\\ \hline \end{array}\qquad \begin{array}{r} \frac{4}{10}\\ 2\\ \hline \end{array}\qquad \begin{array}{r} \frac{4}{5}\\ 2\\ \hline \end{array}\qquad \begin{array}{r} \frac{4}{100}\\ 2\\ \hline \end{array}\qquad \begin{array}{r} 0.4\\ 2\\ \hline \end{array}\qquad \begin{array}{r} 0.04\\ 2\\ \hline \end{array}$$

State rapidly the following products:

3. $\frac{1}{10} \times \frac{3}{10}$ $\frac{2}{10} \times \frac{3}{10}$ $\frac{2}{10} \times \frac{5}{10}$ $\frac{5}{10} \times \frac{9}{10}$
4. 0.1×0.3 0.2×0.3 0.2×0.5 0.5×0.9

135. Multiplying by a Decimal Fraction. This is best understood from the following examples:

$$\boxed{\begin{array}{r} 6.2\\ 3.5\\ \hline 3\,10\\ 18\,6\\ \hline 21.70\\ 21.7 \end{array}}$$

1. Multiply 6.2 by 3.5.

If we were to multiply 62 by 35 the result would be 2170. Since tenths multiplied by tenths is equal to hundredths, the result is 21.70; that is, $\frac{2170}{100}$ or $21\frac{70}{100}$ or 21.7.

So we see that we multiply as with whole numbers, and, beginning at the right, point off as many decimal places in the product as there are decimal places in both factors together.

$$\boxed{\begin{array}{r} 0.42\\ 0.35\\ \hline 210\\ 126\\ \hline 0.1470 \end{array}}$$

2. Multiply 0.42 by 0.35.

If we were to multiply 42 by 35 the result would be 1470. Since hundredths multiplied by hundredths is equal to ten-thousandths, the result is 0.1470; that is, $0.42 \times 0.35 = \frac{42}{100} \times \frac{35}{100} = \frac{1470}{10000} = 0.1470 = 0.147$.

136. Decimal Places in the Product. In multiplying decimals,

Multiply as with whole numbers, and then, beginning at the right, point off as many decimal places in the product as there are decimal places in all the factors together.

ORAL EXERCISE

Multiply the following:

1. 0.7 × 3 0.9 × 4 0.8 × 5 0.3 × 7
2. 0.2 × 4 0.4 × 8 0.9 × 9 0.8 × 7
3. 0.1 × 0.1 0.1 × 0.2 0.2 × 0.3 0.2 × 0.4
4. 0.3 × 0.4 0.3 × 0.5 0.4 × 0.4 0.4 × 0.5
5. 0.6 × 0.7 0.6 × 0.8 0.7 × 0.8 0.8 × 0.9

6. Why will there be three decimal places in the product of 0.75 multiplied by 0.3? of 2.325 multiplied by 7?

7. How do you know the number of decimal places in the product of 0.62 and 2.79? of 2.756 and 3.123?

Without multiplying, tell the orders of the decimal parts of:

8. 2.8 × 7.9 2.04 × 6.09 1.28 × 2.67
9. 1.7 × 1.44 1.23 × 3.456 3.421 × 0.713

WRITTEN EXERCISE

Multiply the following:

1. 2 × 0.79.
2. 0.82 × 8.
3. 3 × 1.75.
4. 3 × 0.96.
5. 0.67 × 9.
6. 5 × 2.42.
7. 4 × 0.78.
8. 0.83 × 4.
9. 6 × 7.23.
10. 7 × 0.93.
11. 0.27 × 7.
12. 8 × 9.23.
13. 0.9 × 84.
14. 0.3 × 9.9.
15. 4.7 × 35.
16. 8.1 × 8.1.
17. 48 × 0.8.
18. 6.7 × 1.8.
19. 0.7 × 96.
20. 0.8 × 9.8.
21. 1.5 × 67.
22. 2.7 × 3.4.
23. 0.8 × 74.
24. 0.7 × 4.9.
25. 1.4 × 75.
26. 1.9 × 2.6.
27. 0.6 × 48.
28. 0.9 × 4.7.
29. 3.8 × 46.
30. 4.7 × 3.6.

Multiply the following:

31. 7.3 × 4.8.

32. 96 × 9.6.

33. 5.4 × 3.5.

34. 43.2 × 98.6.

35. 0.823 × 2946.

36. 3.49 × 80.23.

37. 2.99 × 124.63.

38. 89.1 × 49.3.

39. 72.9 × 87.09.

40. 36.4 × 49.87.

41. 0.600 × 0.500.

42. 6.50 × $28.43.

43. 0.3 × $333.33⅓.

44. 8.001 × 234.56.

45. 0.25 × $648.36.

46. At $17.50 per front foot (that is, per foot fronting the street), what will a piece of land with a frontage of 28.3 ft. cost?

47. A certain tank when full holds 42.8 gal. How much does it contain when 0.37 full?

48. A certain rectangle is 0.78 as wide as it is long. It is 4.9 in. long. What is the perimeter?

49. A certain rectangle is 3.42 times as long as it is wide. It is 4.8 ft. wide. What is the perimeter?

50. A city lot is 45.8 ft. wide and 76.3 ft. deep. What is the area of the lot?

51. The circumference of a circle is 3.1416 times the diameter. If the diameter is 42.27 ft., what is the circumference?

52. How many square rods of land are there in a piece of land 80 rd. wide and 114 rd. long? How many acres? What is it worth at $87.50 an acre?

53. There are three towns, A, B, C, forming the three corners of a triangle. From A to B is 27.32 mi., from B to C is 1.4 times as far, and the perimeter of the triangle is 92.8 mi. How far is it from C to A?

In all applied problems of this kind, as well as in all abstract work, teachers should urge their pupils to estimate the results in advance, for the reason given on page 26. Every business man does this so as to avoid any absurd result due to an error in computation. In Ex. 53 a fair estimate would be 20 mi.

ORAL EXERCISE

State the values of:

1. $12 \times \frac{1}{12}$ $3 \times \frac{1}{3}$ $10 \times \frac{1}{10}$ 10×0.1
2. 10×0.2 10×0.6 $10 \times \frac{1}{100}$ 10×0.01
3. $100 \times \frac{1}{100}$ 100×0.01 100×0.02 100×0.09
4. $100 \times \frac{25}{100}$ $100 \times \frac{75}{100}$ 100×0.25 100×0.75
5. 100×0.38 100×0.75 $1000 \times \frac{25}{1000}$ 1000×0.025

6. We have found that to multiply a decimal fraction by 10, we need only move the decimal point one place in which direction? How is it when we multiply by 100? by 1000?

137. Multiplying by Multiples of 10. On page 22 we found how to multiply by 10, 100, 1000, and so on for other powers of 10. We also found how to multiply by such numbers as 40, 700, and so on. To multiply decimal fractions by such numbers is substantially the same as to multiply dollars and cents. Therefore, to multiply 487.7 by 600 is the same as to multiply by 6, moving the decimal point two places to the right.

To multiply 89.07 by 740 is the same as to multiply by 74, moving the decimal point one place to the right.

WRITTEN EXERCISE

Multiply the following:

1. 40 × 6.73.
2. 30 × 4.87.
3. 60 × 9.81.
4. 90 × 86.4.
5. 80 × 36.5.
6. 70 × 625.3.
7. 80 × 65.42.
8. 900 × 37.08.
9. 400 × 61.8.
10. 1000 × 1.25.
11. 2000 × 6.725.
12. 6970 × 43.2.
13. 9250 × 67.25.
14. 8220 × 350.4.
15. 6700 × 495.4.
16. 2500 × 374.65.
17. 3500 × 408.08.
18. 35,000 × 1.25.
19. 75 × 12.48.
20. 750 × 1.248.
21. 7500 × 0.1248.

138. Bills. If goods are not paid for by cash, which usually means that they are not paid for when they are purchased but at some later date, the dealer sends a *bill* showing the purchases, the prices, and the amount due. We studied such bills in the fourth grade. The following is a model bill, receipted by the dealers to show that it has been paid:

Columbus, Ohio, April 6, 1918

Mrs. M. A. Olds, 179 Cairo Street

Bought of DOE & ROE, Grocers,
8296 MAIN ST.

Apr.	3	2 tins sliced smoked beef	.25		50
	4	3 cans corned beef	.13		39
	5	6 cans tomato soup	.22	1	32
				2	21
		Received payment Doe & Roe			

WRITTEN EXERCISE

Make out bills for the following items, inserting the names of the purchaser and the dealer, and receipting the bills:

1. 1 baseball cap, 40¢; 1 catcher's mitt, $1.50; 1 baseball mask, $1.40; 1 catcher's body protector, $1.20.

2. 1 league ball, 85¢; 3 bats @ 60¢; 2 fielder's mitts @ 75¢; 3 canvas bases @ $1.10; 1 pitcher's toe plate, 13¢.

3. 4 hunting caps @ 30¢; 1 hunting vest, 69¢; 2 pr. corduroy hunting pants @ $1.45; 2 hunting coats @ $1.50.

Make out bills for the following, as on page 158:

4. 3 qt. Lima beans @ 15¢, 4 cans tomatoes @ 14¢, 6 cans French peas @ 22¢, 5 cans California asparagus @ 42¢, 2 cans imported asparagus @ 85¢, 4 cans sardines @ 18¢.

5. 5 boxes Smyrna figs @ 60¢, 4 boxes stuffed dates @ 40¢, 6 lb. California prunes @ 20¢, 3 lb. Malaga raisins @ 22¢, 4 tins crystallized ginger @ 40¢, 6 jars marmalade @ 28¢.

6. 6 heads lettuce @ 6¢, 9½ lb. butter @ 32¢, 7½ gal. oil @ 18¢, 12 lb. raisins @ 12½¢, 6 lb. coffee @ 32¢, 4½ doz. eggs @ 36¢, ½ doz. oranges @ 60¢, 6 lb. powdered sugar @ 7½¢.

7. 14 yd. flannel @ 45¢, 32 yd. sheeting @ 30¢, 4½ yd. velvet @ $1.60, 2½ yd. ribbon @ 40¢, 2 pr. gloves @ $1.75.

8. 5 gal. molasses @ 45¢, 10 lb. coffee @ 18¢, 15 lb. sugar @ 7¢, 6 boxes crackers @ 8¢, 6½ doz. eggs @ 34¢, 9 lb. butter @ 33¢, 6½ lb. cheese @ 12¢.

9. 36 yd. carpet @ $1.12½, 15 yd. border @ $1.25, 15½ yd. cotton @ 38¢, 22 yd. cloth @ 37½¢, 7½ yd. silk @ $1.40.

In making out a bill take the next higher cent when there is a fraction of a cent in a product.

10. 75 gro. bone buttons @ 22¢, 675 yd. cashmere @ 85¢, 840 yd. cambric @ 17½¢, 560 yd. chambray @ 22½¢, 620 yd. flannel @ 37½¢, 950 yd. sheeting @ 24½¢.

11. 7 gro. toothbrushes @ $9.65, 8 doz. brushes @ $17.75, 2½ gro. nailbrushes @ $26.80, ½ doz. clothes brushes @ $2.92, 6 doz. boxes tooth powder @ $2.04, 4½ doz. atomizers @ $19.60.

12. 3 rifles @ $12.50, 4 shot guns @ $17.50, 400 loaded paper shells @ $2.69 per 100, 3 hunting knives @ $1.85, 4 redwood cleaning rods @ 75¢, 2 trap targets @ $6.50.

13. 3 gun cases @ $4.25, 4 hunting coats @ $3.50, 3 hunting caps @ 33¢, 5 fishing rods @ $2.50, 7 silk casting lines @ 80¢.

139. Price List. The following is a price list of articles interesting to boys and girls. It is used in the problems on pages 160 and 161.

Boys' storm-proof boots, $1.50
Girls' school shoes, $1.45
Girls' skating cap, 60¢
Boys' sweater, $1.50
Boys' silver watch, $3.50
Girls' watch, $3.75
Rubber-stamp outfit, 40¢
Cartridge belt, $1.35
Baseball bat, 60¢
Football pants, $1.50
Bicycle, $16.75
Scout camera, $1.75
Hammock, $1.75

Small typewriter, $5
School drawing set, 30¢
School-book satchel, 50¢
Postcard projecting lantern, $2.50
Postcard album, 65¢
Single-barreled shotgun, $6.25
Rifle, $13.50
Catcher's mitt, $1.50
Catcher's mask, $2
Basket ball, $3.25
Banjo, $9.50
Guitar, $4.75

WRITTEN EXERCISE

Find the cost of the following:

1. 2 sweaters, 3 skating caps, a boys' watch, and 2 hammocks.

2. A bicycle, a guitar, and 2 catcher's mitts.

3. A pair of boys' storm-proof boots, 2 pairs of girls' shoes, and a school drawing set.

4. A rubber-stamp outfit, a shotgun, and a cartridge belt.

5. 4 baseball bats, a catcher's mitt, a basket ball, 2 catcher's masks, and a rifle.

6. 4 pairs of football pants, 2 basket balls, and a catcher's mask.

7. Make out three different lists of things you would like to buy, six items in each, and find the cost of each list.

Make out bills for the following:

8. 2 pairs of girls' shoes, 2 pairs of boys' boots, 1 school drawing set, 1 rubber-stamp outfit, 2 postcard albums.

9. 3 girls' skating caps, 2 boys' sweaters, 2 pairs of football pants, 1 catcher's mitt, 1 catcher's mask.

10. 1 boys' watch, 1 girls' watch, 1 basket ball, 3 school drawing sets, 3 postcard albums.

11. 2 cartridge belts, 1 shotgun, 1 rifle, 2 scout cameras, 3 baseball bats, 2 hammocks.

12. 2 bicycles, 3 typewriters, 2 projecting lanterns, 3 basket balls, 2 guitars, 3 banjos.

13. 6 hammocks, 2 typewriters, 4 girls' watches, 5 boys' watches, $\frac{1}{2}$ doz. girls' skating caps.

14. $\frac{1}{2}$ doz. boys' sweaters, 8 pairs of boys' boots, 4 cartridge belts, $\frac{1}{2}$ doz. baseball bats, 4 pairs of football pants.

15. $\frac{1}{2}$ doz. scout cameras, $\frac{1}{2}$ doz. catcher's masks, 8 catcher's mitts, 4 rifles, 2 shotguns.

16. 8 projecting lanterns, 9 postcard albums, 25 school drawing sets, 7 typewriters, 3 school satchels.

17. 5 basket balls, 4 banjos, 4 guitars, 8 hammocks, 9 baseball bats, 7 pairs of football pants, 8 rubber-stamp outfits.

18. 15 rifles, 16 single-barreled shotguns, 9 cartridge belts, 9 baseball bats, 6 pairs of football pants, 2 bicycles, 8 basket balls, 4 scout cameras, 6 hammocks, 2 postcard projecting lanterns.

Teachers should have the pupils obtain price lists, either at the local stores or from catalogues, and make bills of goods in which they are interested. The pupils should see that some of the bills in this exercise are such as might represent goods purchased by them or by their parents, while some of the bills include such numbers of articles as would be purchased only by dealers. An effort should always be made to relate a topic of this kind to the personal and home interests of the pupil.

140. Marketing. The following is a price list of household supplies. It may be used in the examples on pages 162 and 163, or the current prices in the various places may be used.

Oranges, per dozen, 40¢
Butter, per pound, 38¢
Eggs, per dozen, 44¢
Sugar, 25-pound bag, $1.30
Oysters, per dozen, 25¢
Coffee, per pound, 30¢
Celery, per bunch, 10¢
Apples, per peck, 40¢
Cheese, per pound, 14¢
Cranberries, per quart, 14¢
Turkey, per pound, 26¢
Halibut, per pound, 15¢

WRITTEN EXERCISE

Find the cost of the following:

1. $2\frac{1}{2}$ doz. oranges and $3\frac{1}{2}$ lb. of butter.

2. $3\frac{1}{2}$ doz. eggs and a 25-pound bag of sugar.

3. 2 doz. oysters and 5 lb. of coffee.

4. 3 bunches of celery and $\frac{1}{2}$ bu. of apples.

5. $2\frac{1}{2}$ lb. of cheese and 4 lb. of coffee.

6. An 8-pound turkey and a quart of cranberries.

7. 4 lb. of halibut and $2\frac{1}{2}$ doz. eggs.

8. $\frac{1}{2}$ doz. oranges, $2\frac{1}{2}$ lb. of butter, 2 doz. eggs, and a 25-pound bag of sugar.

9. 3 doz. oysters, a 9-pound turkey, 2 qt. of cranberries, and a peck of apples.

10. 6 lb. of coffee, 4 doz. eggs, $6\frac{1}{2}$ lb. of butter, 4 bunches of celery, and 6 lb. of halibut.

11. 3 doz. oranges, $1\frac{1}{2}$ lb. of butter, $2\frac{1}{2}$ doz. eggs, 25 lb. of sugar, 1 pk. of apples, 3 lb. of cheese.

12. 2 doz. oysters, 7 lb. of coffee, 4 doz. eggs, $2\frac{1}{2}$ pk. of apples, 3 qt. of cranberries, and a 12-pound turkey.

Make out bills for the following, inserting the names of purchaser and dealer and receipting the bills:

13. 4 bunches of celery, 2 pk. of apples, 2 lb. of butter, 2 doz. oysters, 4 lb. of halibut.

14. 4 qt. of cranberries, a 14-pound turkey, 2 lb. of cheese, 25 lb. of sugar, 3 doz. oranges.

15. A 15-pound turkey, 3 qt. of cranberries, 1 pk. of apples, 4 bunches of celery, 3 doz. eggs, 1 doz. oranges.

16. 2 lb. of butter @ 38¢, 2 lb. of domino sugar @ 8½¢, 2 packages of puffed rice @ 14¢, 3 doz. eggs @ 34¢, 2 lb. of powdered sugar @ 6¢, 2 lb. of coffee @ 30¢.

17. 2 doz. oranges @ 60¢, 3 doz. eggs @ 33¢, 2 lb. of butter @ 44¢, 7 lb. of granulated sugar @ 5¢, ½ lb. of tea @ 90¢, 2 grapefruits @ 15¢, 4 tins of sugar wafers @ 25¢.

18. 2 bottles of malt vinegar @ 22¢, 3 packages of corn flakes @ 10¢, 1½ doz. oranges @ 60¢, 3 doz. eggs @ 44¢, 2½ lb. of butter @ 42¢, 4 cans of pea soup @ 29¢.

19. 1 pt. of olive oil, 50¢, 2 boxes of domino sugar @ 17¢, 1½ lb. of tea @ 90¢, ½ doz. cakes of soap @ $1.08 per doz., 3 doz. oranges @ 40¢, ½ lb. of milk chocolate @ 50¢.

20. 2 baskets of figs @ 30¢, 3 doz. cakes of soap @ $1.06 a dozen, 2 bottles of vinegar @ 22¢, 2 lb. of chocolate mints @ 80¢, 2 boxes of layer figs @ 55¢.

21. 4 bottles of ammonia @ 24¢, 3 boxes of salt @ 10¢, ½ gal. of olive oil @ $3 per gallon, 3 cans of mustard @ 25¢, 2 bags of flour @ 85¢, 4 lb. of confectioners' sugar @ 6½¢.

22. 1½ lb. of pork tenderloin @ 38¢, 1¾ lb. of French lamb chops @ 28¢, 2½ lb. of steak @ 26¢, 8 lb. of roast beef @ 34¢, 1½ lb. of fish @ 22¢, 2 packages of sausage @ 25¢.

141. Simple Accounts. We have already learned (pages 15–17) how to keep simple accounts. We shall now review the subject briefly. The following is a sample page from a boy's account book:

1916		Receipts			1916		Payments		
March	1	On hand . .	1	18	March	2	Papers . .		25
"	6	From father		25	"	3	Car fare .		10
"	14	Errand. . .		10	"	4	Pencil . .		05
"	20	Paper route .		75	"	31	On hand .	2	03
"	29	Errand. . .		15					
			2	43				2	43

In order to train pupils to keep simple accounts, an accomplishment that every one should possess, teachers are requested to have at least one exercise a week in which the children make a record of their income and expenses. This exercise need not occupy an entire period. Except when class instruction is needed, the records may be made before school or at odd times.

The accounts should be of actual moneys belonging to the pupils, if that is possible. It is better to have a briefer account and have it a real one than to fill it with imaginary transactions. The accounts should indicate briefly but adequately the sources and dispositions of income: not "Rec. from Aunt Mary," but "Gift from Aunt Mary"; not "Paid out," but "Paid for Candy"; not "Earned," but "Shoveling Snow." The headings at the top of the pages should be uniform through each book, as "Received," "Paid"; or "Income," "Outgo"; or "Receipts," "Payments." Balances should be computed at the close of each month. These balances should be properly brought down by the use of red lines, if possible, and carried forward to the next month's account.

Ask the pupils to bring to school bills that have been used in business transactions. This is review work, and pupils should become proficient in making and receipting bills of various sorts that are used in the common business of life. Make the work as practical as possible.

The examples in any textbook must necessarily be imaginative, and those given on page 165 may be used in case it is not possible to carry out the work in the spirit above set forth.

The dollar sign and the decimal point are commonly omitted when the ruling of the account book shows clearly, as in the above example, which column is dollars and which is cents.

WRITTEN EXERCISE

Rule a sheet of paper, or use an account book properly ruled, and write the following accounts, balancing each:

1. April 2, on hand, \$1.25. Receipts: April 9, gift from Uncle John, 30¢; April 20, errand, 15¢; April 24, errand, 10¢. Payments: April 13, ball, 15¢; April 14, bat, 60¢.

2. June 1, on hand, \$1.35. Receipts: June 8, from Rob for money borrowed, 10¢; June 18, paper route, 60¢; June 23, care of horse, 50¢. Payments: June 13, catcher's mask, \$1.15; June 24, 2 bats @ 60¢.

Write the following accounts, inserting dates and items, and balancing each:

3. On hand, \$1.60. Receipts: \$2.20, 20¢, 10¢, 15¢. Payments: \$1.50, 25¢, 75¢, 10¢.

4. On hand, \$1.25. Receipts: 20¢, 40¢, 75¢, 30¢. Payments: 20¢, 30¢, 5¢, 60¢.

5. On hand, \$3.60. Receipts: \$1, 45¢, 15¢, 25¢, 5¢. Payments: \$1.35, 25¢, 60¢, 10¢, 10¢.

6. On hand, \$4.30. Receipts: 60¢, \$1.30, 40¢, 20¢, 10¢, 5¢. Payments: \$1.75, 30¢, 40¢, 60¢, 10¢, 2¢, 1¢.

7. On hand, \$2.30. Receipts: \$5, 20¢, 30¢, 2¢, 10¢. Payments: \$1.25, \$1, 40¢, 65¢, 75¢, 2¢, 10¢.

8. On hand: \$3.35. Receipts: 5¢, 25¢, 60¢, \$1, \$2.25. Payments: \$2, \$1.20, 60¢, 2¢, 35¢, 4¢, 5¢.

9. On hand: \$2.75. Receipts: \$1, 75¢, 60¢, \$2, 40¢, 5¢. Payments: 40¢, 10¢, 3¢, 20¢, 25¢, 10¢, 15¢.

10. On hand: \$4.40. Receipts: 30¢, 70¢, \$1, 10¢, 15¢, 20¢. Payments: 25¢, \$2.50, 30¢, 45¢, 5¢, 15¢, 2¢.

142. Dividing by a Whole Number. The following examples will illustrate the division of a decimal fraction by a whole number.

1. In 13 da. a man traveled 925.6 mi. in his automobile. What was the average distance per day?

We have to divide 925.6 mi. by 13.

We see that 92 tens ÷ 13 = 7 tens, and there remains 15.6 to be divided.

We see that 15 ÷ 13 = 1, and there remains 2.6 to be divided.

We see that 2.6 ÷ 13 = 0.2, so that the quotient is 71.2, and the average rate was 71.2 mi. per day.

$$\begin{array}{rr}
 & 71.2 \\
13 & \overline{)925.6} \\
 & \underline{91} \\
 & 15 \\
 & \underline{13} \\
 & 26 \\
 & \underline{26}
\end{array}$$

2. If the man in Ex. 1 had traveled 814 mi. in 12 da., what would have been the average distance per day?

We have to divide 814 mi. by 12.

Since 814 = 814.0 = 814.00, we may place a decimal point after the 814, followed by as many zeros as we please. Dividing 814.0 by 12 as in Ex. 1, we have for the quotient 67.8, and 0.4 remainder.

We may write the quotient $67.8\frac{1}{3}$, or we may annex more zeros and continue to divide as far as we please. The quotients $67.8\frac{1}{3}$, $67.83\frac{1}{3}$, $67.833\frac{1}{3}$, and so on, all have the same value. For practical purposes such a result would be written simply 67.8.

$$\begin{array}{rr}
 & 67.8 \\
12 & \overline{)814.0} \\
 & \underline{72} \\
 & 94 \\
 & \underline{84} \\
 & 100 \\
 & \underline{96} \\
 & 4
\end{array}$$

ORAL EXERCISE

Divide, in turn, by 3, by 4, and by 5:

1. 360	$\frac{360}{100}$	0.60	0.6	1.2	0.12
2. 240	$\frac{240}{100}$	0.24	2.4	24	0.024

Divide, in turn, by 6 and by 7:

3. 42	4.2	0.42	420	0.042	$\frac{42}{10000}$
4. 84	8.4	0.84	840	8400	0.084

WRITTEN EXERCISE

1. The perimeter of a triangle whose sides are all equal is 1237.8 in. What is the length of each side?

2. The perimeter of a six-sided figure, with equal sides, is 7.62 in. What is the length of each side?

3. Four boys are camping on the shore of a lake and pay $17.64 for the outfit. How much is each boy's share?

4. The distance from Chicago to Cedar Rapids is 219.4 mi. How many miles in $\frac{1}{4}$ of the distance?

Divide, carrying each quotient to one decimal place:

5. 2087.2 ÷ 5.
6. 6039.3 ÷ 7.
7. 4276.8 ÷ 6.
8. 3748.1 ÷ 9.
9. 5263.7 ÷ 8.
10. 843.2 ÷ 72.
11. 342.8 ÷ 66.
12. 802.7 ÷ 69.
13. 534.8 ÷ 92.
14. 417.6 ÷ 87.
15. 52.3 ÷ 48.
16. 62.9 ÷ 69.
17. 34.1 ÷ 128.
18. 46.7 ÷ 293.
19. 99.8 ÷ 487.

Divide, carrying each quotient to two decimal places:

20. 274.1 ÷ 36.
21. 428.3 ÷ 41.
22. 720.6 ÷ 84.
23. 293.4 ÷ 73.
24. 820.9 ÷ 29.
25. 873.4 ÷ 37.
26. 607.4 ÷ 49.
27. 700.6 ÷ 58.
28. 801.1 ÷ 63.
29. 803.7 ÷ 96.
30. 42.38 ÷ 46.
31. 73.42 ÷ 68.
32. 82.37 ÷ 49.
33. 64.86 ÷ 38.
34. 42.73 ÷ 19.
35. 68.42 ÷ 37.
36. 72.26 ÷ 35.
37. 90.71 ÷ 54.
38. 82.32 ÷ 94.
39. 72.08 ÷ 33.
40. 4287 ÷ 693.
41. 2909 ÷ 426.
42. 4702 ÷ 328.
43. 5309 ÷ 772.
44. 6027 ÷ 428.
45. 5009 ÷ 762.
46. 8121 ÷ 406.
47. 9303 ÷ 500.
48. 7000 ÷ 228.
49. 6280 ÷ 600.

ORAL EXERCISE

1. Divide 10 by 2; 100 by 20. Compare the results.

2. Divide $2.50 by 5; $25 by 50. Compare the results.

3. If multiplying both dividend and divisor by the same number does not change the quotient, how does the quotient of 360 ÷ 4 compare with that of 36 ÷ 0.4?

143. Division by a Decimal. Since both divisor and dividend may be multiplied by the same number without changing the quotient, therefore,

In the division of decimal fractions, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer, and divide as in United States money.

Divide 6.25 by 2.5.

Multiplying 2.5 and 6.25 by 10 we have 25 and 62.5. The result is the same if we divide 62.5 by 25.

Dividing as in United States money, 62.5 ÷ 25 = 2 with 12.5 still to be divided. Then 12.5 ÷ 25 = 0.5. Therefore the quotient is 2.5.

We might, if we desired, multiply both 2.5 and 6.25 by 100, giving 625 to be divided by 250. The result would be the same.

$$\begin{array}{r} 2.5 \\ 25\overline{)62.5} \\ 50 \\ \hline 12\,5 \\ 12\,5 \\ \hline \end{array}$$

Instead of moving both decimal points to the right, we may draw a line to show the position of the decimal point in the quotient, as in the annexed solution. That is,

In the division of decimal fractions, draw a vertical line as many places to the right of the decimal point in the dividend as there are decimal places in the divisor. Then divide as with whole numbers, placing the decimal point in the quotient above this vertical line.

$$\begin{array}{r} 2.5 \\ 2.5\overline{)6.2|5} \\ 5\,0 \\ \hline 1\,2\,5 \\ 1\,2\,5 \\ \hline \end{array}$$

Both of these plans are in common use. The school should adopt either plan and adhere to it in teaching.

ORAL EXERCISE

1. How long will it take a railway train traveling 0.7 mi. a minute to go 14.7 mi.? to go 28 mi.?

2. How long will it take a railway train traveling 0.8 mi. a minute to go 32.8 mi.? to go 40 mi.?

3. A drawing 9.6 in. long and 0.8 in. wide is how many times as long as wide?

4. How many times is 0.1 in. contained in 3.8 in.?

Divide the following:

5. \$6 ÷ \$3.
6. $\frac{6}{10} \div \frac{3}{10}$.
7. 0.6 ÷ 0.3.
8. 0.06 ÷ 0.03.
9. 1.2 ÷ 0.4.
10. 2.5 ÷ 0.5.
11. 3.6 ÷ 0.9.
12. 3.5 ÷ 0.7.
13. 2.8 ÷ 0.7.
14. 8.1 ÷ 0.9.
15. 7.2 ÷ 0.8.
16. 5.6 ÷ 0.7.

WRITTEN EXERCISE

1. How long will it take an aeroplane traveling at the rate of 1.2 mi. a minute to go 16.8 mi.?

2. The speedometer of an automobile shows that the car has gone 23.9 mi. on a trip of 143.4 mi. What part of the total distance has the car gone?

Divide the following:

3. 125 ÷ 25.
4. 12.5 ÷ 2.5.
5. \$1.25 ÷ \$0.25.
6. \$125 ÷ \$0.25.
7. \$1250 ÷ \$2.50.
8. 6.232 ÷ 0.82.
9. 462 ÷ 22.
10. 46.2 ÷ 2.2.
11. 4.62 ÷ 2.2.
12. 0.462 ÷ 0.21.
13. 11.56 ÷ 3.4.
14. 17.466 ÷ 7.1.
15. 625 ÷ 0.25.
16. 600 ÷ 0.06.
17. 629 ÷ 0.37.
18. 7.14 ÷ 4.2.
19. 30.5 ÷ 0.61.
20. 62.32 ÷ 8.2.

ORAL EXERCISE

State the following results:

1. 10 × 0.4	4 ÷ 10	10 × 1.4	14 ÷ 10
2. 10 × $0.04	$0.40 ÷ 10	10 × 0.04	0.4 ÷ 10
3. 10 × $0.25	$2.50 ÷ 10	10 × 0.25	2.5 ÷ 10
4. 10 × 12.5	125 ÷ 10	100 × 0.04	4 ÷ 100

144. Dividing by Powers of 10. From the above exercise and from the work on page 30, we find the following:

To divide a number by 10, move the decimal point one place to the left.

Any whole number may be thought of as having a decimal point after units. That is, 6 is the same as 6. or 6.0 or 6.00.

To divide a number by 100, move the decimal point two places to the left.

Thus 7 ÷ 100 = 0.07 or .07.

To divide a number by 1000, move the decimal point three places to the left.

$$\boxed{\begin{array}{r} 6\cancel{0}\cancel{0})\underline{2.16} \\ 0.36 \end{array}}$$

To divide 216 by 600 is the same as to divide 2.16 by 6.

WRITTEN EXERCISE

Divide the following:

1. 133 ÷ 70.
2. 10.8 ÷ 60.
3. 32.9 ÷ 70.
4. 3745 ÷ 500.
5. 2291 ÷ 290.
6. 296.1 ÷ 470.
7. 273.6 ÷ 120.
8. 2984 ÷ 4000.
9. 7344 ÷ 9000.
10. 625 ÷ 25,000.
11. 961 ÷ 31,000.
12. 882 ÷ 21,000.
13. 1728 ÷ 1200.
14. 144 ÷ 60,000.
15. 872 ÷ 22,000.
16. 7260 ÷ 2500.
17. 154 ÷ 12,000.
18. 145.7 ÷ 4700.
19. 57.19 ÷ 190.
20. 571.9 ÷ 1900.
21. 162.5 ÷ 250.

145. Illustrative Problems. 1. Divide 16.35 by 500.

Here it is better to divide both divisor and dividend by 100, canceling the zeros and moving the decimal point two places to the left.

$$\begin{array}{r} 5)\underline{0.1635} \\ 0.0327 \end{array}$$

2. Divide 3.82 by 2.5, carrying the quotient to one decimal place.

Multiplying both numbers by 10, we have 38.2 ÷ 25 = 1.5 with .7 remainder. If we do not write the fraction $\frac{7}{25}$ in the quotient, the quotient may be written 1.5 +, meaning 1.5 and a fraction less than $\frac{1}{2}$.

3. Divide 0.049 by 0.17, carrying the quotient to two decimal places.

Multiplying both numbers by 100, we have 4.9 ÷ 17 = 0.28 with .14 remainder. If we do not write the fraction $\frac{14}{17}$ in the quotient, the quotient may be written 0.29 −, meaning 0.28 and a fraction $\frac{1}{2}$ or greater than $\frac{1}{2}$.

WRITTEN EXERCISE

Divide the following:

1. 17.25 ÷ 500.
2. 3.248 ÷ 200.
3. 817.6 ÷ 700.
4. 82.48 ÷ 8000.
5. 0.7364 ÷ 7000.
6. 74.34 ÷ 9000.
7. 15.72 ÷ 12,000.
8. 133.1 ÷ 11,000.
9. 404.3 ÷ 13,000.
10. $6844.50 ÷ 900.
11. $7337.75 ÷ 500.
12. $1247.01 ÷ 300.

Divide, carrying the quotients to three decimal places:

13. 17.78 ÷ 1.5.
14. 62.23 ÷ 0.23.
15. 19.26 ÷ 3.1.
16. 21.43 ÷ 3.7.
17. 33.43 ÷ 6.8.
18. 423.6 ÷ 0.27.
19. 167.8 ÷ 3.7.
20. 8.348 ÷ 2.68.
21. 41.73 ÷ 2.18.
22. 44.87 ÷ 23.7.
23. 97.634 ÷ 0.125.
24. 483.62 ÷ 42.7.
25. 826.3 ÷ 12.5.
26. 42.78 ÷ 3.162.
27. 75.41 ÷ 2.807.

146. Estimating Results. In the following exercise pupils should state the approximate answers orally, and these should be written down. The problems should then be solved on paper and the real result compared with the approximate one. This is one of the best checks against absurd results, and should frequently be made a subject of class drill with other written exercises.

Requiring pupils to estimate results carries out the policy already referred to on page 26. Teachers should not expect to receive the same approximations from all pupils. A natural approximation for Ex. 1 is $3\frac{1}{2} \times 22$¢.

WRITTEN EXERCISE

1. At 22¢ a pound, how much will $3\frac{5}{8}$ lb. of steak cost?

2. At 18¢ a gallon, how much will 6.25 gal. of oil cost?

3. At 26¢ a pound, how much will $6\frac{7}{8}$ lb. of butter cost?

4. At $12\frac{1}{2}$¢ a yard, how much will $8\frac{3}{4}$ yd. of ribbon cost?

5. At $2 a yard, how much will 2.8 yd. of velvet cost?

6. A dealer buys 200 lb. of raisins for $19. How much do they cost a pound?

7. A dealer pays $18.83 for 7 rocking-chairs. How much does he pay for each?

8. A dealer sells 17 pails of mackerel for $40.80. What is the price per pail?

9. A man pays $1560 a year for the rent of a city house. How much does he pay a month?

10. Traveling at the rate of 47 mi. an hour, how far will a train go between 8.45 A.M. and 11.30 A.M.?

11. A grocer buys a 50-gallon barrel of Porto Rico molasses for $11.50. How much does it cost per gallon?

12. A grocer sells a half chest of tea, 140 lb. to the chest, for $19.60. What is the price per pound?

147. Cancellation with Decimals. A dealer has two sizes of boxes in which he sells candy. The larger box is 4.2 in. by 8.6 in. by 2.1 in., and the smaller one is 4.2 in. by 4.3 in. by 1.4 in. The larger box holds how many times as much as the smaller one?

$$\boxed{\frac{\cancel{4.2} \times \overset{\cancel{2}}{\cancel{8.6}} \times \overset{3}{\cancel{2.1}}}{\cancel{4.2} \times \cancel{4.3} \times \underset{\cancel{2}}{\cancel{1.4}}}}$$

We see that we must divide 4.2 × 8.6 × 2.1 by 4.2 × 4.3 × 1.4.

Writing the work like a fraction, as here shown, we cancel common factors.

Evidently 4.2 may be canceled at once.

We also see that 8.6 = 2 × 4.3, so we may cancel the factor 4.3, leaving 2 over the 8.6 in the dividend.

We now have left 2 × 2.1 in the dividend and 1.4 in the divisor. We may now cancel the factor 0.7 from 1.4 and 2.1, and then cancel the 2.

Therefore the dividend is 3 times the divisor, and so the larger box is 3 times the smaller one.

If we wish to do so, in this example, we may multiply both dividend and divisor by 1000, thus having 42 × 86 × 21 to be divided by 42 × 43 × 14, and avoiding decimals entirely. This is often the better plan.

WRITTEN EXERCISE

Find the values of the following:

1. $\frac{3.9 \times 0.84}{15.6}$.
2. $\frac{1.4 \times 0.6}{0.63}$.
3. $\frac{2.2 \times 2.4}{4 \times 7.7}$.
4. $\frac{5.5 \times 0.42}{0.77}$.
5. $\frac{4.5 \times 6.3}{8.1}$.
6. $\frac{5.6 \times 0.3}{0.63 \times 0.8}$.
7. $\frac{2.1 \times 0.4}{0.015}$.
8. $\frac{0.21 \times 0.6}{0.126}$.
9. $\frac{0.8 \times 1.5 \times 0.6}{3.6 \times 0.2 \times 10}$.
10. $\frac{1.2 \times 0.39 \times 1.4}{2.6 \times 0.42 \times 2}$.
11. $\frac{0.2 \times 3.6 \times 1.43}{9.9 \times 0.4 \times 2.6}$.
12. $\frac{6.3 \times 4.2 \times 0.17}{2.7 \times 3.4 \times 0.49}$.

13. A block of marble 3.5 ft. by $4\frac{1}{4}$ ft. by 2.2 ft. weighs how many times as much as one that is $2\frac{1}{2}$ ft. by 7.7 ft. by 0.85 ft.?

14. If 4 tennis rackets cost $8, how much will 2 rackets cost? Solve by one division.

15. If 12 fishing reels cost $7.80, how much will 3 fishing reels cost? How much will 4 fishing reels cost? In each case solve by one division.

16. If 3 shotguns cost $29.25, how much will 15 shotguns cost? Solve by one multiplication.

17. If 3 cameras cost $14.25, how much will 4 cameras cost? Solve by one division and one addition, or else by one multiplication by a mixed number.

18. If 4 spoon hooks cost $1.76, find how much 2 spoon hooks will cost, and then find how much 6 spoon hooks will cost.

19. If 6 trout baskets cost $5.40, find how much 3 trout baskets will cost, and then find how much 9 trout baskets will cost.

20. If 2 folding camp cots cost $4.50, find how much one will cost, and then find how much 3 cots will cost.

21. If 6 telegraph outfits for learners cost $17.10, how much will 2 outfits cost? How much will 4 outfits cost?

22. If 3 music bags cost $2.94, how much will 9 music bags cost? How much will 11 music bags cost?

23. If 2 German accordeons cost $7.30, how much will 3 German accordeons cost?

24. If 2 violins cost $9.50, how much will 5 violins cost?

25. If 3 banjos cost $6.75, how much will 7 banjos cost?

26. If 4 mandolins cost $11.40, how much will 6 mandolins cost? How much will 9 mandolins cost?

27. If 6 boxes of writing paper cost $1.80, how much will 9 boxes cost? How much will 12 boxes cost? How much will 15 boxes cost? How much will 24 boxes cost?

WRITTEN REVIEW

1. A man has two farms, one containing 175.4 A., and the other 163.8 A. How many acres of land has he in both farms?

2. At $75 an acre, how much will 62.4 A. of land cost?

3. An automobile travels 28.6 mi. the first hour and 22.9 mi. the second hour. How far does it travel in the two hours?

4. In going from Denver to Pueblo, a distance of 119.4 mi., the train passes through Colorado Springs, 74.9 mi. from Denver. How far is it from Colorado Springs to Pueblo?

5. A train leaving Denver at 7.45 P.M. reaches Santa Fe the next day at 4.25 P.M., a distance of 405.9 mi. Find the average rate per hour. Remember that 4.25 P.M. means 4 hr. 25 min. P.M.

6. In traveling from Chicago to Winona, a distance of 296.1 mi., we pass through Madison, 129.7 mi. from Chicago. The distance from Chicago to Madison lacks how much of being half the distance from Chicago to Winona?

7. One train leaves Chicago at 10.30 A.M. and arrives at Omaha at 11.59 P.M. Another train leaves Chicago at 7 P.M. and arrives at Omaha at 7.20 the next morning. The distance being 491.3 mi., find the average rate of each train per hour.

8. A man, on an automobile trip, stops for luncheon when the indicator on his car shows that he has traveled 37.8 mi. At the end of the trip he finds he has traveled 113.4 mi. What part of the trip had he made when he stopped for luncheon?

9. Two cousins, Frank and Tom, visit their uncle in Denver. Frank lives in Colorado Springs, 74.9 mi. from Denver, and Tom lives in Carbondale, 373 mi. from Denver. Tom tells his uncle that Carbondale is five times as far from Denver as Colorado Springs is, but Frank says it is not. Which is right?

Find the cost of the following:

10. 27.9 A. of land at $62.50 per acre.

11. 72.8 A. of land at $87.50 per acre.

12. A city lot having a frontage of 62.8 ft., at $375 per foot.

13. 165.48 A. of land at $125 per acre; at $140 per acre.

14. 27.5 T. of hay at $16.40 per ton; at $17.50 per ton.

15. 48.6 T. of hay at $14.75 per ton; at $15.25 per ton.

16. 23.48 mi. of road at $1875 per mile; at $2350 per mile.

17. 148.72 mi. of railway at $21,748.50 per mile.

18. 61.82 mi. of canal excavating at $164,872.25 per mile.

19. Kate's brother Fred is in college. Last year his entire expenses were $625.75. At the same rate, how many boys could be kept in college for a year for the amount paid for a single battleship that cost $12,515,000?

20. A passenger train leaves Clinton at 5.10 P.M. and reaches Anamosa, a distance of 71.4 mi., at 8.05 P.M. A freight train leaves Clinton at 4.15 A.M. and reaches Anamosa at 11 A.M. By how many miles an hour does the average rate of the passenger train exceed that of the freight train?

21. Marquette is 71.7 mi. from Hancock. A train leaves Marquette at 4.45 and reaches Hancock at 8.10, while another leaves at 7.45 and reaches Hancock at 10.30. Which train makes the greater average distance per hour, and how much greater?

22. The distance from Milwaukee to St. Paul is 325.6 mi. The distance from Milwaukee to Wyeville, 153.9 mi., lacks how much of being half the distance from Milwaukee to St. Paul?

23. A farmer buys three adjoining pieces of land, the first containing 142.7 A., the second 115.6 A., and the third 168.7 A. He then sells half of the land. How many acres does he sell?

24. A man has three adjacent lots with a frontage of 148.5 ft. One of the lots has a frontage of 52.75 ft. and another of 49.5 ft. What is the frontage of the third lot?

25. A man who had a farm of 215.8 acres bought an adjacent farm of $192\frac{3}{4}$ acres. He sold a field of $72\frac{1}{3}$ acres and another of 68.25 acres. How much land had he left?

26. The perimeter of a triangle is 27.25 ft., and two of the sides are $8.33\frac{1}{3}$ ft. and $9.75\frac{2}{3}$ ft. What is the third side?

27. The distance from New Orleans to Baton Rouge is 89.4 mi., and from New Orleans to Natchez by way of Baton Rouge, 214.1 mi. How far is it from Baton Rouge to Natchez?

28. The distance from Chicago to Lincoln is 561.4 mi., and from Chicago to Colorado Springs by way of Lincoln, 1072.2 mi. How far is it from Lincoln to Colorado Springs?

29. If the four sides of a field are 22.75 rd., 17.65 rd., 28.6 rd., and 18.35 rd., what is the perimeter of the field?

30. From the sum of 218.7 lb. and 306.4 lb. subtract the sum of 175.4 lb. and 163.7 lb.

31. How many bushels of potatoes will 24 acres produce if an average acre produces 125 bu.?

32. If a dealer buys three barrels of sugar containing respectively 313.6 lb., 314.8 lb., and 315.5 lb., how many pounds does he buy in all?

33. A man bought three barrels of sugar containing respectively 312.75 lb., 314.75 lb., and 316.5 lb. How many pounds did he buy in all?

34. A man has to make a railway journey of 254.7 miles. After he has gone $\frac{1}{3}$ of the distance he receives a telegram calling him halfway back to his starting point. After he gets there, how many miles is it to the end of his journey?

LITTLE EXAMINATIONS

I. 1. Change to common fractions or to mixed numbers:

$0.6 \qquad 0.75 \qquad 0.33\frac{1}{3} \qquad 0.87\frac{1}{2} \qquad 3.25$

2. Change to decimals, carrying the results to thousandths:

$\frac{2}{3} \qquad \frac{3}{8} \qquad \frac{3}{5} \qquad 2\frac{3}{16} \qquad 32\frac{5}{8} \qquad 33\frac{1}{3}$

3. $72.49 + 36.84 + 90.73 + 86.87 + 230.4 + 2.75.$

4. $52 + 78.26 + 140.3 + 9.625 + 41.7 - 129.967.$

5. $25 \times 37.42.$

6. $8.42 \times 264.$

7. $5.36 \times 21.7.$

8. $84.76 \div 46.$

9. $16.074 \div 4.8.$

10. $12.446 \div 0.23.$

II. 1. Change to common fractions or to mixed numbers:

$0.37\frac{1}{2} \qquad 0.66\frac{2}{3} \qquad 0.16\frac{2}{3} \qquad 0.06\frac{1}{4} \qquad 4.75$

2. Change to decimals, carrying the results to thousandths:

$\frac{5}{8} \qquad 3\frac{3}{8} \qquad 7\frac{7}{8} \qquad 3\frac{5}{16} \qquad 2\frac{1}{32} \qquad 5\frac{3}{64}$

3. $26.98 + 4.764 + 314.9 + 34.69 + 2.786.$

4. $27.99 + 6.783 + 427.6 + 96.49 - 75.496.$

5. $75 \times 68.64.$

6. $5\frac{3}{4} \times 29.48.$

7. $6.31 \times 41.82.$

8. $78.75 \div 2.5.$

9. $9.648 \div 0.75.$

10. $234.8 \div 1.25.$

III. 1. $2\frac{3}{8} \times 74.64.$

2. $0.12\frac{1}{2} \times 85.6.$

3. $3\frac{2}{3} \times 42.27.$

4. $7.3\frac{1}{3} \times 45.9.$

5. $8 \div 1.25.$

6. $0.014 \div \frac{7}{8}.$

7. $3.75 \div \frac{3}{8}.$

8. $52.83 \div 0.9.$

9. $5.001 \div 0.03.$

10. Multiply 2.008 by $5\frac{3}{4}$ and divide the product by $2\frac{7}{8}$.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. A corner stone in a building is dated MDCCCLXXXIX. In what year was the building erected?

2. Write in figures seven million, ninety-six and five tenths.

Add the following:

3.	**4.**	**5.**	**6.**	**7.**
$27.36	$52.37	$61.72	$82.73	$48.73
4.83	26.42	3.89	61.54	60.07
23.49	3.58	55.70	20.09	2.96
17.06	47.63	6.32	17.27	40.37
5.27	73.58	12.47	38.46	6.84

8. Multiply 17 ft. 3 in. by 4; by 7; by $2\frac{1}{2}$.

9. Divide 11 ft. 8 in. by 4; by 5; by 2.5.

10. Find the sum of 17 rd. $9\frac{1}{2}$ ft. and 26 rd. $8\frac{1}{3}$ ft.

11. From 3 A. 40 sq. rd. subtract $2\frac{1}{2}$ A.

12. From the sum of $296.40 and $37.65 subtract $198.79.

Write as rapidly as you can the results of the following:

13. $\frac{1}{2} + \frac{1}{4}$.
14. $\frac{1}{2} \times \frac{1}{4}$.
15. $\frac{1}{2} \div \frac{1}{4}$.
16. $\frac{3}{4} - \frac{1}{2}$.
17. $\frac{3}{4} \times \frac{1}{2}$.
18. $\frac{3}{4} \div \frac{1}{2}$.
19. $\frac{1}{2} + \frac{1}{8}$.
20. $\frac{1}{2} \times \frac{1}{8}$.
21. $\frac{1}{2} \div \frac{1}{8}$.
22. $\frac{7}{8} - \frac{1}{2}$.
23. $\frac{5}{8} \times \frac{3}{4}$.
24. $\frac{7}{8} \div \frac{3}{4}$.

Add the following:

25.	**26.**	**27.**	**28.**	**29.**
240.4	64.28	33.21	169.28	147.26
802.2	85.	150.5	3.7	38.09
79.8	6.32	7.32	45.94	140.6
388.7	108.51	46.48	2.33	3.98

30.	**31.**	**32.**	**33.**	**34.**
4.028	0.387	1.043	0.2909	0.7864
.396	.0426	.427	.0628	.3429
.285	.0539	6.091	.0703	.0008
4.72	.2064	.5	.5076	.703
3.942	.0708	.709	.0289	.0682
6.587	.0684	.792	.0587	.5209

Add, and also subtract, the following:

35.	**36.**	**37.**	**38.**	**39.**
90.32	209.8	6.273	81.04	50.93
4.86	49.2	.992	6.98	27.98

40.	**41.**	**42.**	**43.**	**44.**
1.035	0.2809	82.481	64.035	70.068
.876	.1589	2.909	37.029	39.579

Multiply or divide as indicated:

45. 1.4 × 6.64.

46. 3.6 × 72.08.

47. 888.16 ÷ 2.8.

48. 103.6 ÷ 0.56.

49. 1.5 × 553.35.

50. 5.8 × 439.002.

51. 825.468 ÷ 4.2.

52. 1131.264 ÷ 4.8.

53. 1.6 × 140.64.

54. 6.2 × 53.134.

55. 20.72 ÷ 0.28.

56. 38.8692 ÷ 5.4.

57. 388.692 ÷ 5.4.

58. 3.88692 ÷ 5.4.

59. 3886.92 ÷ 0.54.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you are multiplying dollars and cents by a fraction, where do you place the decimal point in the product?

2. If you are dividing dollars and cents by a whole number, how do you know where to place the decimal point in the quotient?

3. If you are multiplying one decimal fraction by another, how many decimal places must you have in the product? Why is this the case? Illustrate by an example.

4. If you are dividing a decimal fraction by a whole number, how can you tell where to put the decimal point in the quotient? Illustrate by an example.

5. If you are dividing a decimal fraction, and the divisor is not a whole number, how may you make it such without altering the quotient? Give your reason.

6. If you know the length of each side of a triangle, how do you find the perimeter, or distance around the triangle?

7. If you know the length of only one side of a square, how do you find the perimeter? Illustrate by an example.

8. If you know the length of only one side of a square, how do you find the area of the square? Illustrate by an example.

9. If you know the cost of several baseball bats, how do you find the cost of one?

10. If you know the cost of baseballs by the dozen, how do you find the cost of each ball? Could you probably buy one at this price? If not, why?

11. If you know the cost of a certain number of tennis balls, how do you find the cost of a certain other number?

III. INDUSTRIAL PROBLEMS

DOES IT PAY TO HAVE GOOD ROADS?

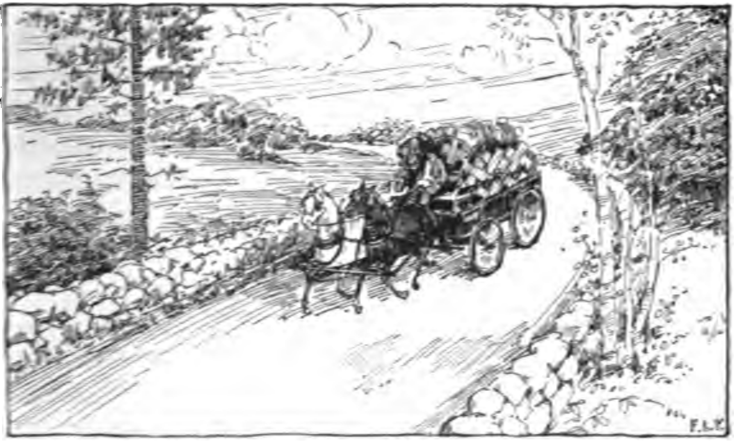
1. Do good roads pay? Here is a picture of a bad road in the country. A teamster had to haul $7\frac{1}{2}$ tons of barbed wire from the railway, a distance of 23 mi. He found that he could haul only 1500 lb. to a load and that it took him $1\frac{1}{2}$ da. to do it. How long did it take him to haul the $7\frac{1}{2}$ tons of wire, and how much did it cost at $4 per day for man and team?

2. In Ex. 1, what was the cost of hauling 1 ton 1 mi., or the cost of 1 ton-mile, as it is ordinarily called?

3. The next picture shows the same road after improvement. The teamster finds that with the same team he can now easily haul $2\frac{1}{2}$ tons to the load and do the work in 1 da. What does it now cost to haul a ton of wire? What is the cost per ton-mile?

4. Comparing the results in Exs. 2 and 3, what common fraction less is the cost per ton-mile in Ex. 3, owing to good roads?

5. If a teamster has to haul 30 tons of merchandise 23 mi. over this road, what is the cost if the road is in the bad condition referred to in Ex. 1? What is the cost if the road is in the good condition referred to in Ex. 3 and shown in the following picture?



6. A farmer lives 10 mi. from the railway. The road was formerly so bad that with a two-horse team he could haul only 40 bu. of wheat, and it took a day to make the round trip. If the farmer reckoned $4 a day for man and team, how much did it cost per bushel to haul the wheat?

7. The roads were recently improved. The farmer can now haul 75 bu. with his team. Find the cost per bushel of hauling the wheat now.

8. Comparing Exs. 6 and 7, how much more money does the farmer really get per bushel now that he has good roads?

9. Taking 60 lb. as the weight of a bushel of wheat and comparing Exs. 6 and 7, what has been the reduction in cost of cartage per ton-mile?

10. The average cost of hauling cotton to market in the Southern states is 80¢ per bale. If these states market 16,250,000 bales a year, what is the cost of hauling it?

11. A state often borrows money to build good roads. New York State has borrowed $100,000,000 for this purpose, and pays each year as interest 0.04 of this sum. How much interest does the state pay per year? New York City, because of its wealth, pays $\frac{7}{10}$ of this. What is left for the rest of the state to pay?

12. Sometimes a state neglects its roads after they are improved. In this way a certain road went to pieces in a single year, and it cost $1400 a mile to restore it. If it had been kept in repair, the total cost would have been only $608 a mile. On a road 78 mi. long, how much did the state waste by its carelessness?

13. Automobiles help to pay for the roads they use. In a recent year there were 1,900,000 motor vehicles of all kinds in this country, and they paid on an average $10 a year for registration and licenses. How much did they pay to the states?

14. A county produces an average of 30 bu. of wheat to the acre, and this wheat has to be hauled an average of 5 mi. If good roads reduce the hauling cost 5¢ per ton-mile, how much do they increase the value of the wheat per acre? At 60 lb. to the bushel, what part of a ton is produced to the acre?

15. Having solved Ex. 14, let us find the saving on this part of a ton for 5 mi. at 5¢ per ton-mile.

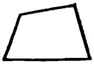
16. Following the work in Ex. 15, let us now consider how much this means per square mile. There being 640 acres to a square mile, how much is the saving per square mile?

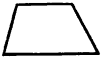
17. If the county mentioned in Exs. 14–16 contains 300 sq. mi., and 0.6 of the land is used for growing wheat, how much does the county gain in value of wheat per year by having good roads?

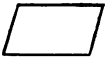
CHAPTER V

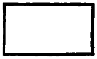
PRACTICAL MEASUREMENTS

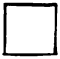
148. Figures commonly Measured. We are already familiar with most of the figures commonly measured. We shall now review some of these and learn how to measure others.


Quadrilateral


Trapezoid


Parallelogram


Rectangle


Square

149. Quadrilateral. A figure which is bounded by four straight lines is called a *quadrilateral.*

150. Trapezoid. A quadrilateral which has one pair of opposite sides parallel is called a *trapezoid.*

151. Parallelogram. A quadrilateral which has its two pairs of opposite sides parallel is called a *parallelogram.*

152. Rectangle. A parallelogram which has all its angles right angles is called a *rectangle.*

153. Square. A rectangle which has its sides all equal is called a *square.*

154. Polygon. A figure which is bounded by straight lines is called a *polygon.* The figures pictured above are all special kinds of polygons.

As a preliminary to this chapter the tables on pages 108–119 should be reviewed; and in order that this review should be thorough, much oral work is provided on pages 186–187.

ORAL EXERCISE

1. Express 640 rd. as miles.
2. Express 64 oz. as pounds.
3. Express 2 A. as square rods.
4. Express 33 ft. as rods; as yards.
5. Express 54 cu. ft. as cubic yards.
6. Express 9 ft. as inches; as yards.
7. Express 11 da. as hours; as weeks.
8. Express 17 qt. as pints; as gallons.
9. Express 9 pk. as quarts; as bushels.
10. How many cubic inches in 10 cu. ft.?
11. How many rods in 3 mi.? in 10 mi.?
12. How many yards in 8 rd.? in 10 rd.?
13. How many cubic feet in 2 cd. of wood?
14. Express 300 min. as hours; as seconds.
15. How many quarts in a barrel of $31\frac{1}{2}$ gal.?
16. How many pints in a 60-gallon hogshead?
17. Express 80 sq. rd. as a fraction of an acre.
18. How many right angles in 270°? in 180°?
19. How many days from May 23 to June 25?
20. How many days from June 11 to July 17?
21. What part of a 16-carat ring is pure gold?
22. Express 11 yd. as rods; as feet; as inches.
23. Express 30 pt. as quarts; as gills; as gallons.
24. How many feet in 2 rd. 8 ft.? in 2 rd. 10 ft.?
25. How many feet in 10 rd.? in 5 rd.? in 2 rd.?
26. How many degrees in 540′? in 420′? in 660′? in 720′?

27. How many mills in $2.75?

28. How many days from August 15 to October 15?

29. How many acres in a tract of land 10 mi. square?

30. How many days from January 17 to February 17?

31. How many square feet in 10 sq. yd.? in 5 sq. yd.?

32. Using 6 qt. a day, how long will 3 bu. of oats last a horse?

33. What is the perimeter of a square that is 4½ ft. on a side?

34. What is the side of a square whose perimeter is 14 ft. 4 in.? of a square whose perimeter is 4½ ft.?

35. Since a gallon contains 231 cu. in., how many gallons in 693 cu. in.?

36. How many square feet in a surface having an area of 720 sq. in.?

37. Express a long ton (2240 lb.) as a short ton and hundredths of a short ton.

38. What is the perimeter of an equilateral triangle that is 3 ft. 8 in. on a side?

39. If school closes June 21, how many days from that day to the Fourth of July?

40. How many quarts will a 10-bushel bin hold? a 5-bushel bin? How many pecks in each?

41. A man buys a building lot that has a frontage of 2 rd. How many square feet in his sidewalk if it is 5 ft. wide?

42. A cellar 15 ft. by 21 ft. is to be excavated to a depth of 6 ft. How many loads (cubic yards) of earth must be removed?

43. If a man buys a load of coal weighing 1 T. 500 lb., how many pounds of coal does he buy? Express this weight in tons and tenths of a ton.

WRITTEN EXERCISE

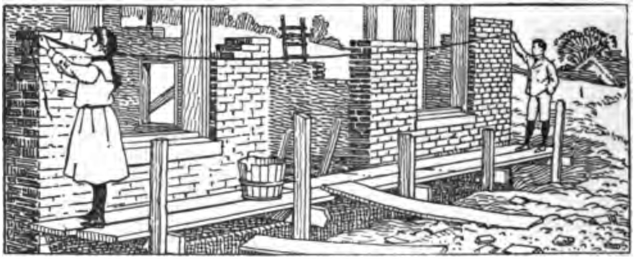
1. A baseball "diamond," or "infield," is a square 90 ft. on a side. Draw this to the scale of 1 in. to 20 ft.

As we found on page 110, this means that we represent 20 ft. by 1 in. Therefore 90 ft., which is 4½ × 20 ft., will be represented by 4½ in. Such work is very practical.

2. On the infield the pitcher's plate is 60.5 ft. from the point where two sides of the diamond meet at the home base. Draw another plan of the infield, using the scale of 1 in. to 30 ft., and mark the position of the pitcher's plate.

3. The first, second, and third bases are placed in squares 15 in. on a side. Draw one of these squares to the scale $\frac{1}{5}$.

This means that 5 in. is to be represented by 1 in.



4. Fred and Rose measure a house that is being built near the school. They find it to be 20 ft. by 30 ft. Draw a ground plan of the house to the scale of 1 in. to 5 ft.

5. They find that the front door is 4 ft. by 7 ft. 6 in. Draw a plan of the door to the scale $\frac{1}{12}$.

6. A brick is 8 in. long and 4 in. wide. Draw a plan of the top to the scale $\frac{1}{2}$.

Teachers should give much work in measuring and in drawing to scale.

7. Rob and Mary are measuring a cellar that is being dug for a house. They use a tape that is 100 ft. long. It is divided into feet, inches, half inches, and quarter inches. How many inches in the tape? How many half inches? How many quarter inches?

8. They find that the workmen have dug only part of the cellar, this part being 6 ft. wide. Express the width in yards. Express the width in inches.

9. They find that the excavation is 22 ft. 6 in. long. Express this in feet and a fraction; in yards and a fraction.

10. The distance from where Rob is to the house is 412 ft. 6 in. A rod being 16½ ft., express the distance in rods.

11. If a field is 17 rd. long, what is the length in feet?

12. If a field is 19 rd. long and 12 rd. wide, what length of fence is needed to inclose it?

13. If Jack steps 27 in. each time, and takes 450 steps in coming from his home to the school, how many feet does he come?

In each half year the teacher should give plenty of work in practical measuring. The pupils should measure the lengths of their ordinary steps, and should learn to pace such distances as the length of the room and the width of the playground. They should become perfectly familiar with the divisions of the ruler, and should have practice in estimating the height of the room, of trees, and of the schoolhouse; the width of the street and of pieces of land; and the distances they travel in coming to school. In order to be real, this work must apply to local conditions and must therefore be given by the teacher.

DRAWING TO SCALE

1. Draw a plan of a floor 16 ft. long and 12 ft. wide, representing 1 ft. by $\frac{1}{8}$ in.

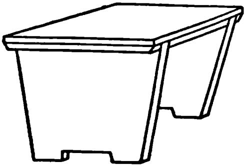
2. What part of 1 ft. is $\frac{1}{8}$ in.? Then the drawing in Ex. 1 is what part as long as the floor that it represents?

3. Draw a plan of a box cover 6 in. by 10 in., to the scale $\frac{1}{4}$.

4. Draw a plan of a school garden 56 ft. long by 48 ft. wide, to the scale of $\frac{1}{8}$ in. to 1 ft.

5. Draw a plan of a rug that the class is weaving. The rug is to be 10 in. by 16 in. Use the scale $\frac{1}{4}$.

6. This is a drawing of a doll's table made by the class. The top is to be 10 in. by 12 in. The end supports are to be 10 in. at the top, 7 in. at the bottom, and 8 in. high. Draw plans for the top and ends, using the scale $\frac{1}{4}$. The end supports are indented 1 in. at the distance of 2 in. from each bottom corner.



7. Draw a plan of a book cover that the boys are making. The cover is to be 5 in. by 8 in. Use the scale $\frac{1}{2}$.

8. A corridor in a school building is 12 ft. by 128 ft. Draw a floor plan of this corridor, using the scale of $\frac{1}{16}$ in. to 1 ft.

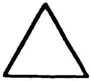
9. A playground near our school is 65 ft. by 125 ft. Draw a plan of the playground, using the scale of $\frac{1}{16}$ in. to 1 ft.

10. A schoolroom is 40 ft. long and 20 ft. wide. Draw a plan of the floor, using the scale $\frac{1}{80}$.

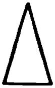
11. A sitting room is 14 ft. by 18 ft. Draw a plan of the floor, using any convenient scale.

12. A bedroom is 12 ft. 6 in. by 16 ft. Draw a plan of the floor, using any convenient scale.

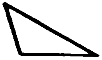
155. Equilateral Triangle. A triangle in which the three sides are equal is called an *equilateral* triangle.



Equilateral



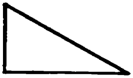
Isosceles



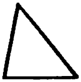
Scalene

156. Isosceles Triangle. A triangle in which two of the sides are equal is called an *isosceles* triangle.

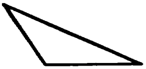
157. Scalene Triangle. A triangle in which no two sides are equal is called a *scalene* triangle.



Right



Acute



Obtuse

158. Right Triangle. A triangle in which one of the angles is a right angle is called a *right* triangle. The side opposite the right angle is called the *hypotenuse.*

159. Acute Triangle. A triangle in which all the angles are acute angles is called an *acute* triangle.

160. Obtuse Triangle. A triangle in which one of the angles is an obtuse angle is called an *obtuse* triangle.

ORAL EXERCISE

1. How long is the side of an equilateral triangle if the perimeter is 45 in.? 9 ft. 6 in.? 9½ yd.? 63 ft. 9 in.?

2. How long is the side of a square if the perimeter is 84 in.? 8 ft. 4 in.? 16 yd. 8 in.? 60½ rd.? 120 in.?

3. How long is the fourth side of a quadrilateral if the perimeter is 85 in. and three of the sides are 10 in., 20 in., 30 in.?

WRITTEN EXERCISE

1. What is the perimeter of an equilateral triangle $275\frac{2}{3}$ ft. on a side?

Draw figures of the correct shape in all examples on this page, not to scale but only approximately.

2. A triangle has a perimeter of 342 ft. 8 in. One side is 90 ft. and another is half as much longer; how long is the third?

3. The base of an isosceles triangle is 35 ft., and each of the equal sides is $\frac{1}{5}$ longer. What is the perimeter?

4. Cut off the three angles of a triangle and place them together. The sum is equal to how many right angles?

Find the sides of equilateral triangles of which the perimeters are as follows:

5. 113 ft. 3 in.
6. 127 yd.
7. 88 ft. 3 in.

Find the sides of squares of which the perimeters are as follows:

8. 110 ft. 4 in.
9. 42 ft. $3\frac{1}{2}$ in.
10. 61 yd. 2 ft.
11. 2 ft. 4 in.
12. 118 ft. 8 in.
13. 227 mi. 40 rd.

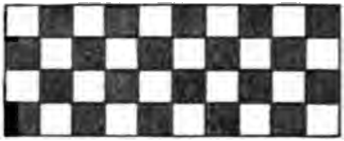
14. The perimeter of an isosceles triangle is 100 ft., and the unequal side is 33 ft. Find the length of one of the equal sides.

15. The perimeter of an isosceles triangle is 88 ft., and the unequal side is $\frac{1}{5}$ of the perimeter. Required the sides.

16. The perimeter of a triangle is 24 ft. One side is $\frac{1}{3}$ of the perimeter and another is $\frac{3}{8}$ of the perimeter. How many feet are there in each of the three sides?

17. The perimeter of a triangle is 120 ft. One side is $\frac{1}{4}$ of the perimeter and another is $\frac{1}{3}$ of the perimeter. How many feet are there in each of the three sides?

161. Area of a Rectangle. A school building has an entrance hall 10 ft. long and 4 ft. wide. The floor is made of marble squares 1 ft. on a side. What is the easiest way of finding the number of squares?



There are 10 squares in each row, and there are 4 rows. Hence there are 4 × 10 squares.

In the same way we see that the area of the floor is

4 × 10 sq. ft. = 40 sq. ft.

162. Base and Height. The line on which a figure appears to stand is called the *base.* The distance from the highest point above the base to the base is called the *height* or *altitude* of a figure.

163. Finding the Area of a Rectangle. From the above example and definitions we see that

The area of a rectangle equals the product of the base and height.

It must always be understood that such a rule refers to abstract numbers. We do not mean that we multiply 10 ft. by 4 ft. in § 161, but that

4 × 10 = 40, the *number* of square feet.

WRITTEN EXERCISE

Find the areas of the following rectangles, using short methods:

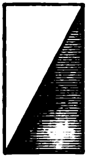
1. $37\frac{1}{2}$ ft. by 48 ft.
2. $33\frac{1}{3}$ yd. by 42 yd.
3. 75 ft. by 96 ft.
4. $66\frac{2}{3}$ yd. by 48 yd.
5. 125 ft. by 160 ft.
6. 250 ft. by 324 ft.
7. 625 ft. by 720 ft.
8. $87\frac{1}{2}$ in. by 96 in.
9. Find the area of the floor of a room 24 ft. by 28 ft. 6 in.

The teacher should give plenty of practical work in finding the areas of floors, platforms, desks, the school grounds, the sidewalk, and the like.

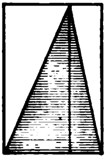
ORAL EXERCISE

1. How is the area of a rectangle found?

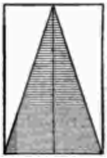
2. How do the areas of these triangles compare with the areas of the rectangles? Then a triangle is what part of a rectangle of the same base and the same height?



A



B



C

3. If the rectangle *A* is 2 in. wide and 4 in. high, what is its area? What is the area of the shaded triangle in *A*?

4. If the rectangle *B* is 3 ft. wide and 5 ft. high, what is its area? What is the area of the shaded triangle in *B*?

5. If the rectangle *C* is 4 yd. wide and 6 yd. high, what is its area? What is the area of the shaded triangle in *C*?

6. What is the area of a rectangle with base 6 ft. and height 9 ft.? of a triangle with base 6 ft. and height 9 ft.?

7. How do you find the area of a triangle?

8. Do you know of some triangular piece of ground the area of which you might wish to find? What measurements would you take to find the area of such a piece?

9. What is the area of a triangular flower bed with base 20 ft. and height 10 ft.?

Find the areas of triangles with bases and heights as follows:

10. 4 in., 3 in.
11. 8 in., 6 in.
12. 5 in., 4 in.
13. 7 in., 6 in.
14. 8 in., 10 in.
15. 9 ft., 4 ft.

164. Area of a Triangle. As shown on the preceding page,

The area of a triangle equals half the product of the base and altitude.

For example, what is the area of a triangle of base 18 in. and altitude 7 in.?

$\frac{1}{2}$ of 18 × 7 sq. in. = 63 sq. in.

ORAL EXERCISE

Using aliquot parts, state the areas of triangles with bases and altitudes as follows:

1. 80 in., 75 in.
2. 16 in., 125 in.
3. 60 in., $33\frac{1}{3}$ in.
4. 80 in., 25 in.
5. 44 in., 50 in.
6. 120 in., $16\frac{2}{3}$ in.

WRITTEN EXERCISE

Find the areas of triangles with bases and altitudes as follows:

1. 43 ft., 32 ft.
2. 7.9 in., 8.7 in.
3. 8.7 in., 6.3 in.
4. 15.2 in., 9.4 in.
5. $37\frac{1}{4}$ ft., $42\frac{1}{2}$ ft.
6. 2 yd. 32 in., 27 in.
7. 3 ft. 4 in., 2 ft. 8 in.
8. 2 ft. $2\frac{1}{2}$ in., 1 ft. $8\frac{1}{4}$ in.

9. What is the area of an equilateral triangle with perimeter 37 ft. 6 in. and altitude 10.8 ft.?

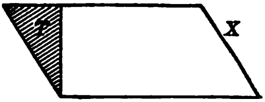
10. What is the area of a triangle with altitude 6.4 rd. and base 17 rd.? altitude 7.2 in. and base 17 in.?

11. The two sides forming the right angle of a right triangle are 30 rd. and 40 rd. What is the area in square rods? in acres?

12. What is the number of square feet of slating required for a steeple, the surface of which is made up of 6 triangles, each with base 20 ft. and altitude 65 ft.?

ORAL EXERCISE

1. If we cut triangle T from this parallelogram and place it where X is, what kind of a figure do we have? What does this tell us about the area of a parallelogram?



This work should actually be done, a parallelogram being cut from paper and then transformed into a rectangle as suggested.

State the areas of parallelograms with bases and heights as follows:

2. 11 ft., 9 ft.
3. 2 yd., $1\frac{1}{2}$ ft.
4. 6 ft., $9\frac{1}{2}$ ft.
5. 4 in., $6\frac{3}{4}$ in.
6. 8 ft., $10\frac{1}{4}$ ft.
7. 10 ft., $5\frac{1}{2}$ ft.

State the areas of triangles with bases and altitudes as follows:

8. 7 ft., 6 ft.
9. 8 ft., $4\frac{1}{2}$ ft.
10. 3 ft., 6 in.

165. Area of a Parallelogram. *The area of a parallelogram equals the product of the base and altitude.*

WRITTEN EXERCISE

Find the areas of parallelograms with bases and heights as follows:

1. $19\frac{3}{4}$ in., 16.8 in.
2. 171.4 ft., 27.2 ft.
3. 6 ft. 4 in., 4 ft. 6 in.
4. 14.7 ft., 16.5 ft.
5. 3.75 in., $2.33\frac{1}{3}$ in.
6. 16 ft. 6 in., 8.25 ft.

Find the areas of triangles with bases and altitudes as follows:

7. 31.2 in., 27.3 in.
8. 32 ft. 6 in., 12 ft.
9. 16.75 in., 14.2 in.
10. 17.3 ft., 1 rd.

11. A man has a piece of land in the form of a parallelogram. The base of the parallelogram is 36 rd. and the height is 24 rd. How many acres are there in the piece of land?

166. Altitude of a Parallelogram. What is the altitude of a parallelogram with base 10 ft. and area 60 sq. ft.?

If the base is 10 ft. and the altitude is 1 ft., the area is 10 sq. ft.

Therefore the number of feet in the height is 60 sq. ft. ÷ 10 sq. ft., or 6. In practical computation we do not label the numbers; we simply divide 60 by 10.

Teachers should review with care the statement on page 18 with respect to abstract and concrete numbers. They should also encourage the pupils to find the areas of pieces of land in the forms of triangles and parallelograms if such can be found in the vicinity of the school. Only part of the examples on pages 196 and 197 need be taken by any one class.

WRITTEN EXERCISE

Find the areas of parallelograms with bases and heights as follows:

1. 6.7 in., 3.9 in.
2. 42.5 ft., 27.8 ft.
3. 16.9 ft., 7.2 ft.
4. 3 ft., 2 ft. 7 in.
5. 5 rd. 6 ft., $2\frac{1}{2}$ rd.
6. 17 ft. 8 in., 27 in.
7. 9 ft. 6 in., 6 ft. 9 in.
8. 345 ft., 127 ft.
9. 2 yd. 17 in., 2 ft. 3 in.
10. 17 yd. 23 in., 5 yd. 2 in.

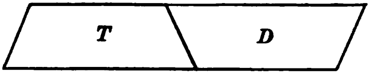
Find the heights of parallelograms with bases and areas as follows:

11. $15\frac{1}{2}$ in., 108.5 sq. in.
12. 25 ft., 275 sq. ft.
13. 22 ft. 4 in., 268 sq. ft.
14. 24 ft. 6 in., 588 sq. ft.
15. 129 ft., 10,320 sq. ft.
16. 31 ft. 2 in., 187 sq. ft.
17. 16 ft. 8 in., 100 sq. ft.
18. 17 ft. 6 in., 105 sq. ft.

Find the heights of triangles with bases and areas as follows:

19. 4 in., 6 sq. in.
20. 9 in., 36 sq. in.
21. 10 in., 45 sq. in.
22. 12 in., 60 sq. in.
23. 14 ft., 70 sq. ft.
24. 12 yd., 72 sq. yd.

167. Area of a Trapezoid. If a trapezoid T has its double cut from paper and turned over and fitted to it, like D, the two together make a parallelogram. This is easily seen by cutting figures out of paper. Therefore



The area of a trapezoid equals half the area of a rectangle with the same altitude and with a base equal to the sum of the two parallel sides of the trapezoid.

For example, what is the area of a trapezoid of altitude 4 in. and parallel sides 8 in. and 10 in.?

The area is ½ of $4 \times (8 + 10)$ sq. in., or ½ of 4×18 sq. in., or 36 sq. in.

ORAL EXERCISE

State the areas of trapezoids with altitudes first given below, followed by the two parallel sides:

1. 6 in., 7 in. and 13 in.
2. 10 in., 12 in. and 13 in.
3. 14 in., 14 in. and 16 in.
4. 15 in., 8 in. and 12 in.
5. 17 in., 9 in. and 11 in.
6. 11 in., 11 in. and 19 in.

WRITTEN EXERCISE

Find the areas of trapezoids with altitudes first given below, followed by the two parallel sides:

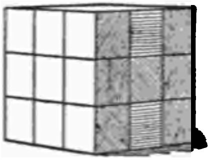
1. 13½ in., 6¼ in. and 14 in.
2. 22 in., 11½ in. and 9¾ in.
3. 1.5 in., 1.5 in. and 3.4 in.
4. 2.5 in., 2.1 in. and 4.2 in.
5. 3½ ft., 4 ft. and 5½ ft.
6. 17 rd., 16 rd. and 23 rd.

7. If the area of a trapezoid is 8¼ sq. in. and the parallel sides are 3½ in. and 4¾ in., what is the altitude?

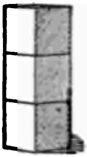
8. If the altitude of a field in the form of a trapezoid is 10 rd., and the parallel sides are 31¼ rd. and 16¾ rd., what is the area?

ORAL EXERCISE

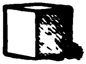
1. If $C = 1$ cu. in., what is the volume of B? the volume of A?



A



B



C

2. What is the volume of a work box 3 in. by 4 in. by 2 in.?

State the volumes of solids of the following dimensions:

3. 4 in., 5 in., 6 in.
4. 3 in., 3 in., 7 in.
5. 2 in., 3 in., 10 in.
6. 6 in., 8 in., 10 in.

168. Rectangular Solid. A solid having six sides, each side being a rectangle, is called a *rectangular solid.* If the sides are all squares, the solid is called a *cube.*

169. Volume of a Rectangular Solid. *The volume of a rectangular solid equals the product of the three dimensions.*

In finding volumes we do not write 3 in. × 4 in. × 2 in., but 3 × 4 × 2 cu. in. It is customary, however, in giving dimensions, to write 3 in. × 4 in. × 2 in., or 3″ × 4″ × 2″, reading either of these expressions "3 inches by 4 inches by 2 inches."

ORAL EXERCISE

Find the volumes of solids of the following dimensions:

1. 3″ × 4″ × 7″.
2. 6′ × 8′ × 10′.
3. 3′ × 8′ × 11′.
4. 2″ × 8″ × 9″.
5. 2′ × 3′ × 6′.
6. 5′ × 6′ × 10′.
7. 4′ × 7′ × 10′.
8. 6′ × 9′ × 10′.
9. 20′ × 30′ × 5′.
10. 11″ × 11″ × 10″.
11. 6 yd. × 8 yd. × 2 yd.
12. 6 ft. × 8 ft. × 11 ft.

WRITTEN EXERCISE

1. How many cubic inches in an aquarium 16 in. long, 8½ in. wide, and 9½ in. deep?

2. How many cubic feet of masonry in a wall 40 ft. long, 30½ ft. high, and 1½ ft. thick?

3. A cellar 24 ft. by 39 ft. by 6 ft. is to be excavated. How much will it cost at 40¢ a load (cubic yard)?

4. How many cords in a pile of 4-foot wood 5 ft. high and 56 ft. long? 6 ft. high and 128 ft. long?

5. How many loads of earth must be taken out in excavating a cellar 24 ft. by 18 ft., and 7 ft. deep?

6. How much will it cost to excavate a ditch 150 ft. by 2 ft. by 6 ft., at 50¢ per cubic yard? at 62½¢ per cubic yard?

7. A schoolroom is 32 ft. 8 in. long, 18⅓ ft. wide, and 12½ ft. high. What is the number of cubic feet in the room?

8. A tunnel 720 ft. long has a cross-section area of 180 sq. ft. How much earth and rock were excavated? (Imagine the cross section 10 ft. by 18 ft.)

9. The dimensions of a room are 14 ft., 12 ft., and 9 ft. How many square feet of floor? of walls? of ceiling?

10. In preparing a flower bed in a park it was necessary to fill in a space 60 ft. by 100 ft. to an average depth of 1½ ft. The earth cost 33⅓¢ a load. What was the total cost of the earth used?

11. The interior of a freight car is 8 ft. wide, 34 ft. long, and 7 ft. high. How many cubic feet does it contain? If it is filled with grain to a height of 5 ft., what is the weight of the grain at 60 lb. to the bushel, allowing 1¼ cu. ft. to the bushel?

Teachers should frequently require the pupils to estimate results in advance.

170. Practical Equivalents. In the problems given below, refer to these measurements without memorizing them.

$31\frac{1}{2}$ gal. = 1 bbl.
231 cu. in. = 1 gal.
$24\frac{3}{4}$ cu. ft. = 1 perch

$7\frac{1}{2}$ gal. = 1 cu. ft.
$4\frac{1}{5}$ cu. ft. = 1 bbl.
$1\frac{1}{4}$ cu. ft. = 1 bu.

The three values in the right-hand column are the ones usually taken, but they are not exact. For example, a bushel is 2150.42 cu. in.

In estimating brickwork, take 22 bricks with the mortar as 1 cu. ft.

A ton of hay varies from about 450 cu. ft. to 550 cu. ft., depending on the quality and on how much the hay has settled, but 500 cu. ft. may be taken in solving the problems. The volume of a ton of coal depends upon the size and kind of the coal, but 35 cu. ft. may be taken for our purposes.

In examples like the following, all measures refer to inside dimensions.

WRITTEN EXERCISE

1. How many bushels in a bin 6 ft. by 7 ft. by 4 ft.?

2. How many bushels in a bin 5 ft. by 9 ft. by 3 ft.?

3. How many bushels in a box 18 in. wide, 3 ft. long, and 2 ft. deep?

4. How many gallons in a tank 5 ft. by 6 ft. by 4 ft.?

5. How many gallons in an aquarium 14 in. long, 9 in. wide, and 11 in. deep?

6. How many gallons in a tank $6\frac{1}{4}$ ft. long, $3\frac{1}{2}$ ft. wide, and $2\frac{1}{3}$ ft. deep?

7. A cistern is 6 ft. square at the bottom and 5 ft. deep. How many barrels will it hold?

8. An iron tank is 8 ft. long, $6\frac{1}{2}$ ft. wide, and $4\frac{3}{4}$ ft. deep. How many barrels will it contain?

9. A watering trough is 6 ft. long, 18 in. wide, and 14 in. deep. How many barrels will it contain?

10. A coal bin is 21 ft. long, 13 ft. wide, and 8 ft. deep. About how many tons of coal will it hold?

11. A coal bin is 14 ft. long, 8½ ft. wide, and 6¼ ft. deep. About how many tons of coal will it hold?

12. A haymow is 22 ft. long, 14 ft. wide, and 9 ft. high. About how many tons of hay will it hold?

13. A barn has two haymows, each 24 ft. long, 15 ft. 6 in. wide, and 10 ft. high. About how many tons of hay will it hold?

14. How many cords of wood in a pile 4 ft. wide, 7 ft. high, and 32½ ft. long?

15. How many cords of wood in a pile 4 ft. wide, 6½ ft. high, and 44 ft. long?

16. If a schoolroom is 30 ft. long, 15 ft. wide, and 12 ft. high, and 180 cu. ft. of air should be allowed to each person, how many will the room accommodate?

17. A tank holds 82,500 gal. About how many cubic feet does it contain?

18. An aquarium contains 3⅘ cu. ft. About how many gallons does it hold?

19. An aquarium is 18 in. long, 9¼ in. wide, and 10⅛ in. deep. How many gallons does it hold?

20. A can containing 1732.5 cu. in. is full of milk. What is the milk worth at 6¢ a quart?

21. A wall is 32½ ft. long, 2 ft. thick, and 32½ ft. high. How many perches of masonry does it contain?

22. A brick wall is to be built 42 ft. long, 1½ ft. thick, and 6¼ ft. high. About how many bricks will be needed?

23. How much will a mason charge for laying a wall 20 ft. long, 1½ ft. thick, and 5½ ft. high, at $1.87½ a perch?

24. The standard Florida orange box for shipping is 12 in. by 12 in. by $26\frac{5}{8}$ in. How many cubic inches does it contain?

Use cancellation in all such cases.

25. In shipping Georgia peaches a crate 8 in. by $12\frac{1}{2}$ in. by 22 in. is used. This crate contains how many cubic inches more than 1 bu.?

26. In New Jersey a bushel cranberry box is 12 in. by $8\frac{3}{4}$ in. by 22 in. How does this differ from the standard bushel?

27. California apples and pears for local use are shipped in a 50-pound box 22 in. by 10 in. by 12 in. How does this differ in capacity from the standard bushel?

28. A California cantaloupe crate is 38 in. long, 16 in. wide, and 15 in. deep. How many cubic inches does it contain?

29. The California orange box is sometimes $26\frac{1}{2}$ in. by $11\frac{1}{4}$ in. by $11\frac{1}{4}$ in., and sometimes 22 in. by $7\frac{3}{4}$ in. by $17\frac{1}{2}$ in. Which holds the more, and how many cubic inches more?

30. Dried fruits are shipped in 50-pound or 25-pound boxes. The 50-pound box is $15\frac{1}{4}$ in. by 9 in. by 9 in. How many cubic inches in the box?

31. The 25-pound box mentioned in Ex. 30 is sometimes $13\frac{3}{4}$ in. by $9\frac{3}{4}$ in. by $5\frac{3}{4}$ in., and sometimes 16 in. by 9 in. by 5 in. Which box holds the more, and how much more?

32. For shipping fruit out of the state, the sizes of some of the boxes used in California are as follows:

Plums, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by 4 in.
Cherries, $16\frac{1}{2}$ in. by $10\frac{3}{8}$ in. by $2\frac{1}{2}$ in.
Pears, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $8\frac{1}{2}$ in.
Peaches, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $4\frac{7}{8}$ in.

Find the number of cubic inches in each of these boxes.

171. Other Problems Relating to Volumes. Instead of having the three dimensions of a rectangular solid given to find the volume, we may have the volume and certain dimensions given to find other dimensions.

1. How thick is a box 5 ft. long and 4 ft. wide, containing 60 cu. ft.

$$\boxed{\frac{60}{5 \times 4} = 3}$$

Since it is 5 ft. long and 4 ft. wide, the area of the base is 20 sq. ft. If the box were 1 ft. high, the volume would be 20 cu. ft.

Therefore the box is as many times 1 ft. high as 60 cu. ft. ÷ 20 cu. ft., or 3. Therefore the box is 3 ft. high.

The number of cubic units of volume, divided by the product of the numbers of units of two dimensions, equals the number of units of the third dimension.

This is sometimes expressed thus: *The volume divided by two dimensions equals the third dimension.*

2. What is the area of the base of a block $2\frac{1}{2}$ in. high, containing 70 cu. in.?

If the block were only 1 in. high, the volume would be 70 cu. in. ÷ $2\frac{1}{2}$, or 28 cu. in. But the area of the base of such a block is 28 sq. in. Therefore the area of the base is 28 sq. in.

$$\boxed{70 \div \frac{5}{2} = \frac{2 \times 70}{5} = 28}$$

This may be briefly stated as follows: *The volume divided by the height equals the area of the base.*

3. What is the thickness of a block containing 100 cu. in., the base containing 30 sq. in.?

If the block were 1 in. thick, it would contain 30 cu. in. Therefore it is as many times 1 in. thick as 100 cu. in. ÷ 30 cu. in., or $3\frac{1}{3}$. Hence it is $3\frac{1}{3}$ in. thick.

$$\boxed{\frac{100}{30} = 3\frac{1}{3}}$$

This may be briefly stated as follows:

The volume divided by the area of the base equals the height.

WRITTEN EXERCISE

1. How many cubic feet in a hall 60 ft. by 80 ft. by 23 ft.?

The pupils should measure the schoolroom and then apply Ex. 16, page 202.

2. A tank with base 9 ft. by 16⅔ ft. contains 900 cu. ft. How deep is it?

3. A box with base 13 in. by 20 in. contains 3900 cu. in. How deep is it?

4. A tank 4 ft. deep contains 450 cu. ft. What is the area of the base?

5. A hall 22 ft. high and 30 ft. wide contains 28,600 cu. ft. Find the length.

6. A block containing 700 cu. ft. has a base area of 56 sq. ft. How high is it?

7. A hall contains 41,341.3 cu. ft. It is 41.3 ft. long and 36.4 ft. wide. How high is it?

8. A box contains 42,159 cu. in. The area of the bottom is 1081 sq. in. How deep is the box?

9. A block of granite is 14 in. long and 8.3 in. wide. It contains 836.64 cu. in. How thick is it?

10. The floor of a hall contains 1386 sq. ft. The hall contains 15,246 cu. ft. Required the height.

11. A storeroom is 14.3 ft. long, 13 ft. 6 in. wide, and has a capacity of 2316.6 cu. ft. How high is the room?

12. The floor of a bedroom is square and contains 144 sq. ft. If the room contains 1368 cu. ft., find the three dimensions.

First find the two equal factors of 144.

13. A room containing 2110.68 cu. ft. is 16.4 ft. long, and has a floor area of 234.52 sq. ft. What are the three dimensions of the room?

14. How many cubic feet in a bin that is 6 ft. 4 in. long, 3 ft. wide, and 3 ft. 6 in. deep?

15. Express in bushels the volume of a bin 4 ft. by 6 ft. by 3 ft. 6½ in., allowing 2150 cu. in. to the bushel.

16. How many bars of soap, each 4 in. long, 2¾ in. wide, and 1½ in. thick, can be packed in a box 16 in. long, 13¾ in. wide, and 1 ft. deep? Will the bars fit in the box?

17. At $9 a thousand, what is the value of the bricks in a kiln 60 ft. long, 25 ft. wide, and 9 ft. high, each brick being 4 in. by 8 in. by 2 in.?

18. The length of a tank is twice as great as the width, and the width is twice as great as the depth. If the width is 2 yd., how many gallons does the tank hold?

19. A cord of stone having the same volume as a cord of wood, what is a pile of cut stones 27 ft. long, 5 ft. wide, and 6 ft. high worth at $4.20 a cord?

20. If it takes 550 cu. ft. of clover hay to make a ton, how many tons of clover hay will fill a mow 32 ft. long, the width being double the depth and half the length?

21. How many gallons of water are there in a watering trough that is 6 ft. long and 16 in. wide, the depth of the water being three fourths of the width?

22. A bin 21 ft. 6 in. long and 10 ft. wide is filled with wheat to the depth of 5 ft. Allowing 2150 cu. in. to the bushel, what is the wheat worth at 80¢ a bushel?

23. A cellar is 32 ft. long, 17 ft. wide, and 8 ft. 6 in. deep. How much will it cost to build the wall, 1 ft. thick, at 35¢ per cubic foot, no allowance being made for openings, and the length of the wall being determined by the outside measure, thus doubling the corners?

172. Capacity and Weight. The laws of the various states usually specify the weight of a bushel, requiring it to weigh at least the amount specified below. This table should not be committed to memory, but used for reference.

Corn (in the ear),	70 lb.	Oats,	32 lb.
Barley,	48 lb.	Dried Apples,	26 lb.
Corn (shelled), Rye,	56 lb.	Timothy Seed,	45 lb.
Wheat, Clover Seed, Peas,	60 lb.	Dried Peaches,	33 lb.
Beans, White Potatoes,	60 lb.	Sweet Potatoes,	55 lb.
Buckwheat,	48 lb.	Onions,	48 lb.

WRITTEN EXERCISE

1. How many tons to 1000 bu. of wheat? to 1000 bu. of oats?

2. How many bushels of white potatoes to the ton?

3. How many bushels to a ton of corn in the ear? How many bushels to a ton of shelled corn?

4. How many tons to 1000 bu. of corn in the ear? How many tons to 1000 bu. of shelled corn?

Find the number of bushels to a ton of the following:

5. Oats.
6. Buckwheat.
7. Clover seed.
8. Onions.
9. Sweet potatoes.
10. Timothy seed.

11. A bushel of rye weighs how many times a bushel of oats?

12. A bushel of oats is what part as heavy as a bushel of shelled corn?

13. A bushel of timothy seed is what part as heavy as a bushel of buckwheat? It is what part as heavy as a bushel of peas?

LITTLE EXAMINATIONS

I. 1. Find the area of a rectangle 47 ft. 6 in. by 32 ft. 6 in.

2. Find the area of a triangle with base 16 ft. and height 6 ft.

3. Find the area of a parallelogram with base 7 ft. 6 in. and height 4 ft. 2 in.

4. Find the area of a trapezoid with parallel sides 4 ft. 6 in. and 3 ft. 4 in., and height 2 ft. 6 in.

5. Find the volume of a rectangular solid 2 ft. 3 in. long, 1 ft. 8 in. wide, and 13 in. deep.

II. 1. Find the area of a rectangle 1 yd. 9 in. by 2 yd. 16 in.

2. Find the area of a triangle with base 14 rd. 7 ft. and height 9 rd. 3 ft.

3. Find the area of a rectangle $8\frac{1}{4}$ ft. by $4\frac{1}{2}$ rd.

4. Find the area of a parallelogram with base 127 rd. $8\frac{1}{4}$ ft. and height 64 rd.

5. Find the volume of a rectangular solid 6 ft. 8 in. by 5 ft. 4 in. by 2 ft. 6 in.

III. 1. Find the area of a rectangle 84 rd. $4\frac{1}{8}$ ft. by 126 rd.

2. Find the area of a triangle with base $12\frac{1}{2}$ ft. and height 17 in.

3. Find the area of a rectangle $6\frac{1}{2}$ rd. by 146 ft.

4. Find the area of a triangle with base 37 rd. $4\frac{1}{8}$ ft. and height 36 ft.

5. Find the volume of a rectangular solid 3 ft. $9\frac{1}{2}$ in. long, 2 ft. $7\frac{1}{4}$ in. wide, and 16 in. deep.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write in our common numerals the number sixty million, two hundred seventy-seven thousand, eight hundred nine.

2. Write in our common numerals the number seventy-two and sixty-four thousandths.

Add the following:

3.	4.	5.	6.	7.
\$2.96	\$8.61	\$5.42	\$8.87	\$6.43
4.83	7.43	6.84	2.93	4.21
5.77	9.36	.76	8.40	8.36
6.09	2.08	1.20	1.20	5.60
.73	1.39	9.08	.67	3.57
4.86	2.57	6.77	.42	5.79

8. From the sum of \$72.86 and \$47.63 subtract \$107.59.

Perform the following operations:

9. $\frac{2}{3} + \frac{1}{2}$.
10. $\frac{3}{4} - \frac{3}{8}$.
11. $\frac{7}{8} + \frac{3}{4}$.
12. $\frac{7}{8} - \frac{1}{4}$.
13. $\frac{1}{2} - \frac{1}{3}$.
14. $\frac{3}{16} + \frac{1}{2}$.
15. $\frac{9}{16} - \frac{1}{2}$.
16. $\frac{1}{2}$ of $\frac{9}{16}$.
17. $\frac{7}{16} \div \frac{1}{2}$.
18. $\frac{9}{16} \div \frac{3}{4}$.
19. $1\frac{1}{2} + 2\frac{7}{8}$.
20. $3\frac{1}{3} + 2\frac{1}{2}$.
21. $6\frac{1}{4} - 3\frac{7}{8}$.
22. $7\frac{2}{3} - 2\frac{1}{2}$.
23. $2\frac{1}{2} \times 2\frac{1}{2}$.
24. 0.7×1.2.
25. 1.8×2.3.
26. 1.6×0.9.
27. 0.6×0.91.
28. $1.44 \div 1.2$.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. I have measured the length, width, and height of this room. How do I find the number of square feet in the walls and ceiling, including all openings?

2. I have measured the length and width of a room. How do I find the number of square yards of linoleum needed to cover a floor?

3. I know the dimensions of this room and the dimensions of all openings. How do I find the number of square yards of plastering needed for the walls and ceiling, allowing for the doors and windows?

4. How do you find the number of square feet of flooring needed to cover the floor of your schoolroom?

5. How do you find the number of square yards of plaster needed for this room?

6. If you know the cost of shingles by the bunch and the number of bunches necessary to cover a given area, how do you find the cost of shingles for a roof of given dimensions?

7. How do you find the cost of a sidewalk to be laid in front of the schoolhouse?

8. Given the length, breadth, and thickness of a rectangular solid, how do you find the volume?

9. Given the volume of a rectangular solid and two dimensions, how do you find the third dimension?

10. If you know the number of bricks needed for a cubic foot of wall, how do you estimate the number of thousand bricks needed to build a wall?

III. INDUSTRIAL PROBLEMS

GETTING HORSES READY FOR MARKET

1. To one lot of horses a dealer fed corn. In this case the average weight increased from 1436 lb. to 1674 lb. in 112 da. What was the average increase per day?

2. To another lot he fed three parts corn to one of oats, and the average weight increased from 1468 lb. to 1734 lb. in 112 da. What was the average increase per day?

3. To a third lot he fed half corn and half oats, the increase being from 1463 lb. to 1736 lb. in 112 da. What was the average increase per day in this case?

Examples like this, representing scientific experiments, not only give the pupils exercise in computation but also give them some idea of the way in which the government is trying to solve the problem of how to improve stock. In rural schools similar problems may be brought to class by the pupils.

THE PARCEL POST

1. The captain of our school baseball team finds that the catalogue from which he orders supplies tells the weight of the articles. The parcel post rates (1915) for shipping in four zones are:

Pounds	*Local Rate*	*First Zone*	*Second Zone*	*Third Zone*	*Fourth Zone*
1	$0.05	$0.05	$0.05	$0.06	$0.07
2	.06	.06	.06	.08	.11
3	.06	.07	.07	.10	.15
4	.07	.08	.08	.12	.19
5	.07	.09	.09	.14	.23
6	.08	.10	.10	.16	.27
7	.08	.11	.11	.18	.31
8	.09	.12	.12	.20	.35
9	.09	.13	.13	.22	.39
10	.10	.14	.14	.24	.43

The first zone is from 1 mi. to 50 mi., the second is from 50 mi. to 150 mi., the third is from 150 mi. to 300 mi., and the fourth is from 300 mi. to 600 mi. Our school is 75 mi. from the city where the captain buys the goods. He buys three bats at 45¢, the shipping weight of each being 50 oz. How much money does he have to send for the bats and postage?

Any weight between 1 lb. and 2 lb. is counted as 2 lb., and so on. The limit of weight in the first and second zones is 50 lb., in other zones 20 lb.

2. The captain wishes two catcher's mitts. They cost $2 each, and the shipping weight of each is 25 oz. How much money does he have to send for the mitts and postage?

3. Another team is 170 mi. from the city where it buys supplies. The captain of that team sends for two baseball masks. They cost $1.40 each, and the weight of each is 24 oz. How much money must he send for the masks and postage?

CHAPTER VI

INTRODUCTION TO PERCENTAGE

173. The Need of knowing about Per Cents. Fred Roberts is captain of the baseball team of the sixth grade of our school. He finds that he can buy at the store a catcher's mitt for $2. The next day he sees that it has been marked down 10 per cent. He now begins to think that it is necessary that he should know what per cent means.

If any one in the class knows, he may tell the meaning, otherwise it may be taken up after § 174. This page is merely a kind of preparatory reading lesson.

Mrs. Roberts, Fred's mother, wishes to buy a suit of clothes for Fred. The price was $15, but the suit has been marked down 20 per cent. If she wishes to buy the suit, she must know what this means. Do you know what it means?

The teacher has to make a report of the number of pupils tardy or absent last week. She says in the report that 5 per cent of the pupils were tardy and 2 per cent were absent. Do you know what this means?

A man wishes to buy an automobile. He can buy a new one of the kind he likes for $1160, but one that is nearly new is offered for 25 per cent less than this price. Do you know how to find what the man would have to pay for this car?

It is not necessary that any pupil should answer these questions. What is of importance is that all the class should see that per cents are frequently met and that every one must know what they mean.

174. Per Cent. Another name for "hundredths" is *per cent.* For example, instead of saying "ten hundredths" we may say "ten per cent." The two expressions mean exactly the same.

If Fred Roberts finds that a catcher's mitt which is marked \$2 can be bought for 10 per cent less than the marked price, this means that it can be bought for $\frac{10}{100}$, or $\frac{1}{10}$, less than \$2; that is, it can be bought for \$2 less $\frac{1}{10}$ of \$2, or \$2 – \$0.20, or \$1.80.

Such simple illustrations should be drawn from local cases, even before the subject of discount is taken up, as it will be a little later.

175. Symbol for Per Cent. There is a special symbol for per cent, %. Thus we write 20% for 20 per cent, and it means the same as $\frac{20}{100}$, or 0.20.

176. Hundredths written as Per Cents. Because hundredths and per cents are the same, any fraction with 100 for its denominator may be written in the form of per cent. Thus

$$\frac{3}{100} = 3\% \qquad \frac{1}{100} = 1\% \qquad \frac{21}{100} = 21\% \qquad \frac{75}{100} = 75\%$$

ORAL EXERCISE

Read the following as per cents:

1. $\frac{2}{100}$.
2. $\frac{4}{100}$.
3. $\frac{5}{100}$.
4. $\frac{7}{100}$.
5. $\frac{9}{100}$.
6. $\frac{11}{100}$.
7. $\frac{25}{100}$.
8. $\frac{32}{100}$.
9. $\frac{48}{100}$.
10. $\frac{62}{100}$.
11. $\frac{68}{100}$.
12. $\frac{72}{100}$.
13. $\frac{80}{100}$.
14. $\frac{90}{100}$.
15. $\frac{100}{100}$.

Read the following as hundredths:

16. 10%.
17. 25%.
18. 30%.
19. 42%.
20. 50%.
21. 72%.
22. 92%.
23. 99%.
24. 100%.
25. 200%.

26. Read $\frac{125}{100}$ as per cent, and 250% as hundredths.

27. Read as per cents: $\frac{12\frac{1}{2}}{100}$, $\frac{37\frac{1}{2}}{100}$, $\frac{\frac{1}{2}}{100}$, $\frac{33\frac{1}{3}}{100}$, $\frac{\frac{1}{3}}{100}$, and $\frac{66\frac{2}{3}}{100}$.

177. Halves and Fourths as Per Cents. Mr. Roberts says that he sold an old automobile for 50% of what it cost him. What does this mean? He sold the car for how many hundredths of what it cost him? Express this fraction in lowest terms.

If a baseball team plays 32 games and loses only 25% of them, how many games does it lose? The team loses how many hundredths of the games? Express this fraction in lowest terms.

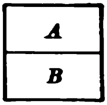
From these two examples we see that

$$50\% = \tfrac{1}{2} \qquad\qquad 25\% = \tfrac{1}{4}$$

Other simple examples having local interest to the pupils should be given. The following oral exercise emphasizes the relations of $\frac{1}{2}$ and $\frac{1}{4}$ to per cents.

ORAL EXERCISE

1. If this square represents a floor containing 100 sq. ft., how many square feet in *B*? Then *B* is how many hundredths of the square? What per cent of the square is *B*? Then *A* is what per cent of the square?

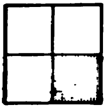


2. If asked to find 50% of a number, you may find what fraction of the number instead?

Find 50% of each of the following numbers:

3. 24. 4. 32. 5. 44. 6. 50. 7. 200.

8. If this square represents a box cover containing 100 sq. in., how many square inches are shaded? How many fourths are shaded? How many hundredths are shaded? What per cent is shaded?



9. If asked to find 25% of a number, you may find what fraction of the number instead?

Find 25% of each of the following numbers:

10. 24. 11. 32. 12. 88. 13. 8. 14. 160.

ORAL EXERCISE

1. Half of a square is what per cent of it?
2. A fourth of a square is what per cent of it?
3. An eighth of a square is what per cent of it?

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178. Important Per Cents. We have found that

$50\% = \frac{1}{2}$ $25\% = \frac{1}{4}$ $12\frac{1}{2}\% = \frac{1}{8}$

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4. How much is 50% of 20? of 40? of 50? of 7?
5. How much is 25% of 32? of 60? of 120? of 5?
6. How much is $12\frac{1}{2}\%$ of 40? of 72? of 88? of 100?
7. How much is $12\frac{1}{2}\%$ of 32? of 16? of 160? of 320?
8. In your class, if 50% are boys, how many are boys?

WRITTEN EXERCISE

Find the values of the following:

1. 50% of 1632.
2. 25% of 2592.
3. 50% of 12,370.
4. 25% of $3416.80.
5. 50% of 978 bu.
6. 25% of 96 ft. 8 in.
7. $12\frac{1}{2}\%$ of $91.28.
8. $12\frac{1}{2}\%$ of $73.68.

9. A class at school contains 48 pupils, and 50% of them are girls. How many are girls? How many are boys?

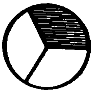
10. A boy had 64 marbles and lost 25% of them. How many marbles did he lose and how many did he have left?

11. A hawk captured $12\frac{1}{2}\%$ of 152 chickens. How many were captured? How many were left?

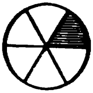
12. Of 1408 sheep owned by a farmer $12\frac{1}{2}\%$ were sold. How many were sold? How many remained?

ORAL EXERCISE

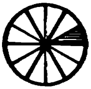
1. If this circle is called 100, how much is the shaded part? how much is the unshaded part? If the circle is called 1, what per cent is the shaded part?



2. Read $0.33\frac{1}{3}$, using the words "per cent"; using the word "hundredths." How many thirds are there in 1? Then how many times is $33\frac{1}{3}\%$ contained in 1?



3. If this circle is called 100, the shaded part is what part of $33\frac{1}{3}$? How much is this? Then the shaded part is what per cent of the circle?



4. How much is $\frac{1}{2}$ of $16\frac{2}{3}\%$? The shaded part of this circle is what per cent of the whole circle? Then $\frac{1}{12}$ is what per cent?

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179. Important Per Cents. We have found that

$16\frac{2}{3}\% = \frac{1}{6}$ $\qquad 33\frac{1}{3}\% = \frac{1}{3}$ $\qquad 66\frac{2}{3}\% = \frac{2}{3}$

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5. How much is $33\frac{1}{3}\%$ of 90? of 120? of 45? of 60?
6. How much is $66\frac{2}{3}\%$ of 30? of 66? of 90? of 300?
7. How much is $16\frac{2}{3}\%$ of 36? of 24? of 66? of 72?
8. How much is $33\frac{1}{3}\%$ of \$6? of \$15? of \$30?

WRITTEN EXERCISE

Find the values of the following:

1. $33\frac{1}{3}\%$ of 891.
2. $66\frac{2}{3}\%$ of 711.
3. $16\frac{2}{3}\%$ of 504.
4. $16\frac{2}{3}\%$ of \$41.10.
5. $16\frac{2}{3}\%$ of \$159.72.
6. $66\frac{2}{3}\%$ of 747 rd.
7. $16\frac{2}{3}\%$ of 9774 bu.
8. $33\frac{1}{3}\%$ of 222 ft. 9 in.
9. $16\frac{2}{3}\%$ of 726 bu. 6 qt.
10. $66\frac{2}{3}\%$ of 141 lb. 15 oz.

180. Per Cents as Common Fractions. Since per cent means hundredths, we know that

$$62\tfrac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}.$$

To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and reduce this fraction to lowest terms.

For example, 50% means 0.50, or $\frac{50}{100}$, and this equals $\frac{1}{2}$.

It is often convenient to use one form, and often another. Thus, if we are multiplying by 13.5%, it is more convenient to think of the multiplier as 0.135; but if we are multiplying by $33\frac{1}{3}\%$, it is better to think of it as $\frac{1}{3}$ instead of $0.33\frac{1}{3}$.

We know that $\frac{2}{3}$ may be reduced to hundredths by multiplying each term by $33\frac{1}{3}$. We then have

$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} = 66\tfrac{2}{3}\%.$$

To express a common fraction as per cent, reduce it to hundredths, and then write the numerator followed by the symbol for per cent.

We may also reduce $\frac{2}{3}$ to hundredths by dividing 2 by 3. We then have

$$\tfrac{2}{3} = 0.66\tfrac{2}{3} = 66\tfrac{2}{3}\%.$$

WRITTEN EXERCISE

Express as common fractions in lowest terms:

1. 32%.
2. 68%.
3. 24%.
4. $6\frac{1}{4}\%$.
5. 36%.
6. 125%.
7. 17%.
8. 375%.
9. $82\frac{1}{2}\%$.
10. $33.3\frac{1}{3}\%$.

Express the following as per cents:

11. $\frac{3}{8}$.
12. $\frac{5}{8}$.
13. $\frac{7}{8}$.
14. $\frac{1}{5}$.
15. $\frac{2}{5}$.
16. $\frac{3}{5}$.
17. $\frac{4}{5}$.
18. $\frac{1}{16}$.
19. $\frac{3}{16}$.
20. $\frac{15}{16}$.

181. Finding Per Cents. We have now found that certain important fractions like ½, ¼, ⅛, ⅙, ⅓, and ⅔ are easily expressed as per cents; that any fraction can be expressed as a per cent if we first reduce it to hundredths; and that per cents can easily be expressed as common fractions.

If we wish to find 50%, 25%, 12½%, 16⅔%, 33⅓%, or 66⅔% of a number, it is easier to use the equivalent common fractions, as in the following example:

If Ralph wishes to buy a camping suit that is marked $8, and finds that it is to be marked down 12½%, how much will the suit cost him after it is marked down?

Since 12½% = ⅛, we simply take ⅛ of $8, and this tells the amount the suit is to be marked down.

Then we have ⅛ of $8 = $1;

and $8 − $1 = $7, cost.

If we wish to find such a per cent as 15% of a number, it is better to use decimal fractions, as in the following example:

Kate wishes a new dress. The price of the dress she wants is $14, but she finds that to-morrow at a bargain sale this dress is to be marked down 15%. How much will it then cost her?

$$\begin{array}{r} \$14 \\ .15 \\ \hline 70 \\ 14 \\ \hline \$2.10 \end{array} \qquad \begin{array}{r} \$14 \\ 2.10 \\ \hline \$11.90, \text{ cost} \end{array}$$

Because 15% = 0.15, we multiply $14 by 0.15. We then have

15% of $14 = $2.10;

and $14 − $2.10 = $11.90, cost.

To find a per cent of a number, express the per cent as a decimal fraction and multiply the number by this decimal.

As stated above, in cases like 50% it is better to use a common fraction than a decimal. By so doing we can frequently solve a problem mentally that would otherwise require written work.

ORAL EXERCISE

1. If you have 50 words in a spelling test and are marked 90%, how many words did you spell correctly?

2. If you have 10 questions in arithmetic and are marked 80%, how many questions did you answer correctly?

3. If you have 12 questions in geography and are marked 75%, how many questions did you answer correctly?

4. If 60% of a class of 40 have not missed a day this year, how many have been present every day?

5. If a merchant pays 2¢ apiece for pencils and sells them at a profit of 50%, how much does he receive for each?

6. If a milk dealer pays 5¢ a quart for milk and sells it at a profit of 20%, how much does he receive per quart?

7. If a butcher buys meat at 18¢ a pound and sells it at a profit of $16\frac{2}{3}\%$, at what price does he sell it per pound?

WRITTEN EXERCISE

1. In the world's championship series in a certain year, the players' share was $135,000, of which the Giants received 40%. How much money did they receive?

2. In the same year the Athletics received 60% of the players' share, which was $135,000, as stated in Ex. 1. How much money did they receive?

3. The St. Louis club was at bat 4800 times in one season and the base hits were 25%. How many base hits did they make?

Find the values of the following:

4. 22% of 5475.
5. 6% of $2496.
6. 95% of $625.
7. 65% of $9200.
8. 87% of 4700 ft.
9. $66\frac{2}{3}\%$ of $4131.

10. In a school of 350 pupils 8% were in the sixth grade. How many pupils were there in that grade?

11. In a school of 250 pupils 12% are in the third grade and 14% in the first grade. How many pupils are there in each of these grades?

12. Of 320 pupils 5% have been tardy or absent this month. How many pupils have not been tardy or absent?

In such a case, use a common fraction for per cent.

13. If there should be an increase of $3\frac{1}{3}\%$ next year in a school of 330 pupils, how many pupils would there then be?

14. In a school of 246 pupils $16\frac{2}{3}\%$ were not in the school last year. How many new pupils are there this year?

In such a case, use a common fraction for per cent.

15. An ice wagon started out with a load of ice. One cake weighed 190 lb. at starting, but before it was delivered lost 4% by melting. How much did it then weigh?

16. The area of Illinois is 56,600 sq. mi. Of this 63% is underlain with coal beds. How many square miles of coal lands are there in the state?

17. A class in industrial arts bought 36 yd. of cloth, and when the cloth was washed the shrinkage was $\frac{1}{2}\%$. How much did the cloth shrink? What was the length after shrinkage?

Since per cent means hundredths, $\frac{1}{2}\%$ means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$. So we can find $\frac{1}{2}\%$ of a number by taking $\frac{1}{200}$ of it.

18. In the class in cooking it was found that some food costing $8.50 had deteriorated in value 5% by being kept for a week. What was the amount of the deterioration expressed in cents?

When an answer is a certain number of cents and a fraction, leave the fraction in the answer except in the case of bills, which are considered elsewhere.

19. One park has an area of 539 acres, and another has 69% as much. What is the area of the second park?

20. In a village of 975 inhabitants 20% are school children. How many school children are there?

21. A man bought a piece of land for $1500. He sold it so as to gain 8% on what it cost him. How much did he gain?

22. A farmer irrigated 240 acres of land last year and 15% more this year. How many acres did he irrigate this year?

23. In a school of 250 pupils, if there should be an increase of 4% next year, how many pupils would there then be?

24. In a school of 300 pupils $4\frac{2}{3}$% are absent to-day. How many pupils are absent?

25. If $14\frac{1}{2}$% of the pupils in a school of 400 pupils were not in the school last year, how many new pupils are there this year?

26. If 24% of the pupils in a school of 250 pupils are in the sixth and seventh grades, how many are in those grades?

27. If 25% of the pupils in a school of 372 pupils are in the first and second grades, how many are in those grades? in all the other grades?

28. If 55% of the pupils in a school of 280 pupils are girls, how many girls are there in the school? how many boys?

29. How much must be added to 250 ft. to increase it 17%? to increase it 25%? to increase it 100%?

30. How much must be added to 148 lb. to increase it 25%? to increase it 50%? to increase it 75%?

31. A column of mercury in a thermometer is 3.6 in. high. If it increases 10% in height, how high is it then?

32. A book is $\frac{4}{5}$ in. thick. Each of the two parts of the cover is $12\frac{1}{2}$% of this thickness. How thick is each part?

33. A rectangle 4¼ in. long and 3½ in. wide is increased 40% in length and 33⅓% in width. What are the length and width now?

34. A certain water tank holds 60,000 gal. A large pump at the waterworks pumps 5% of this amount in a single stroke. How many gallons does it pump in one stroke?

35. A certain building in San Francisco cost $1,240,000, and another cost 50% as much. How much did the second cost?

36. If the receipts at the St. Louis docks were 125 carloads on one day and 20% less the next day, what were the receipts on the second day?

37. The value of the property in Chicago at the time of the great fire of 1871 was estimated as $600,000,000, and 31% of it was destroyed. Find the value of the property destroyed.

38. If 500 tons of ore are shipped to Hoboken, and 15% of it is loaded on a steamer, how many tons are loaded on the steamer?

39. A large electric company has certain engines of 22,000 horse power, and certain others of 66% as much horse power. Find the horse power of the latter engines.

40. If you have 25 questions in arithmetic and are marked 80%, how many questions did you answer correctly?

41. If you have 30 words in a spelling test and are marked 90%, how many words did you spell correctly?

42. A man bought a bicycle for $40 and sold it at a loss of 35%. How much did he lose?

43. A dealer bought 83 rocking-chairs at $6 apiece and sold them at a profit of 15%. How much did he gain?

44. A class set out a tree on the school grounds. The tree was 12 ft. 6 in. high when it was set out, and the next year it had increased 25% in height. How high was it then?

182. Debtor and Creditor. A man who owes money to another is called a *debtor* (Dr.). The man to whom he owes money is called the *creditor* (Cr.). So if Rob has bought a baseball from Fred and owes him 50¢, Rob is the debtor and Fred is the creditor.

183. Debit and Credit. The amount owed on a bill is called the *debit*, and any payments that have been made form the *credit*.

184. Bill. A bill usually begins about as follows:

Mr. A. B. Jefferson, 17 Monroe St.

Bought of ROE & DOE, Grocers
No. 4 Washington Street

It may also read: "To Roe & Doe, Dr." This means that Mr. Jefferson is debtor to Roe & Doe for the goods mentioned in the bill.

The following is an example of a bill filled out, footed, and receipted:

Indianapolis, Ind.,..........[*Date*]..........

Mrs. John Doe, 47 Tenth Ave.

To RICHARD ROE & CO., Dr.
Dry Goods Dealers, No. 10 Grand Street

.....[*Year*]..... ......[*Date*]......	$\frac{1}{2}$ yd. silk	.75		38
"	$\frac{3}{4}$ " "	1.25		94
"	$7\frac{1}{2}$ " velour	3.50	26	25
			27	57

Received payment

Richard Roe & Co.

To fill out the bill by inserting in the two columns at the right the amount of each of the various items is called "making the extensions."

185. Receipt. A receipt need not necessarily be on a bill for goods or services. If Robert owes George $1.25 for a catcher's mask that he bought from him last week, and pays the money to-day, he may ask for a receipt so that the fact of the payment shall not be forgotten. Such a receipt would read as follows:

$1.25 Davenport, Ia., April 6, 1916

Received from .... Robert Montgomery ....

One and $\frac{25}{100}$ ~~~~~~~~~~~~~~~~~~~~~~~~~~~~ Dollars

for a catcher's mask.

George Sinclair

A receipt should contain the place and date, the name of the one paying the money (the *payer*), the amount written both in figures and in words, a statement of the reason for paying the money (the nature of the debt), and the signature of the one to whom the money is paid (the *payee*).

WRITTEN EXERCISE

Write receipts for the following, inserting dates and names:

1. $1.15 for a pair of skates.
2. $1.20 for three baseball bats.
3. $0.75, balance due on a football.
4. $1.35, balance due on a mandolin.
5. $0.80, balance due on a tennis racket.
6. $1.60, balance due on a tennis racket and a tennis net.
7. $6.50 for two days' labor on sidewalk and fence.
8. $0.72 for 26 papers at 2¢ each, and 4 papers at 5¢ each.

WRITTEN EXERCISE

Date, fill, foot, and receipt the following bills, inserting the name and address of some purchaser and of some dealer whom you know:

1. Bill for furniture: 6 chairs @ $3.00; 2 armchairs @ $6.25; 1 rocking-chair @ $5.50; 1 table @ $22.75.

An item like "6 chairs @ $3.00" means $3.00 each.

2. Bill for dry goods: 3½ yd. satin @ $1.50; 2 yd. flannel @ 50¢; 1½ yd. canvas @ 20¢; 1½ yd. ribbon @ 23¢.

3. Bill for meat: 2½ lb. steak @ 32¢; 3½ lb. fish @ 20¢; 4¾ lb. roast beef @ 36¢; 8 lb. turkey @ 31¢.

4. Bill for dry goods: 17½ yd. embroidery @ 10¢; 3 yd. lawn @ 20¢; 7½ yd. percale @ 25¢; ⅛ yd. satin @ $1.50.

5. Bill for crockery: ½ doz. cups and saucers @ $6.00; 3 doz. plates @ $4.25; 1½ doz. tumblers @ 90¢; 2 doz. individual butter plates @ 70¢; ½ doz. bowls @ $2.30.

An item like "½ doz. cups @ $6.00" means $6.00 a dozen.

6. Bill for groceries: 10 lb. sugar @ 8¢; 2 qt. berries @ 11¢; 4 doz. eggs @ 26¢; 5½ lb. butter @ 32¢; 1½ lb. cheese @ 22¢; ½ doz. bars soap @ 44¢.

7. Bill for dry goods: 7 yd. muslin @ 18¢; 6 handkerchiefs @ 30¢; 3 pairs kid gloves @ $1.50; 12 yd. sheeting @ 12½¢; 16 yd. pongee @ 70¢.

8. Bill for stationery: 500 envelopes @ $3.50 per M (that is, per 1000); 1 box writing paper @ $2.25; ½ doz. pencils @ 35¢; 3 penholders @ 5¢; 1 bottle ink @ 30¢.

9. Bill for groceries: 5 lb. coffee @ 32¢; 2 lb. tea @ 45¢; 8 lb. ham @ 15¢; 7 lb. lard @ 8¢; 3½ lb. butter @ 33¢; 3 qt. currants @ 7¢; 2 doz. eggs @ 31¢; 1¾ lb. cheese @ 16¢; ½ gal. sirup @ 65¢.

186. Per Cents off. If you have a catcher's glove or a tennis racket that you have used for a year, you may be glad to sell it at half price so as to get part of the money that it cost and have this much toward buying a new one. In that case you would sell it at a *discount* of 50%.

Sometimes merchants have bargain sales to dispose of odd pieces or goods that are going out of style. At such times they usually sell goods at a certain "per cent off."

For example, goods which are marked \$2.50, but are sold at 10% off, sell at \$2.50 – 10% of \$2.50, or \$2.25.

$$\begin{array}{r} \$2.50 \\ .10 \\ \hline \$0.25 \end{array} \qquad \begin{array}{r} \$2.50 \\ 0.25 \\ \hline \$2.25 \end{array}$$

Merchants who sell at wholesale, that is, in large quantities to dealers, sell at a certain per cent off the price as stated in their catalogues, or off the *list price*, as it is called.

187. Discount. A reduction made on a price or amount is called a *discount*. Discounts are usually reckoned at some rate per cent of the amount from which the discount is made. If you should offer to sell for \$2 a tennis racket that cost \$3, you would be offering to sell it at a discount of $\frac{1}{3}$, or a discount of $33\frac{1}{3}\%$.

188. Rate of Discount. The per cent of discount is called the *rate of discount*. The amount of a price or bill after the discount has been made is called the *net price* or the *net amount*.

What is the discount and the selling price, or net price, when the list price of a shipment of furniture is \$245 and the rate of discount is 12%?

12% of \$245 = \$29.40, discount.
\$245 – \$29.40 = \$215.60, selling price.

$$\begin{array}{rl} \$245 & \text{list price} \\ 0.12 & \text{rate of discount} \\ \hline \$29.40 & \text{discount} \end{array}$$

$$\begin{array}{rl} \$245. & \\ 29.40 & \\ \hline \$215.60 & \text{selling price} \end{array}$$

189. Order for Goods. The following is a model order for goods:

WOOD & ROBERTS
Booksellers

Madison, Wis., Feb. 15, 1918

Messrs. Ginn and Company, Publishers
2301 Prairie Ave., Chicago

Dear Sirs: Please send at once, by express,
75 Wentworth-Smith Complete Arithmetics, Pt. I,
60 ‘‘ ‘‘ ‘‘ Arithmetics, Pt. II.

Yours truly

WOOD & ROBERTS

190. Model Invoice. The following is a form of the invoice that Ginn and Company would send in reply:

Feb. 16, 1918

Messrs. Wood & Roberts
Madison, Wis.

Bought of GINN AND COMPANY
Educational Publishers
2301 PRAIRIE AVENUE, CHICAGO

Terms: *Cash*

75	W.-S. Complete Arithmetics, Pt. I	$.40	$30.00	
60	‘‘ ‘‘ Arithmetics, Pt. II	.40	24.00	
			$54.00	
		Less 1/6	9.00	
				$45.00

It is customary to state per cents like $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, 20%, and 25% in the common-fraction form, as above. In this case, $16\frac{2}{3}\%$ is stated as $\frac{1}{6}$. If the discount had been $12\frac{1}{2}\%$, it would have been stated as $\frac{1}{8}$.

ORAL EXERCISE

1. How much is $200 less $50? $100 less 25% of itself?

2. If from 1 we take $\frac{1}{4}$, how much is left? If from 1 we take 25%, how much is left?

3. How much is 1 less $\frac{1}{5}$? $\frac{100}{100}-\frac{20}{100}$? $1-0.20$? $1-20\%$?

4. If you buy a $1 book for 10% less than the list price, how much does it cost you?

Find the price that must be paid for goods marked as stated in each of these examples, and discounted at the rate given:

5. $2, 10%.
6. $6, 20%.
7. $5, 20%.
8. $4, 25%.
9. $50, 30%.
10. $60, $33\frac{1}{3}$%.
11. $1.60, 10%.
12. $100, 30%.
13. $6, $33\frac{1}{3}$%.
14. $9, $33\frac{1}{3}$%.
15. $10, 75%.
16. $1.50, 10%.

WRITTEN EXERCISE

Find the price that must be paid for goods marked as stated in each of these examples, and discounted at the rate given:

1. $635, 6%.
2. $350, 15%.
3. $275, 15%.
4. $425, 35%.
5. $21.25, 8%.
6. $45.50, 20%.
7. $375.25, 4%.
8. $175.50, 20%.
9. $225.75, 20%.

10. How much will a set of Scott's works, marked $14, cost at 20% discount? Check by taking 80% of $14.

11. If you can buy a $7 suit of clothes at 15% discount, how much will you save on the marked price?

12. At a bargain sale an $8 chair was sold at 30% discount. For how much did it sell?

13. If the list price of your books, pencils, and paper is $2.50 a year, how much will they cost at 12% discount?

Find the discount on goods of which the list price and rate of discount in each case are as follows:

14. \$430, 15%.
15. \$281, 20%.
16. \$425, 10%.
17. \$1235, 17%.
18. \$2350, 14%.
19. \$43.26, $33\frac{1}{3}\%$.
20. \$42.64, 25%.
21. \$6336, $6\frac{1}{4}\%$.
22. \$34.54, $16\frac{2}{3}\%$.

Find the selling price in each case, given the list price and rate of discount as follows:

23. \$275, 20%.
24. \$575, 5%.
25. \$462, 3%.
26. \$650, $12\frac{1}{2}\%$.
27. \$425, 12%.
28. \$320, 14%.
29. \$12.75, 20%.
30. \$6202, 14%.
31. \$52.84, $16\frac{2}{3}\%$.
32. \$26.32, 50%.
33. \$4235, 22%.
34. \$14.43, $33\frac{1}{3}\%$.

35. A \$17.50 set of china is marked down 15%. What is the selling price?

36. If you buy \$1.80 worth of fireworks at 15% discount, how much do they cost?

37. A book dealer buys a 65¢ arithmetic at 22% discount. How much does it cost him?

38. A \$31 overcoat is marked "25% off" at a bargain sale. What is the selling price?

39. If you buy a \$1.20 sled at 10% discount, how much does it cost? Suppose the discount is 20%; 25%.

40. How much will 2 dozen blackboard erasers cost at 45¢ a dozen, discount 10%? discount 20%?

41. A man paid \$150 for a carriage, and after using it for a year sold it at a discount of $16\frac{2}{3}\%$. What did he receive for it?

42. A man bought a \$900 automobile at 15% discount. After using it for a season he sold it for 15% less than he paid for it. How much did he receive for it?

ORAL EXERCISE

1. What is $2.50 with 10% added? with 10% subtracted?

2. What is $1200 with 20% added? with 25% subtracted?

3. A dealer buys some books for $100 and sells them at a profit of 20%. How much does he gain?

4. A dealer buys some cloth for $50 and sells it at a profit of 20%. How much does he gain? What is the selling price?

5. A grocer bought $15 worth of berries, but left them in the sun so long that he had to sell them at a loss of 10%. How much did he lose?

191. Profit and Loss. The per cent of profit or loss is reckoned on the *cost* of the goods.

In practical business it is essential that the cost should be fully considered, including freight charges, storage, share of rent and sale charges, and all of what the business man calls "overhead charges." The principle is the same as in the simpler problems adapted to work in the school.

192. Illustrative Problems. 1. A man buys goods for $40 and sells them at a profit of 20%. What is the selling price?

$$\begin{array}{r}\$40\\ \underline{0.20}\\ \$8\end{array}\ \text{profit}\qquad\qquad\begin{array}{r}\$40\\ \underline{8}\\ \$48\end{array}\ \text{selling price.}$$

2. A man buys goods for $60 and sells them for $40. He loses what per cent?

$$\begin{array}{r}\$60\\ \underline{40}\\ \$20\end{array}\ \text{loss}\qquad\qquad 6\cancel{0})\underline{2\cancel{0}\quad}\atop 0.33\tfrac{1}{3}$$

Since $20 is lost on $60, we wish to find what per cent $20 is of $60. Consequently we have the product (20) and one factor (60) given, to find the other factor.

$$\$20 \div \$60 = 0.33\tfrac{1}{3}.$$

Therefore the loss is 33⅓%.

ORAL EXERCISE

1. A hat dealer buys hats at $48 a dozen and sells them at $5 each. What is his per cent of profit?

2. A grocer buys starch at 50¢ a box and sells it at 10% profit. What is the selling price?

3. A dealer buys stove polish at 3¢ a cake and sells it at 4¢. What is his per cent of profit?

4. A grocer buys matches at 48¢ a dozen boxes and sells them at 54¢. What is his per cent of profit?

5. A druggist buys some soap at the rate of $2.40 a dozen cakes and sells it at 25¢ a cake. Find his per cent of profit.

6. Some soap, bought at $1.50 a dozen cakes, is damaged by dampness and is sold at 10¢ a cake. Find the per cent of loss.

WRITTEN EXERCISE

1. A grocer pays $12 a gross for boxes of starch. He sells the starch at 10¢ a box. What is his per cent of profit?

2. He buys washing powder in 1-pound packages at $5.88 a gross and sells it at 5¢ a package. What is his per cent of profit?

3. Some of this washing powder was damaged and had to be sold at 4¢ a package. What was the per cent of loss on each?

4. A dealer bought 150 bags of salt, 56 lb. to the bag, for $63. He sold $\frac{2}{3}$ of the salt at the rate of 1¢ a pound, and the rest at the rate of 2 lb. for a cent. Did he gain or lose on the transaction, and what per cent?

5. Macaroni in packages costs a grocer $15 a gross. He sells it at 12¢ a package. What per cent does he gain?

6. Gelatin in packages costs a grocer $11.52 a gross. At what rate per package must he sell it to gain 25%?

ORAL EXERCISE

1. How much is \$20 plus 10% of itself? plus 1% of itself?

2. How much is \$3 plus 30% of itself? plus 6% of itself?

3. How much is \$15 plus 1% of itself? plus 33⅓% of itself?

4. If you bought \$10 worth of eggs for a dealer, receiving 5% for your work, how much would you receive?

5. If you bought \$2 worth of eggs for a dealer and he paid you 10% of the cost, how much would you receive?

193. Commission. The pay received by one person for buying or selling for another is often reckoned at a certain per cent of the price, and is called *commission*.

For example, if a man buys \$1000 worth of oats for another man, at 5% commission, he would receive 5% of \$1000, or \$50.

WRITTEN EXERCISE

1. Required the commission on an \$850 sale, at 2½%.

2. Required the commission on a \$1200 sale, at 5½%.

3. Required the commission on a \$2000 purchase, at 4½%.

4. A commission merchant sells for a factory 7850 lb. of cheese at 13½¢ a pound, charging 5% commission. What sum should he remit to the factory?

5. A man sells at an auction \$292 worth of property for another man and receives 10% commission. How much does he get? How much is left for the other man?

6. A dealer in fruit in a city sends a man to the fruit region to buy for him. He pays him 2% commission. The buyer purchases \$2500 worth in one place, \$3200 worth in another, and \$4100 worth in another. What is his total commission?

ORAL EXERCISE

1. If you sold $2 worth of papers on a commission of 20%, how much would you make?

2. At 30%, how much is the commission on a $120 sale?

3. At 10% commission, how much does a collection agent receive for collecting a debt of $95.70?

4. An agent sells 100 school desks at $3.75 each and receives a commission of 10%. How much does he receive?

WRITTEN EXERCISE

1. How much is an architect's commission on a house costing $9472, at 5%?

2. How much is an auctioneer's commission for selling $358 worth of furniture, at 6%?

3. A real-estate dealer sells 168 acres of land at $40 an acre. How much is his commission at 2½%?

4. An agent sells our school $65 worth of maps, and his firm pays him 7½% commission. How much is this?

5. A farmer sends some butter and eggs to the city, and they are sold for $72.50. He is charged 8% by his agent. What is the net amount received by the farmer?

6. A real-estate dealer rents 21 city apartments for the owner, at $1500 each, and charges a commission of 2½%. How much is the commission?

7. A salesman's sales in a department store in a city amounted during the six days of one week to $257.70, $103.10, $175.02, $132.30, $302.25, $50.63. He receives a commission of 2½% on his sales. How much did he receive that week?

ORAL EXERCISE

1. If a man borrows money of another, how does he pay him for the loan, besides returning the money?

2. If you put money in a savings bank and leave it 6 mo. or 1 yr., will they pay you back only what you put in?

3. If any one where you live wished to borrow $100 for a year, what interest would he have to pay?

194. Interest. The money paid for the use of money is called *interest*. For example, if I borrow $50 for a year and pay $3 for the use of it, the $3 is interest.

195. Principal. The sum of money on which interest is charged is called the *principal*.

196. Reckoning Interest. Interest is ordinarily reckoned as a certain per cent of the principal. If the interest is 6% of the principal, and the principal is $200, the interest for 1 yr. is 6% of $200, or $12.

197. Rate of Interest. The per cent of interest is called the *rate*. That is, if the interest is 6% on the principal the rate is 6%.

WRITTEN EXERCISE

1. What is the interest on $375 for 1 yr. at 6%? at 4%?

2. What is the interest on $950 for 1 yr. at 5%? at 3%?

3. What is the interest on $235 for 1 yr. at 4%? at 5%?

4. What is the interest on $750 for 1 yr. at 6%? at 5%?

5. If you had $27 in a bank that paid 3% a year, how much interest would you receive for a year?

6. How much are the interest and principal together on $475 at the end of 1 yr., the rate of interest being 6%?

ORAL EXERCISE

Find the interest on the following sums for 1 yr.:

1. $100 at 4%.
2. $300 at 6%.
3. $60 at 3%.
4. $500 at 5%.
5. $900 at 3%.
6. $50 at 4%.
7. $400 at 6%.
8. $1200 at 3%.
9. $70 at 5%.
10. $1000 at 4%.
11. $3000 at 4%.
12. $90 at 6%.

Find the interest on the following sums for 6 mo., first finding the interest for 1 yr., and then dividing by 2:

13. $100 at 6%.
14. $200 at 5%.
15. $40 at 6%.
16. $400 at 3%.
17. $250 at 4%.
18. $60 at 4%.
19. $600 at 5%.
20. $400 at 5%.
21. $70 at 3%.
22. $800 at 4%.
23. $1000 at 3%.
24. $80 at 5%.

WRITTEN EXERCISE

Find the interest on the following sums for 1 yr.:

1. $652 at 4%.
2. $375 at 5%.
3. $27 at 3½%.
4. $325 at 3%.
5. $433⅓ at 3%.
6. $42 at 4½%.
7. $275 at 4%.
8. $450.25 at 4%.
9. $75 at 2½%.
10. $725 at 6%.
11. $275.20 at 5%.
12. $84 at 4½%.
13. $535 at 5%.
14. $625.20 at 5%.
15. $72 at 3⅓%.

Find the interest on the following sums for 6 mo.:

16. $82 at 5%.
17. $175 at 4%.
18. $350 at 2½%.
19. $75 at 6%.
20. $325 at 5%.
21. $675 at 3%.
22. $45 at 4%.
23. $450 at 6%.
24. $450 at 4½%.
25. $35 at 2%.
26. $395 at 3%.
27. $864 at 3%.
28. $475 at 6%.
29. $750 at 5%.
30. $890 at 6%.

198. Finding Interest. We have found the interest on money, at various rates, for one year, and also for half a year. We shall now find interest for other periods of time.

What is the interest on $250 for 10 mo. at 6%?

The interest for 1 yr. is 6% of $250, and for 10 mo. it is $\frac{10}{12}$, or $\frac{5}{6}$, of this sum. We therefore have

$$\tfrac{5}{6} \text{ of } 6\% \text{ of } \$250 = \$12.50.$$

In general, use cancellation. Thus, in this example we have

$$\tfrac{10}{12} \text{ of } 6\% \text{ of } \$250 = \frac{\cancel{10} \times \cancel{6} \times \$\overset{25}{\cancel{250}}}{\underset{2}{\cancel{12}} \times \underset{\cancel{10}}{\cancel{100}}} = \$\tfrac{25}{2} = \$12.50.$$

In this particular case we see that $\frac{5}{6}$ of 6% is 5 × 1%, or 5%, and so we simply have 5% of $250 = 5 × $2.50 = $12.50. Advantage of such short methods should always be taken. In finding interest, consider 30 da. as a month.

WRITTEN EXERCISE

1. If the interest on a certain sum is $93.84 for 1 yr., how much is it for 3 mo.? for 6 mo.? for 9 mo.?

2. If the interest on a certain sum is $36.48 for 1 yr., how much is it for 3 mo.? for 6 mo.? for 9 mo.?

3. What is the interest on $436, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 10 mo.?

4. What is the interest on $360, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 9 mo.?

5. What is the interest on $540, at 4%, for 1 yr.? for 1 mo.? for $\frac{1}{2}$ mo.? for 5 da.? for 1 da.?

6. What is the interest on $175.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 3 mo.? for 1 mo.? for 10 da.?

7. What is the interest on $1550, at 6%, for 1 yr.? for 6 mo.? for 3 mo.? for 9 mo.? for $1\frac{1}{2}$ mo.?

ORAL EXERCISE

Find the interest on the following:

1. \$100, at 5%, for 1 yr.
2. \$300, at 6%, for 1 yr.
3. \$500, at 4%, for 1 yr.
4. \$600, at 5%, for 1 yr.
5. \$800, at 5%, for 1 yr.
6. 100, at 6%, for 6 mo.
7. \$600, at 3%, for 6 mo.
8. \$800, at 5%, for 6 mo.

199. Interest for Years and Months. We have seen that to find the interest for 2 yr. 6 mo., we find the interest for 1 yr., and multiply by $2\frac{1}{2}$. That is,

Find the interest for 1 year, and multiply by the number of years.

Thus, to find the interest on \$125 for 2 yr. 3 mo. at 5% we proceed as follows:

2 yr. 3 mo. = $2\frac{1}{4}$ yr.
5% of \$125 = \$6.25, interest for 1 yr.
$2\frac{1}{4}$ × \$6.25 = \$14.06, interest for 2 yr. 3 mo.

$$\boxed{\begin{array}{r} \$125 \\ .05 \\ \hline \$6.25 \\ 2\frac{1}{4} \\ \hline 1\ 56\frac{1}{4} \\ 12\ 50 \\ \hline \$14.06 \end{array}}$$

In all such cases cancellation will be found very convenient. Here, for example, since 2 yr. 3 mo. = $2\frac{1}{4}$ yr., we have

$$2\tfrac{1}{4} \times 5\% \text{ of } \$125 = \frac{9 \times 5 \times \overset{5}{\cancel{\$125}}}{4 \times \underset{4}{\cancel{100}}} = \$\tfrac{225}{16} = \$14.06.$$

WRITTEN EXERCISE

Find the interest on the following:

1. \$250, at 6%, for 6 mo.
2. \$300, at $5\frac{1}{2}$%, for 8 mo.
3. \$8200, at 5%, for $2\frac{1}{2}$ yr.
4. \$625, at 4%, for $3\frac{1}{2}$ yr.
5. \$35, at 6%, for $1\frac{1}{2}$ yr.
6. \$45.50, at 4%, for 2 yr. 3 mo.
7. \$750, at 5%, for 2 yr. 4 mo.
8. \$75.50, at 6%, for 2 yr. 6 mo.
9. \$225, at 6%, for 3 yr. 6 mo.
10. \$4250, at $4\frac{1}{2}$%, for 1 yr. 6 mo.

Find the interest on $250, at 6%, for:

11. 3 yr. 6 mo.
12. 2 yr. 4 mo.
13. 1 yr. 9 mo.
14. 3 yr. 3 mo.
15. 4 yr. 1 mo.
16. 3 yr. 7 mo.

Find the interest on $375, for 3 yr. 4 mo., at:

17. 6%. **18.** 5%. **19.** 4%. **20.** 3%. **21.** 2%.

Find the interest for 2 yr. 7 mo., at 6%, on:

22. $175. **23.** $325. **24.** $475. **25.** $685. **26.** $750.

27. A man borrows $575, agreeing to pay 6% interest. He pays the principal and interest in 1 yr. 8 mo. How much money does he pay?

28. A man buys 7 head of cattle at $85 a head, and agrees to pay for them in 8 mo., with interest at 6%. How much does he pay in all, principal and interest?

29. Mr. St. John hires Mr. Sinclair to paint his house, agreeing to pay him $65 for the work. Mr. Sinclair agrees to wait 3 mo. for his money if Mr. St. John will pay interest at the rate of 5%. How much is due at the end of the 3 mo.?

30. Higgins & Son are dealers in dry goods. They need $1250 with which to buy holiday goods. They borrow it Nov. 1 and agree to pay it back on Feb. 1 with interest at 5%. How much do they owe on Feb. 1?

31. McDowell and Perkins buy $725 worth of goods from a manufacturer, agreeing to pay for the goods in 5 mo., with interest at 6%. How much do they owe at the end of the 5 mo.?

32. On March 1, Mr. Roberts buys a secondhand automobile from Mr. Grant, agreeing to pay $475 for it on July 1, with interest at 6%. How much interest is due on July 1? What is the total amount due?

WRITTEN REVIEW

Write as common fractions, and also as per cents:

1. 0.1.
2. 0.12.
3. 0.62.
4. $0.62\frac{1}{2}$.
5. $0.03\frac{1}{8}$.
6. 0.33.
7. 0.2.
8. 0.02.
9. $0.16\frac{2}{3}$.
10. 0.75.

11. A man had $1640 in the bank and drew out 10% of his money. How much did he draw out? How much was left?

12. If Fred made 40 runs in this season's games, and Frank made 10% more, how many runs did Frank make?

13. Jack has won 35 games at tennis and has lost 20% less than this number. How many games has he lost?

14. A man bought a bicycle for $30 and sold it at a loss of 25%. How much did he lose?

15. A man bought 75 cows at $65 apiece and sold them at a profit of 18%. How much did he gain?

16. A dealer buys 45 sheep at $7.50 each and sells them at a profit of 10%. How much does he gain on them all?

17. A man bought an automobile for $750, and after using it two seasons sold it for 60% less than it cost. What was the selling price?

18. If there are 350 pupils in a school and 48% are boys, what decimal fraction of the pupils are boys? How many boys are there? How many girls are there?

19. If a city has a population of 26,200, and $22\frac{1}{2}$% are pupils in school, what decimal fraction of the population is in school? How many are in school?

20. A merchant has $16,000 invested in his business. The first year he made a profit of $9\frac{1}{2}$%, the second year $14\frac{1}{2}$%, and the third year 18%. How much did he make in the three years?

ORAL REVIEW

1. From these columns show that 20% = $\frac{1}{5}$, 40% = $\frac{2}{5}$, 60% = $\frac{3}{5}$, and 80% = $\frac{4}{5}$.

				20%
			20%	20%
		20%	20%	20%
	20%	20%	20%	20%
20%	20%	20%	20%	20%
20%	40%	60%	80%	100%

2. Show that 20% is $\frac{1}{2}$ of 40%, $\frac{1}{3}$ of 60%, $\frac{1}{4}$ of 80%, and $\frac{1}{5}$ of 100%.

3. Show that 40% is $\frac{2}{3}$ of 60%, $\frac{1}{2}$ of 80%, and $\frac{2}{5}$ of 100%.

4. Show that 60% is $1\frac{1}{2}$ times 40%, 3 times 20%, $\frac{3}{4}$ of 80%, and $\frac{3}{5}$ of 100%.

These columns should be written on the board. Columns of other per cents, such as $12\frac{1}{2}$%, $16\frac{2}{3}$%, 25%, and $33\frac{1}{3}$%, are also useful for rapid drill work.

5. A foot is what fraction of a yard? A foot is what per cent of a yard?

6. A pint is what fraction of a gallon? A pint is what per cent of a gallon?

7. Five cents is what fraction of a dollar? Five cents is what per cent of a dollar?

8. If you wish to find $33\frac{1}{3}$% of a number, what fractional part of the number do you wish to find?

9. If a man spends one fifth of his income for rent, what per cent does he spend for this purpose?

10. What per cent of the perimeter of a square is one side of the square? What per cent are two sides? three sides?

11. If there are 30 pupils in a class and 50% are boys, what fraction are boys? How many boys are there?

12. Find the interest on $500 for 1 yr. at 6%; at 5%; at 4%.

13. Find the interest on $800 for 2 yr. at 5%; at 4%; at 3%.

WRITTEN REVIEW

Write as common fractions, and also as per cents:

1. 0.01.
2. 0.37.
3. $0.37\frac{1}{2}$.
4. $0.06\frac{1}{4}$.
5. 0.31.
6. $0.31\frac{1}{4}$.
7. $0.33\frac{1}{3}$.
8. 0.66.
9. $0.66\frac{2}{3}$.
10. 0.87.
11. $0.87\frac{1}{2}$.
12. 0.16.
13. $0.07\frac{1}{2}$.
14. $0.08\frac{1}{3}$.
15. $0.03\frac{3}{4}$.

16. From a barrel of flour weighing 196 lb., 75% has been sold. How many pounds are left?

17. A man agreed to build a stone wall 225 ft. long. He has already built $33\frac{1}{3}\%$ of it. How many feet has he built, and how many feet has he still to build?

18. A trolley car has a run of 64 mi. a day. When it has run 75% of this distance, how many miles has it still to run?

Find the values of the following:

19. 25% of 168.
20. 75% of 184.
21. 20% of 135.
22. 30% of 140.
23. 5% of 34.
24. 5% of 48.
25. 6% of 250.
26. 6% of 320.
27. 0.25×484.
28. 25% of 536.
29. 0.75×368.
30. 75% of 460.

31. How much will $62\frac{1}{2}\%$ of a cubic foot of steel weigh if 1 cu. ft. of steel weighs 490 lb.?

32. If a steel car when full carries 96,000 lb. of coal, how much does it carry when it carries $87\frac{1}{2}\%$ of its capacity?

33. If out of 320 problems in arithmetic you solve 95% of them without errors, how many do you solve correctly?

34. If a locomotive weighing 118 tons can exert a pull equal to $22\frac{1}{2}\%$ of its weight, how many tons of pull can it exert?

35. A ball team has played 48 games this season and has won $62\frac{1}{2}\%$ of them. How many games has it lost?

ORAL REVIEW

State the values of the following:

1. $\frac{1}{2}$ of 48 0.5 of 48 0.50 of 48 50% of 48
2. $\frac{1}{4}$ of 36 0.25 of 36 25% of 36 25% of 80
3. $\frac{3}{4}$ of 20 0.75 of 20 75% of 20 75% of 12
4. $\frac{1}{4}$ of 40 25% of 40 75% of 40 75% of 80

With respect to the figures given below, answer the following:

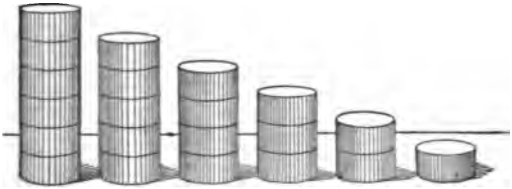
5. *F* is what part as large as *E*? what per cent as large?

6. *F* is what part as large as *D*? what per cent as large?

7. *F* is what part as large as *C*? what per cent as large?

8. *F* is what part as large as *B*? what per cent as large?

9. *F* is what part as large as *A*? what per cent as large?



In each of the following pairs tell what per cent the first is of the second:

10. *E, C.* **12.** *E, D.* **14.** *D, C.* **16.** *D, A.* **18.** *B, A.*
11. *E, A.* **13.** *E, B.* **15.** *C, B.* **17.** *D, B.* **19.** *C, A.*

20. How much is the commission, at 2%, on a sale amounting to $500? on a sale amounting to $5000?

21. How much is the interest on $1000 for 2 yr. at 4%? at 5%? at 6%? at 3%?

WRITTEN REVIEW

1. The diameter of a circle is about 31⅘% as long as the circumference. What is the diameter of a circle whose circumference is 30 ft.?

2. A meter is $9\frac{13}{36}$% longer than a yard. How many inches are there in a meter?

3. A knot, used in measuring distances at sea, is 15⅕% longer than a mile. How many feet are there in a knot?

Find the values of the following:

4. 40% of 160.
5. 60% of 180.
6. 70% of 150.
7. 80% of 120.
8. 4% of 120.
9. 4% of 230.
10. 3% of 125.
11. 3% of 235.
12. ⅕ of 1275.
13. 20% of 95.
14. ¾ of 724.
15. 75% of 724.

16. A man makes a profit of 12½% on a sale of property which cost him $1600. How much was the profit?

Find the values of the following:

17. 50% of $348.
18. 25% of $464.
19. 12½% of $498.
20. 6¼% of $3376.
21. 37½% of $49.76.
22. 62½% of $91.92.
23. 87½% of $42.48.
24. 3⅛% of $739.20.
25. 66⅔% of $372.60.
26. 83⅓% of $42.36.
27. 20% of $926.45.
28. 40% of $732.25.
29. 60% of $821.72.
30. 82½% of $27,500.
31. 35% of $17,500.
32. 63% of $26,200.

33. Find 1¼ times 480; 1.25 times 480; 125% of 480.

34. Find 1⅜ times 960; 1.37½ times 960; 137½% of 960.

WRITTEN REVIEW

Find the values of the following:

1. 50% of 167.
2. 20% of 241.
3. 90% of 340.
4. 2% of $426.
5. 7% of $341.
6. 8% of $468.
7. ¾ of 236.
8. 75% of 236.
9. 0.75 × 236.

Find the net prices of goods billed as follows, with discounts as stated:

10. $450, 20%.
11. $375, 15%.
12. $480, 25%.
13. $675, 8%.
14. $840, 12½%.
15. $925, 28%.
16. $960, 16⅔%.
17. $1760, 12½%.
18. $1575, 33⅓%.

19. A typewriter was marked $80 but was sold at 18% discount. What was the net price?

20. An automobile was marked $950 but was sold at 12% discount. What was the net price?

21. A merchant bought some goods for $1687.50, less 20%. What was the net price?

22. On which will the net amount after deducting the commission be greater, a sale of $1750 less 2% commission or a sale of $1780 less 3% commission? How much greater?

23. A clerk receives $6 a week and 3% commission on goods sold. If he sells $500 worth of goods in a week, what is his income for the week?

24. How much is the interest on $1800 at 6% for 1 yr.? for 2 yr.? for ¼ yr.? for 1¼ yr.? for 2 yr. 3 mo.?

25. A dealer bought a bill of goods amounting to $2750. By paying promptly he was allowed a discount of 6% on this bill. How much was the discount?

LITTLE EXAMINATIONS

I. **1.** 75% of 9.6.
2. 12½% of $1.76.
3. 33⅓% of $2.46.
4. 66⅔% of $9.75.
5. 16⅔% of $8.46.
6. 2½% of $17.50.
7. 104% of 64½ ft.
8. 3% of $4876.
9. 1.7% of $275.
10. 512% of $290.

II. **1.** 25% of $75.
2. 75% of $25.
3. 12½% of $37.50.
4. 37½% of $12.50.
5. 16⅔% of $18.60.
6. 3¾% of 2 ft. 3 in.
7. 2⅞% of 4 yd. 3 in.
8. 8% of $75.50.
9. 1.6% of $4.80.
10. 275% of $500.

III. **1.** 12½% of $16.40.
2. 37½% of $17.60.
3. 62½% of $33.60.
4. 87½% of $42.40.
5. 66⅔% of $48.90.
6. 2½% of $75.
7. 3⅓% of $1.20.
8. 7% of $24.40.
9. 2.3⅓% of $66.
10. 250% of 75 ft.

IV. **1.** How much is $175 less 8% of itself?
2. How much is $250 increased by 15% of itself?
3. Make out a bill of six items and receipt it.
4. How much is the commission at 8% on a sale of goods amounting to $2750? at 8½%? at 12½%?
5. Find the interest on $2450 for 2 yr. 6 mo. at 5%.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write 1921 and 1927 in Roman numerals, and LXIX and LXXXVIII in ordinary numerals.

Find the sum, difference, product, and quotient of each of the following pairs of numbers, dividing the greater by the less:

2. 6531, 2177.
3. 18,846, 3141.
4. 29,547, 4221.
5. 30,195, 6039.
6. 56,080, 7010.
7. 28,404, 7101.
8. 31,104, 3456.
9. 24,258, 4043.
10. 69,993, 7777.

Find the sum, difference, product, and both quotients of each of the following pairs of fractions:

By "both quotients" is meant that we should divide $\frac{2}{3}$ by $\frac{3}{4}$ and $\frac{3}{4}$ by $\frac{2}{3}$.

11. $\frac{2}{3}$, $\frac{3}{4}$.
12. $\frac{1}{3}$, $\frac{3}{8}$.
13. $\frac{2}{3}$, $\frac{5}{8}$.
14. $\frac{2}{3}$, $\frac{1}{8}$.
15. $\frac{3}{4}$, $\frac{1}{8}$.
16. $\frac{3}{4}$, $\frac{5}{8}$.
17. $\frac{1}{2}$, $\frac{3}{16}$.
18. $\frac{1}{3}$, $\frac{5}{16}$.

Find the values of the following:

19. 7% of 48.
20. 9% of 75.
21. 6% of 97.
22. 9% of 7.4.
23. 8% of 6.8.
24. $2\frac{1}{2}$% of 38.
25. $3\frac{1}{3}$% of 69.
26. $1\frac{1}{4}$% of 64.
27. $6\frac{1}{4}$% of 7.2.
28. $3\frac{1}{3}$% of 7.2.
29. 50% of 9 ft. 8 in.
30. $33\frac{1}{3}$% of 10 ft. 9 in.
31. 25% of 8 ft. 6 in.
32. 75% of 3 lb. 4 oz.
33. $37\frac{1}{2}$% of 9 lb. 8 oz.

ORAL EXERCISE

Find 125% of the following:

1. 36.
2. 64.
3. 80 ft.
4. $160.
5. 28.
6. 32.
7. 72 ft.
8. $360.
9. 80.
10. 88.
11. 60 ft.
12. $2.40.

Find 150% of the following:

13. 40.
14. 30.
15. 60 in.
16. $220.
17. 70.
18. 25.
19. 90 yd.
20. $150.
21. 38.
22. 52.
23. 62 rd.
24. $1.20.

25. If a man sells $2400 worth of goods at 25% profit, what is the selling price?

26. If a manufacturer spent $50,000 in advertising and increased this 50%, how much did he then spend?

27. Find $\frac{3}{4}$ of 16; of $160; of 1600 lb.; of 1.6 A.

28. Find $\frac{3}{8}$ of 24; of $240; of 2400 lb.; of 2.4 A.

Find $133\frac{1}{3}\%$ of the following:

29. 36.
30. 48.
31. 60 A.
32. $150.
33. 51.
34. 57.
35. 90 min.
36. $2.70.

Find $116\frac{2}{3}\%$ of the following:

37. 36.
38. 30 yd.
39. 54.
40. 84 rd.
41. 48.
42. $360.
43. 72.
44. $2.40.

45. Goods costing $2400 are sold at $16\frac{2}{3}\%$ above cost. What is the selling price?

46. Goods costing $2700 are damaged to the extent of $33\frac{1}{3}\%$. How much is the loss?

47. Multiply 240 by 100; by 50; by 25.

WRITTEN EXERCISE

Add the following:

1.	2.	3.	4.
$247.60	$382.68	$129.36	$287.62
329.48	592.39	427.96	469.83
272.56	78.62	380.09	48.79
752.40	400.00	870.64	602.76
670.52	32.48	572.04	85.55
727.44	162.96	619.91	328.76

Subtract the following:

5.	6.	7.	8.
$2763.01	$8101.23	$9124.51	$8170.01
1987.62	6472.85	7235.62	5296.83

Multiply as indicated:

9. $\frac{2}{3} \times 468$.
10. $\frac{3}{4} \times 564$.
11. $\frac{7}{8} \times 336$.
12. $2\frac{1}{3} \times 5.28$.
13. $3\frac{3}{4} \times 5.08$.
14. $4\frac{5}{8} \times 7.36$.
15. 294×326.
16. 34.7×289.
17. 42.9×57.6.
18. 5.38×27.9.
19. 7.29×8.26.
20. 6.87×0.79.
21. 0.7×17.09.
22. 0.6×0.072.
23. 2.8×0.009.
24. 7.3×6.007.
25. 5.8×72.01.
26. $6.6\frac{2}{3} \times 8.46$.

Divide, carrying the quotients to the nearest hundredth:

27. $4.5 \div 7.2$.
28. $9.8 \div 0.09$.
29. $0.7 \div 3.4$.
30. $84 \div 67$.
31. $1.42 \div 3.7$.
32. $1.56 \div 0.7$.
33. $28.1 \div 0.06$.
34. $597 \div 0.32$.
35. $8 \div 72.6$.

Perform the operations indicated:

36. $4\frac{3}{8} + 9\frac{5}{16}$.
37. $5\frac{2}{3} \div 3\frac{7}{8}$.
38. $1.7\frac{2}{3} \times 3.1\frac{1}{2}$.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you have a given per cent, how do you express it as a common fraction?

2. If you are to find a certain fractional part of a number, how do you express this as a per cent of the number?

3. If you have a certain rate per cent expressed with the per cent sign, how do you express it as a decimal?

4. If you have a decimal fraction, how do you express the same number with a per cent sign?

5. If you know what a given per cent of a number is, how do you find the number?

6. A man's salary two years ago was increased a certain per cent. Last year it was decreased the same per cent. Is his salary at present greater than or less than it was two years ago?

7. A merchant marks his goods a certain per cent above cost. Owing to hard times he sells them at the same per cent below the marked price. Does he sell them for more than or less than cost?

8. If you know the amount paid for a certain property, and the rate of commission, how do you find the commission?

9. If you know the net amount left after the commission has been deducted, and the price at which the property was sold, how do you find the rate of commission?

10. If you know the time that a sum of money is on interest, the rate, and the sum of money, how do you find the interest?

11. If you know what a dealer paid for some goods, and his selling price, how do you find the per cent of profit or loss?

The boys in class should be asked how to find a baseball player's fielding average, batting average, and so on.

III. INDUSTRIAL PROBLEMS

SHALL IT BE A CAR OR A HORSE?

1. A physician finds that it costs him $4.50 a week to keep his horse at a livery stable. He pays $2.50 twice in every three months for shoeing his horse. Veterinary attention averages $4 a year. What is the total expense of a horse and buggy per year?

2. The horse and buggy cost $275. At 6% interest, how much must be added to the above to obtain the total outlay for a year?

3. Deterioration is to be considered. Figuring that in 15 yr. he could sell the horse and buggy for $50, how much must the physician add to his annual outgo because of deterioration?

4. He decides to see whether it might not be better to buy an automobile. He finds that he can get a serviceable one for $765. At 6%, what would be his annual loss in interest on his money?

5. He finds that every 3 yr. he can, by paying $250 additional, exchange his old car for a new one. How does the deterioration per year compare with that on the horse and buggy (Ex. 3)?

6. Allowing $12.50 a month, what will the tires cost a year?

7. If 15 gal. of gasoline costing 20¢ a gallon are used a week, besides 80¢ worth of lubricating oil, and the garage costs $3 a week, what will this amount to in a year?

8. The automobile will save him 3 hr. a day. If he counts the time at 75¢ an hour, how much will this amount to in 361 da.?

9. He thinks it would be worth $200 a year in comfort to have a car. Now consider the whole question. What are the total expenses of the horse and buggy? What are the total expenses of the car, less the comfort and the saving in time? Which is the better for him to have?

THRIFT IN THE HOME

1. Mrs. Brown finds that she can save at least 50¢ a week by going to the market and buying in person. By doing this, how much a year, at least, will she save?

2. Frank gets interested in the idea of helping to cut down the high cost of living. He says he will raise vegetables. Allowing \$1.25 for plowing, \$3.25 for fertilizer, \$1.30 for seed, and 75¢ a week for 22 wk., which wages his mother insists that he shall have, what is the cost of the season's vegetables?

3. Mrs. Brown used to spend \$54 for these same vegetables at the store. What was the saving in having their own garden?

4. Since Frank's mother no longer buys over the telephone, she decides to cut off this expense. The telephone costs \$2.25 a month. How much does she save in a year by not having one?

5. By buying at a bargain sale an \$18 suit at 15% off, three \$1.50 shirts at 10% off, a dozen 25-cent handkerchiefs at $16\frac{2}{3}\%$ off, a pair of \$4.75 shoes at 25% off, and a hat and some ties amounting to \$5.35 at 20% off, how much does Frank save in all?

6. How much have Frank and his mother together saved on the purchases and telephone in Exs. 1–5, as stated above?

7. They find that they can invest this money at 5% interest. What will it amount to in 10 yr. at simple interest?

8. A year after the investment mentioned in Ex. 7 was made they invested an equal amount again at 5% interest, and they did this every year until the end of the 10 yr. How much did they have at the end of that time, including the interest?

9. Frank will then be old enough to go to college. His mother says that he will need \$1500 if he can earn a free scholarship. Will they have money enough saved to send Frank to college?

IV. EXAMINATION REVIEWS

ORAL EXAMINATION

1. State rapidly the following sums:

87	46	89	67	49	85	74
95	28	98	85	86	49	38

2. Make change from $10, the purchases amounting to:

$6.75 $4.80 $6.92 $3.78 $5.69 $8.17

3. State the results of the following:

3 × 34	6 × 33	294 ÷ 14	198 ÷ 18
7 × 61	8 × 32	330 ÷ 15	264 ÷ 12

4. Express the following as common fractions:

20%   50%   90%   12½%   33⅓%

5. State the results of the following:

12½% of 88   37½% of 88   16⅔% of 24   66⅔% of 24

6. How many rods in a mile? square rods in an acre?

7. How many days from March 7 to April 7?

8. If 16⅔% of the pupils in a school of 180 pupils were not in school last year, how many new pupils are there this year?

9. A rectangle 6 in. by 8 in. was increased 50% in length and 33⅓% in width. What is the area after the increase?

10. In a school of 220 pupils, if there should be an increase of 5% next year, how many pupils would there then be?

Arranged as they are on the cumulative plan and with exactly ten questions in each, these examinations furnish a series of excellent reviews and at the same time they allow for easy rating on the scale of ten or a hundred.

WRITTEN EXAMINATION

1. Add the following:

$4\frac{7}{8} + 1\frac{3}{8}$ $6\frac{5}{8} + 7\frac{3}{4}$ $9\frac{3}{4} + 5\frac{3}{8}$ $16\frac{7}{12} + 15\frac{3}{4}$

2. Subtract the following:

$4\frac{1}{24} - 1\frac{9}{16}$ $9\frac{1}{48} - 3\frac{7}{8}$ $8\frac{5}{24} - 7\frac{1}{4}$ $18\frac{11}{12} - 12\frac{7}{8}$

3. Perform the operations indicated:

$5\frac{3}{4} \times 2\frac{3}{8}$ $7\frac{5}{8} \times 8\frac{3}{4}$ $6\frac{3}{4} \times 7\frac{1}{2}$ $28 \times 75\frac{3}{4}$
$2\frac{1}{2} \div 1\frac{1}{4}$ $13\frac{3}{4} \div 2\frac{3}{4}$ $\frac{9}{16} \div \frac{3}{8}$ $15\frac{1}{4} \div 18\frac{3}{8}$

4. Copy and complete the following:

$2\frac{1}{2}$ mi. = (?) rd. 49 ft. = (?) in.
$82\frac{1}{2}$ ft. = (?) rd. $22\frac{1}{2}$ sq. ft. = (?) sq. yd.
4800 lb. = (?) T. 384 cu. ft. = (?) cd.

5. Find the values of the following:

25% of 844 0.75 × 368 90% of 480
75% of 724 $\frac{3}{4}$ of 5040 69% of 539

6. Find the net prices of goods marked and discounted as follows:

\$850, 10% \$240, $12\frac{1}{2}$% \$450, $66\frac{2}{3}$%
\$900, 25% \$360, $16\frac{2}{3}$% \$270, $33\frac{1}{3}$%

7. Increase each of the following by the per cent indicated:

\$120, 40% \$640, $37\frac{1}{2}$% \$1296, 25%
\$696, $16\frac{2}{3}$% \$1250, 10% \$18.75, $33\frac{1}{3}$%

8. Find the number of which

10 is 10% 12 is 6% 75 is 75%

9. Compare 25% of 50% of \$1000 with 50% of 25% of \$1000, and each with $12\frac{1}{2}$% of 50% of \$2000.

10. The number 85 is 15% less than what number?

WRITTEN EXAMINATION

1. Write 1921 in three different ways.

2. A man has two pieces of land: one of 240 A., which he values at $125 per acre; and one of 65 A., which he values at $80 per acre. What is the average value of all the land per acre?

3. A state road 471 mi. long cost $2,325,798. What was the average cost per mile?

4. A farmer has 160 A. of land worth $150 per acre, 10 cows worth $65 each, 30 T. of hay worth $9.50 per ton, 800 bu. of corn in the crib worth 56¢ per bushel, $125 worth of poultry, farm machinery worth $450, an automobile worth $750, 20 hogs worth $7.50 each, and other property worth $1125. Make an inventory of his property.

5. A careless farmer failed to test his seed corn, and only four grains out of five that were planted grew. The farmer's yield on 40 A. was 1800 bu. If the corn was worth 48¢ per bushel, how much did he lose by his neglect?

6. If corn shrinks 10% in weight when carried over to the next summer, what is the loss on 80 A. of corn yielding 45 bu. (56 lb. each) per acre, the price remaining 50¢ a bushel?

7. Draw a plan of a village lot 200 ft. long and 100 ft. wide, using the scale $\frac{1}{200}$.

8. Draw a plan for a living room 18′ × 24′, with a bedroom 10′ × 12′ adjoining, using the scale $\frac{1}{48}$.

9. In a school of 492 pupils $16\frac{2}{3}\%$ were not there last year. How many of the present number were in the school last year?

10. Using short methods, find the cost of the following bill of goods: 14 pr. gloves @ $1.25, $12\frac{1}{2}$ yd. cloth @ 64¢, 16 rugs @ $12.50, and 4 doz. spools thread @ 15¢ per dozen.

WRITTEN EXAMINATION

1. I bought a hat for $2.40, which was 20% below the marked price. Find the marked price.

2. A farmer had $5\frac{1}{2}$ A. that yielded 12 T. of ensilage to the acre. He put this into a silo, and fed 12 cows an average of 30 lb. a day from November 1 to June 1. About how many tons did he have left in the silo on June 1?

3. By selection of the seed the yield from 80 A. of corn was increased 20% over 45 bu. per acre. Find the total yield.

4. A new road was built along one side of this farm at a cost of $3600 per mile. If the corn is worth 50¢ a bushel, the yield will pay for what part of a mile of this road?

5. A poultry raiser has a lot 10 rd. square which he wishes to fence into four equal square lots, placing the fence posts 1 rd. apart. How many posts will he need? Draw a diagram.

6. A man paid $1200 for an automobile. He sold it after 2 yr. for a price which showed a discount of 25% at the end of 1 yr., and a further discount the next year of 25% on the remainder. What price did he get for his car?

7. A salesman is offered a salary of $1250 a year, or half that amount and 2% commission on his sales. If the sales amount to $30,000 a year, which is the better plan for him?

8. Find the interest on $1250 for 2 yr. 4 mo. at 6%.

9. A teacher who received a salary of $75 a month for 9 mo. lent her brother at the end of the year 20% of her salary. He kept the money 4 yr. 6 mo., and then returned it with 6% interest. How much did he pay her?

10. If an automobile travels at the rate of 24 mi. an hour, how long does it take to go 0.8 mi.?

WRITTEN EXAMINATION

1. Write the following as decimal fractions:

$$\frac{1}{10} \qquad \frac{4}{10} \qquad \frac{232}{1000} \qquad \frac{723}{1000} \qquad \frac{3}{10}$$

2. Write as whole numbers or mixed decimals:

$$\frac{13}{10} \qquad \frac{15}{10} \qquad \frac{225}{100} \qquad \frac{3007}{1000} \qquad \frac{2800}{1000}$$

3. Add, and also subtract the following:

$$\begin{array}{ccccc} 127.42 & 403.61 & 512.71 & 623.42 & 803.40 \\ \underline{109.64} & \underline{362.83} & \underline{329.86} & \underline{435.64} & \underline{637.68} \end{array}$$

4. Divide the following as indicated:

$41.6 \div 0.4$ $\qquad 2.22 \div 0.02$ $\qquad 36.08 \div 3.28$

$0.25 \div 0.5$ $\qquad 3.33 \div 0.03$ $\qquad 1.036 \div 0.56$

5. A man pays $113 for coal at $5.65 a ton. How many tons does he buy?

6. The product of two numbers is 156.25, and one of the numbers is 5000. What is the other number?

7. If wind blows in a hurricane at the rate of 8796.6 ft. per minute, what is the rate per second?

8. A contractor bought a shipment of bricks for $176.40, paying $7.20 a thousand. How many bricks did he buy?

9. In a recent year there were 614 coöperative creameries in Minnesota, and they made 74,934,940 lb. of butter in that year. Find to the nearest pound the average amount per creamery.

10. A farmer found that he could average 2.76 T. of alfalfa per acre on his 40-acre field. At $7 per ton, what was this crop worth? How did the value of the crop compare with that of clover in another 40-acre field averaging 1.85 T. per acre and worth $5.40 per ton?

WRITTEN EXAMINATION

1. Multiply the following:

1.3 × 48   4.25 × 64   3.42 × 286
2.9 × 62   6.75 × 84   4.25 × 66.4

2. Multiply the following:

8.2 × 6.8   8.36 × 4.8   5.33 × 2.96
7.7 × 8.8   9.82 × 6.7   4.56 × 8.21

3. Divide, carrying the quotients to two decimal places:

6.25 ÷ 25   3.49 ÷ 26   8.34 ÷ 26
6.75 ÷ 75   47.2 ÷ 34   342.9 ÷ 0.16

4. Divide, carrying the quotients to three decimal places:

8.48 ÷ 1.7   39.4 ÷ 2.8   426.9 ÷ 0.31

5. If a locomotive costs $23,000, a steel sleeping car $26,000, and a steel day coach $8000, how much will a railway company have to pay for 36 day coaches, 5 sleeping cars, and 2 locomotives?

6. In a year when the Montana mines produced 285,719,918 lb. of copper and Michigan produced 155,715,286 lb., Montana produced how much more than Michigan?

7. A contractor bought a lot of shingles for $717.75, paying $4.12½ a thousand. How many thousand shingles did he buy?

8. In a year when this country produced 304,043,400 lb. of wool, the total value being $75,819,251, what was the average value per pound, to the nearest cent?

9. In a year when Iowa produced 378,766,000 bu. of corn worth 50¢ a bushel, what was the value of the crop?

10. How many times will a roller 4.25 ft. in circumference turn in rolling the length of a lawn 136 ft. long?

CHAPTER VII

HOW TO SOLVE PROBLEMS

200. Necessity for Using Short Methods. We have now learned how to perform all the common operations with numbers, but before we take up further work in business arithmetic we should consider the short methods of computing. If Ralph sells 8 doz. eggs at 25¢ a dozen, it would be absurd for him to take a pencil and some paper to find the amount received. He should calculate this "in his head" by using a short method.

ORAL EXERCISE

1. Multiply by 10: 27, 2.7, and 270.
2. State the short method of multiplying by 10; by 100.
3. Multiply by $\frac{10}{2}$: 20, 44, 88, and 440.
4. State the short method of multiplying by 5.
5. Multiply by $\frac{100}{4}$: 40, 88, and 160.
6. State the short method of multiplying by 25.
7. Multiply by $\frac{100}{3}$: 3, 9, 18, and 66.
8. State the short method of multiplying by $33\frac{1}{3}$.
9. Multiply by $\frac{1000}{8}$: 8, 16, 24, and 32.
10. State the short method of multiplying by 125.

Teachers should observe that Exs. 2, 4, 6, 8, and 10 depend respectively on Exs. 1, 3, 5, 7, and 9. It is desirable to give a few oral exercises applying this work at once to problems of local interest. The work is summarized formally on page 260.

201. Short Methods in Multiplication. We shall now consider those short methods of multiplication which are particularly valuable because of certain multipliers which are so often used.

To multiply by 10, move the decimal point one place to the right, annexing a zero if necessary.

To multiply by 100 or 1000, move the decimal point to the right two or three places respectively, annexing zeros if necessary.

To multiply by 5, multiply by 10 and divide by 2.

To multiply by 25, multiply by 100 and divide by 4.

To multiply by 125, multiply by 1000 and divide by 8.

To multiply by $33\frac{1}{3}$, *multiply by 100 and divide by 3.*

To multiply by 9, multiply by 10 and subtract the multiplicand.

To multiply by 11, multiply by 10 and add the multiplicand.

To find 50% of a number, divide the number by 2.

To find 25% of a number, divide the number by 4.

WRITTEN EXERCISE

Multiply, in turn, by 10, by 100, by 5, by 25, and by 125:

1.	5656	8448	17,184	15,048	14,120
2.	7368	8.776	11,840.8	17.008	50.744
3.	4976	16.424	48,245.6	15.032	30.832

Multiply, in turn, by $33\frac{1}{3}$, *by 9, and by 11:*

4.	16,974	64,050	12,210	14,601	10,803
5.	141.51	136.71	1600.2	154.05	99.819
6.	180.36	160.26	1594.2	17.163	99.603

Find, in turn, 50% and 25% of the following:

7.	11,380	16,816	83,864	396.56	77.768
8.	46,588	43,748	97,144	567.68	47.376

WRITTEN EXERCISE

Multiply the following:

1. 10 × $0.25.
2. 10 × $8251.
3. 10 × $6.25.
4. 10 × $5.50.
5. 100 × $0.37.
6. 100 × $$2.87\frac{1}{2}$.
7. 1000 × $0.25.
8. 1000 × $$6.37\frac{1}{2}$.
9. 50 × $64,224.
10. 50 × $752.20.
11. 50 × $861.55.
12. 25 × $144.
13. 25 × $640.
14. 25 × $325.
15. 675 × $25.
16. 25 × $25.50.
17. 25 × $75.25.
18. $12\frac{1}{2}$ × $6400.
19. $12\frac{1}{2}$ × $4880.
20. 125 × $640.
21. 125 × $6400.
22. 125 × $2800.
23. $33\frac{1}{3}$ × 36.
24. $33\frac{1}{3}$ × 48.
25. $33\frac{1}{3}$ × 144.
26. $33\frac{1}{3}$ × $312.30.
27. $16\frac{2}{3}$ × 12,630.
28. $16\frac{2}{3}$ × $636.30.
29. 5 × 47,664.80.
30. 5 × $210,650.
31. 125 × $84,480.
32. 125 × $844.80.
33. $16\frac{2}{3}$ × 4,800,000.

34. How much will 25 books cost at $1.20 each?

35. How much will 25 wagons cost at $116 each?

36. How much will 50 cows cost at $54 a head?

37. How much will 100 books cost at $62\frac{1}{2}$¢ each?

38. How much will 25 tons of hay cost at $12.40 a ton?

39. How much will $12\frac{1}{2}$ yards of cloth cost at 36¢ a yard?

40. How much will 25 doz. pencils cost at $0.48 a dozen?

41. How much will 48 office desks cost at $25 each?

42. At $5 an acre, how much is a tract of 14,264 acres of mountain land worth?

43. How much will a man earn in 36 weeks at $25 a week? at $$33\frac{1}{3}$ a week?

44. How much will a dozen tables weigh at 25 lb. each? at 50 lb. each?

202. Short Methods in Division. There are certain divisors which are used so frequently in business that it is desirable to know the shortest methods of dividing by them. These include the numbers 5, 10, 25, 100, 125 (with 12.5 or $12\frac{1}{2}$), and $33\frac{1}{3}$.

ORAL EXERCISE

1. Divide by 10: \$420, \$5700, 330, \$425, 375 lb., 6 mi.

2. State the short method of dividing by 10.

3. Since $\frac{1}{5} = \frac{2}{10}$, we may take $\frac{1}{5}$ of a dollar by taking how many tenths of it? State the short method of dividing by 5, or of taking $\frac{1}{5}$ of a number.

4. Divide by 5; \$1110, \$3240, 5280 ft., 4230 mi., 3221.

5. Divide by 100: \$600, 7000, 4800, \$560, \$275, \$48.

6. State the short method of dividing by 100.

7. State the short method of finding $\frac{1}{25}$ (or $\frac{4}{100}$) of a dollar; of dividing a number by 25.

8. Divide by 25: \$2200, \$1200, \$3100, 4100, \$32, \$11.

9. State the short method of dividing by 125. ($\frac{1}{125} = \frac{8}{1000}$.)

10. Divide by 125: \$11,000, 2000, \$4000, \$70, \$600.

11. State the short method of dividing by $33\frac{1}{3}$. (Take $\frac{3}{100}$.)

12. Divide by $33\frac{1}{3}$: \$200, 500, \$7, \$12, \$30,000, \$11,000.

13. In a certain year Delaware had 346.2 mi. of railroad, and the District of Columbia had $\frac{1}{10}$ as many miles. How many miles had the latter?

14. In the same year France had 26,600 mi. of railroad, and Korea had 0.001 as many. How many had Korea?

15. In the same year the United States had 193,300 mi. of railroad, and Ecuador had 0.001 as many. How many had Ecuador?

203. Short Methods in Division. *To divide by 10, move the decimal point one place to the left, prefixing a zero if necessary.*

To divide by 100 or 1000, move the decimal point to the left, two or three places respectively, prefixing zeros if necessary.

To divide by 5, multiply by 2 and divide by 10.

To divide by 25, multiply by 4 and divide by 100.

To divide by 125, multiply by 8 and divide by 1000.

To divide by $12\frac{1}{2}$, multiply by 8 and divide by 100.

To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100.

To divide by $66\frac{2}{3}$, multiply by 3 and divide by 200.

To divide by $16\frac{2}{3}$, multiply by 6 and divide by 100.

WRITTEN EXERCISE

Divide by short methods, writing only the answers:

1. $7945 \div 5$.
2. $3875 \div 25$.
3. $7925 \div 12\frac{1}{2}$.
4. $9676 \div 33\frac{1}{3}$.
5. $3375 \div 125$.
6. $9486 \div 66\frac{2}{3}$.
7. $3739 \div 16\frac{2}{3}$.
8. $8347 \div 33\frac{1}{3}$.
9. $7969 \div 12\frac{1}{2}$.
10. $59{,}625 \div 125$.
11. $3978 \div 16\frac{2}{3}$.
12. $7168 \div 66\frac{2}{3}$.
13. $9375 \div 12\frac{1}{2}$.
14. $4873 \div 33\frac{1}{3}$.
15. $67{,}500 \div 125$.

16. Divide by 5: 125, $375, $67,295, $575,265.

17. Divide by 25: 650, $175, 98,750, $482,000.

18. Divide by 125: $250,125, $375,375, $276,525, $452,625.

19. At 25¢ a yard, how many yards of cloth can be bought for $16.25? for $42.25? for $63.75? for $17.50?

20. At $33\frac{1}{3}$¢ a dozen, how many dozen eggs can be bought for $17? for $56? for $92? for $125?

21. Divide 452,346,125 by 125 and also by 25, using long division, and see how many seconds it takes. Do the same by the short method.

ORAL EXERCISE

1. To multiply by $\frac{1}{2}$ is the same as to divide by what number? To divide by $\frac{1}{2}$ is the same as to multiply by what number? Illustrate each answer on the blackboard.

2. Multiply by $\frac{1}{2}$: 48, 250, 380, 460, 666, 1110, 1234.

3. Divide by $\frac{1}{2}$: 70, 35, 60, 300, 450, 700, 1450, 8112.

4. To multiply by $\frac{1}{3}$ is the same as to divide by what number? To divide by $\frac{1}{3}$ is the same as to multiply by what number? Illustrate each answer on the blackboard.

5. Multiply by $\frac{1}{3}$: 36, 90, 63, 45, 75, 300, 123, 321.

6. Divide by $\frac{1}{3}$: 30, 70, 50, 25, 80, 400, 2000, 4211.

7. Multiply by $\frac{2}{3}$: 30, 60, 90, 33, 63, 300, 600, 1230.

8. Divide by $\frac{2}{3}$: 20, 12, 16, 10, 60, 100, 200, 500, 1000.

9. Multiply by $\frac{1}{4}$: 8, 16, 32, 48, 64, 84, 100, 200, 1000.

10. Divide by $\frac{1}{4}$: 9, 11, 21, 25, 60, 70, 91, 125, 250, 800.

11. Multiply by $\frac{3}{4}$: 8, 16, 20, 32, 40, 44, 48, 60, 80, 444.

12. Divide by $\frac{3}{4}$: 9, 12, 15, 21, 30, 60, 66, 90, 120, 300.

13. Since $\frac{1}{5} = \frac{2}{10}$, what short method do you see of multiplying by $\frac{1}{5}$? of dividing by $\frac{1}{5}$?

14. Multiply by $\frac{1}{5}$: 21, 31, 34, 123, 112, 321, 132, 125.

15. Divide by $\frac{1}{5}$: 22, 300, 34, 160, 444, 664, 486, 1010.

16. Since $\frac{2}{5} = 0.4$, state the short methods of multiplying and dividing by $\frac{2}{5}$. Do the same for $\frac{3}{5}$ and $\frac{4}{5}$.

17. Multiply by $\frac{5}{6}$: 30, 60, 66, 120, 600, 720, 840, 1200.

18. Because $0.12\frac{1}{2} = \frac{1}{8}$, what is the short method of multiplying by $0.12\frac{1}{2}$?

19. At $12\frac{1}{2}$¢ a yard, what will 16 yd. of lawn cost? What will 72 yd. cost at the same rate?

204. Problem Solving. We have now studied the fundamental processes with integral numbers, common fractions, decimal fractions, and denominate numbers, the short methods of working with numbers, and the measurement of simple figures. The remaining part of arithmetic relates chiefly to business problems, and a few directions on the solution of problems are desirable before we proceed. In considering a problem there are three matters which are especially important, namely:

1. *Work rapidly and accurately.* The business man demands this first of all. Rapidity not only means money because of the time saved but it usually means accuracy, for when a man works rapidly he keeps his mind on his work. To insure accuracy, *always check your work.* One of the best checks is that of estimating the answer in advance. If the answer is not about what was expected, there is probably a mistake.

2. *Work as the business man does.* The business man does not write his work in steps, with signs like ÷ and +; he works in columns, because it is quicker and he can more easily check his work. He also makes his work look neat, because he finds that slovenly work leads to inaccuracy.

3. *Analyze the problems.* This means that we are to state the reasons. The business man does not write out his analyses, but he thinks them. We are asked to give analyses both orally and in writing, so that we may learn to think as accurately as the business man thinks.

It should be remembered that the greatest defect in arithmetic is seen in the inability of the pupil to add, subtract, multiply, and divide, these being the processes most frequently employed in business. The defect can be overcome only by constant drill and by attention to rapidity and accuracy. The solution of every type of problem that may be met in after life, and particularly the solution of the technical problems of the shop, the bank, and the broker's office, is not to be expected in the school, but accuracy in the fundamental operations is always possible and should be attained.

205. Oral Analysis. Many problems are so easy that they may be solved without pencil and paper. Merchants solve their problems in this way, and we should solve our own problems in this way whenever it is possible. In analyzing a problem, speak briefly and accurately. Use your own language; do not feel that you must use the exact language of the book or of the teacher.

1. If a dealer pays \$10 for 8 books of the same kind, what is the cost of each book?

No one would think of solving such a problem on paper. If asked to analyze (explain) it, we should say, "Since 8 books cost \$10, 1 book costs $\frac{1}{8}$ of \$10, or \$1.25."

2. If a stationer buys 100 sheets of blotting paper for \$3.50 and retails it at 5¢ a sheet, how much does he gain?

Since he sells 1 sheet for 5¢, he sells 100 sheets for 100 times 5¢, or \$5. (Do not stop to say "500¢"; reduce this to \$5 mentally.) He therefore gains the difference between \$3.50 and \$5, or \$1.50.

3. A dealer pays \$24 for 120 doz. pencils. How much does one pencil cost?

Since 120 doz. pencils cost \$24, 1 doz. costs $\frac{1}{120}$ of \$24, or 20¢. Therefore 1 pencil costs $\frac{1}{12}$ of 20¢, or $1\frac{2}{3}$¢.

4. If a dealer pays \$3.60 for a dozen knives and sells them at 50¢ each, how much does he gain? His gain is what part of the cost?

Since he sells 12 knives at 50¢ each, he receives 12 times 50¢, or \$6. He therefore gains \$6 – \$3.60 = \$2.40. Then \$2.40 is $\frac{2.40}{3.60} = \frac{24}{36} = \frac{2}{3}$ of the cost.

5. If you can buy baseballs at \$5.50 a dozen and are allowed the dozen rate for $\frac{1}{2}$ doz., how much do you save in buying 6 baseballs by the half dozen instead of buying at 55¢ each?

Since 6 baseballs cost 6 × 55¢, or \$3.30, if bought singly, and $\frac{1}{2}$ of \$5.50, or \$2.75, if bought by the half dozen, the saving is \$3.30 – \$2.75, or 55¢.

ORAL EXERCISE

Using short processes when possible, find the cost of the following:

1. 25 yd. of felt @ $1.60; @ $1.44; @ $1.20.
2. 12½ yd. of denim @ 16¢; @ 20¢; @ 24¢.
3. 7½ yd. of dimity @ 16¢; @ 20¢; @ 24¢.
4. 12½ lb. of feathers @ 48¢; @ 56¢; @ 64¢.
5. 32 yd. of cambric @ 10¢; @ 12½¢; @ 11¢.
6. 8 yd. of cashmere @ 75¢; @ 80¢; @ 90¢.
7. 16 yd. of gingham @ 10¢; @ 12½¢; @ 11¢; @ 15¢.
8. 22 yd. of buckram @ 15¢; @ 11¢; @ 20¢; @ 22¢.
9. 20 yd. of crinoline @ 11¢; @ 10¢; @ 12½¢; @ 15¢.
10. 11 yd. of percaline @ 20¢; @ 22¢; @ 25¢; @ 21¢.
11. 10 pieces of beading @ 23¢; @ 24¢; @ 26¢; @ 30¢.
12. 25 yd. of eider down @ 60¢; @ 64¢; @ 68¢.
13. 12 yd. of chambray @ 25¢; @ 20¢; @ 30¢.
14. 22 yd. of nainsook @ 20¢; @ 25¢; @ 30¢; @ 21¢.
15. 12 yd. of linen lawn @ 75¢; @ 70¢; @ 80¢.
16. 20 whalebones @ 20¢; @ 22¢; @ 24¢; @ 30¢.
17. 7 gross of bone buttons @ 22¢; @ 21¢; @ 25¢.
18. 10 yd. of jute burlap @ 35¢; @ 37¢; @ 37½¢.
19. 3 doz. spools cotton @ 40¢; @ 41¢; @ 42¢; @ 43¢.
20. 7 yd. of torchon lace @ 10¢; @ 11¢; @ 12¢.
21. 4 doz. spools of basting cotton @ 22¢; @ 23¢; @ 25¢.
22. 11 yd. of cotton scrim @ 22¢; @ 23¢; @ 24¢; @ 25¢.
23. 30 yd. of cheesecloth @ 5¢; @ 6¢; @ 7¢; @ 8¢.
24. 40 yd. of chiffon @ 75¢; @ 80¢; @ 95¢; @ $1.25.
25. 12 yd. of velvet @ $2.50; @ $3; @ $4; @ $5.

206. Problems in Profit and Loss. In business the merchant's chief problem relates to the profit on his goods.

For example, a merchant bought 800 yd. of linen lawn at 67½¢ a yard, and sold 726 yd. at 80¢ a yard, and the rest at a bargain sale at 65¢ a yard. Find his profit.

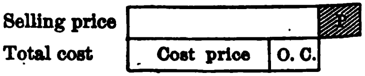
$0.67½	$726	$0.65	$580.80
800	.80	74	48.10
400	$580.80	2 60	$628.90
536		45 5	540.
$540		$48.10	$88.90

The written analysis is as follows:

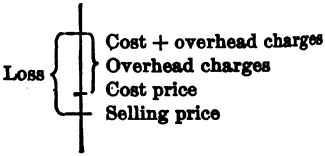
1. 800 × $0.67½ = $540, the cost.
2. 726 × $0.80 = $580.80, received at regular sale.
3. 800 yd. – 726 yd. = 74 yd., sold at bargain sale.
4. 74 × $0.65 = $48.10, received at bargain sale.
5. $580.80 + $48.10 = $628.90, total receipts.
6. $628.90 – $540 = $88.90, profit.

As already stated (§ 191), the merchant has certain "overhead charges," such as rent, help, insurance, freight, and interest on the money invested. These must form part of the total cost. It is not always necessary to mention these in the problems, but they are mentioned often enough to call attention to their importance.

Teachers sometimes find it advantageous to use diagrams to indicate the general conditions of a problem. For example, the accompanying diagram shows the cost price, overhead charges (O. C.), selling price, and profit (P.).



It is also possible to represent these numbers as we would represent temperatures on a thermometer. If we sell above the cost price plus overhead charges, we have a profit; if we sell below this point, we have a loss, as shown in the figure.



WRITTEN EXERCISE

1. A dealer buys 75 sewing machines at \$38 each. He pays \$56.25 freight on the machines and sells them at \$45 apiece. His overhead charges are \$1.20 per machine. Find his profit.

2. A merchant buys 840 yd. of muslin at 6¢ a yard and sells it at the rate of a dozen yards for a dollar. His overhead charges are taken as $\frac{3}{4}$¢ per yard. How much is his profit on the lot?

3. A merchant buys 12 gross of spools of silk at \$5.76 a gross. He sells the spools at 60¢ a dozen. Not counting overhead charges, how much is his profit?

4. A dealer buys 275 lb. of twine at 26¢ a pound. He sells it at 35¢ a pound. His overhead charges are reckoned as $\frac{3}{8}$¢ a pound. How much is his profit per pound? on the whole lot?

5. If a dealer buys 960 yd. of scrim for \$300 and sells three fourths of it at 35¢ a yard and the rest at 30¢ a yard, how much is his profit, not counting overhead charges?

6. A dealer buys 250 reams of paper at \$1.22½ a ream. He sells 7 reams, which were damaged, at 75¢ a ream, and the rest at \$1.35 a ream. Find his profit, not counting overhead charges?

7. A dealer buys 640 yd. of damask linen at 87½¢ a yard. At what price per yard must he sell it to make a profit of \$80 on the whole amount, not counting overhead charges?

8. If a dealer buys 1600 yd. of Victoria lawn for \$300 and the overhead charges are \$40, for how much per yard must he sell it to make a profit of 3$\frac{5}{8}$¢ a yard? Find his whole profit.

9. A dry goods dealer buys 720 yd. of percaline at 18¢ a yard, and the overhead charges are estimated as 1$\frac{5}{8}$¢ a yard. He sells 500 yd. at 25¢ a yard, 120 yd. at 23¢ a yard, and the rest at 15¢ a yard. How much is his profit?

WRITTEN REVIEW

1. A merchant bought 38 pieces of cloth of 36½ yd. each and paid $1.48 a yard for it. What was the total cost?

2. Which will cost the more, 56 cows at $42.50 each or 25 horses at $95 each? how much more?

3. If a cubic foot of ice weighs 58 pounds, what will be the weight of 26 cakes, each containing 3½ cu. ft.?

4. If a man earns $3.75 a day and works 6 days in the week, how much will he earn in 26 weeks?

5. The earth moves about the sun 101,090 ft. in a second. Find to the nearest mile the distance it moves in 15 min.

6. If a steam engine uses 3½ T. of coal a day, how many tons will it consume in February, 1928, if it runs every day?

7. A grocer sold 16 boxes of oranges, each box containing 196 oranges, at an average price of 4½¢ an orange. How much did he receive for all?

8. If electricity travels on a wire at the rate of 288,000 mi. per second, how long would a wire have to be for an electric current to travel from one end to the other in 2½ min.?

9. A man pays $202.95 for coal at $6.15 a ton. How many tons does he buy?

10. A shipper paid $92.16 freight on some goods at $0.64 per thousand pounds. How many tons were there?

11. The product of two numbers is 13.02042, and one of the numbers is 6.414. What is the other number?

12. Wheat that cost 96¢ a bushel is sold for $1.02 a bushel. How much must be sold to realize a profit of $99?

13. A contractor bought a shipment of bricks for $377.40, paying $7.40 a thousand. How many bricks did he buy?

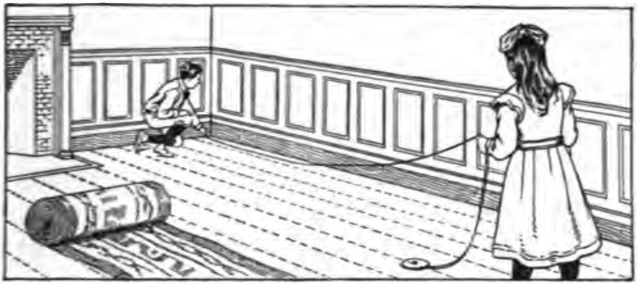
14. The product of two numbers is 468.75 and one of the numbers is 500. What is the other number?

15. How many times will a roller 6.8 ft. in circumference turn in rolling across a street that is 74.8 ft. wide?

16. If the wind in a hurricane blows at the rate of 8796.6 ft. per minute, what is the rate per hour? per second?

17. If a sewing machine makes 360 stitches of $\frac{1}{8}$ in. each in a minute, how many yards of goods can be pulled through in 4 min.?

18. If a sewing machine makes 424 stitches of $\frac{1}{32}$ in. each in a minute, how many yards of goods can be pulled through in 12 min.?



19. This boy and girl have measured the room. They find it to be 24 ft. long and 18 ft. wide. They know that the carpet which their mother bought has a pattern 1 yd. long, that the carpet is 27 in. wide, and that it costs 70¢ a yard, but can be bought at 10% discount. Their mother has $125 in the bank. Now make up a good practical problem and solve it.

20. Farmers have a simple rule for finding the number of bushels in a bin. The rule is to take $\frac{4}{5}$ of the product of the number of feet in the length, breadth, and depth of the bin. How many bushels of corn can be put in a bin $8\frac{1}{2}$ ft. long, $6\frac{1}{4}$ ft. wide, and 5 ft. deep?

21. An Ohio farmer found that he was raising on the average 65 bu. of corn per acre. He determined to see if he could not by proper selection of seed increase the yield. The result was an increase of 15%. The corn being worth 50¢ a bushel, what was the increase in the value of the product on 40 acres?

22. The farmer mentioned in Ex. 21 decided to try still more careful selection. In the first seven years he averaged 77 bu. per acre, and in the last seven he averaged 10% more. What was the latter yield per acre? What was it for the 40 acres? What was the gain in value at 50¢ a bushel?

23. Flies often lay their eggs in the filth of stables. It has been found that 1200 flies will hatch in a pound of this filth. If even $\frac{1}{10}$ of this rate were kept up, how many flies would hatch in a summer from 6 tons of this filth?

24. If a man earns $3.50 a day when he is well, but loses his wages and has to pay $1.30 a day for care and medicines when he is ill, how much more is his net weekly income when he is well and is working 6 da. in the week than when he is ill?

25. If the list prices of some goods are $100, $200, $350, and $50, what is the discount on the whole bill at 10%? at 5%?

26. A bicycle was marked $45, but was sold at 12% discount. What was the net price?

27. A piano was listed at $550, but was sold at 30% discount. What was the selling price?

28. If the drivewheel of a sewing machine has a diameter of 1 ft., and the circumference is $3\frac{1}{7}$ times the diameter, what is the circumference in inches?

29. Water in freezing expands 10% in volume. How many cubic feet of ice would 825 cu. ft. of water make?

207. Interpreting Problems. We have done much work in the interpretation of problems, that is, in finding how to solve them, in estimating the results in advance, in saving labor by determining exactly the thing that should be done, and in using short methods of operation. In the last two years of arithmetic you should give even more attention to this subject. A few examples will now be given to illustrate the study and interpretation of ordinary business problems.

1. A dealer bought some goods for $2500 and sold them for $3000. What is his gain per cent?

Exactly what must I find? The gain per cent on $2500.

How shall I find the sum gained? By taking $2500 from $3000.

Do I need pencil and paper for this work? No, for the gain is $500, and $500 is $\frac{1}{5}$ of $2500, and $\frac{1}{5}$ is 20%.

How shall I check my result? See if $\frac{1}{5}$ of $2500 is $500.

2. A dealer in horses bought a shipment of ponies for $2500. He sold them at a loss of 20%, and with the proceeds he bought some horses which he sold at a profit of 20%. Did he gain or lose by the whole transaction, and how much?

What must I find? First whether he gained or lost, and then how much.

Can I estimate the result in advance? Yes, for he lost 20% on $2500 and gained 20% on a smaller sum, so there was a loss. Furthermore, not only can I estimate the result, but I can easily work it out mentally.

The dealer lost $\frac{1}{5}$ of $2500, or $500; therefore he received $2000 for the ponies. He gained $\frac{1}{5}$ of $2000, or $400, on the horses. He lost $500 and gained $400, so he is $100 out of pocket on the whole deal.

3. A boy is offered a salary of $6.50 a week for the first year in a store, 20% more the second year, and an increase of 50% of this amount the third year. If he works 50 weeks in a year, what is his total salary for the first three years?

How shall I find the boy's income the first year? How much more was it the second year? How much was his salary for the third year?

4. A dealer sold two cars for $960 each. On the first he lost $\frac{1}{5}$ of what he paid, and on the second he gained $\frac{1}{5}$ of what he paid. Did he gain or lose on the whole transaction, and how much?

Problems of this nature are not likely to be met in actual business, but the solution of one such example is interesting. At first reading it looks as if the dealer came out even, but this cannot be true because he did not gain and lose $\frac{1}{5}$ of the same amount.

We are now ready for the work and for the full analysis of the problem.

Analysis

Since he lost $\frac{1}{5}$ of the cost of the first car, and $1 - \frac{1}{5} = \frac{4}{5}$, he sold this car for $\frac{4}{5}$ of the cost.

Therefore $\frac{4}{5}$ of the cost = $960, and $\frac{1}{5}$ of the cost = $960 ÷ 4 = $240.

Therefore the loss on the first car was $240.

Work

$$\begin{array}{ll} 4\overline{)\$960} & \$240 \\ \quad \$240 \text{ loss} & \underline{\quad 160} \\ 6\overline{)\$960} & \ \$80 \text{ net loss} \\ \quad \$160 \text{ gain} & \end{array}$$

Since he gained $\frac{1}{5}$ of the cost of the second car, and $1 + \frac{1}{5} = \frac{6}{5}$, he sold this car for $\frac{6}{5}$ of the cost.

Therefore $\frac{6}{5}$ of the cost = $960,

and $\frac{1}{5}$ of the cost = $960 ÷ 6 = $160.

Therefore the gain on the second car was $160.

Therefore the net loss was $240 – $160 = $80.

5. A dealer bought 32 horses for $4320. How much would 12 horses cost at the same rate?

Analysis

Since 32 horses cost $4320, 1 horse costs $4320 ÷ 32, and 12 horses cost 12 times this amount.

Indicating these operations and canceling as shown, we find that the cost of the 12 horses would be $1620.

Work

$$\frac{\overset{3}{\cancel{12}} \times \overset{540}{\cancel{\$4320}}}{\underset{\cancel{8}}{\cancel{32}}} = \$1620$$

WRITTEN EXERCISE

1. If a man works 312 da. a year at $3.75 a day, and saves $117, what per cent of his income does he spend?

2. A man receiving a salary of $24 a week worked 50 wk. last year. He paid $24 a month rent. How much money had he left for other expenses?

3. A laborer earns $2.25 a day for 6 da. in the week. If he works for 48 wk. and spends 22% of his wages for rent, how much money is left for other expenses?

4. A man works 6 da. in the week for 43 wk., and 5 da. in the week for 5 wk. He takes two rides on the trolley car each day that he works, at 5¢ a ride. How much does he pay in a year for trolley rides?

5. In casting brass hinges the brass shrinks, in cooling, $\frac{1}{64}$ of its length when melted. How long must be the mold for a hinge that is to be $3\frac{5}{16}$ in. long?

6. The wooden pattern from which an iron casting is made weighs $\frac{1}{16}$ as much as the iron. If the pattern weighs 7 lb. 3 oz., how much does the casting weigh?

7. An iron founder knows that an iron casting will weigh 16 times as much as the wooden pattern. If he has four patterns for the parts of a given piece that he wishes to cast, and these weigh 7 lb. 9 oz., 9 lb. 3 oz., 26 lb. 14 oz., and 13 lb. 8 oz., what will the casting weigh?

8. A manufacturer is building a chimney with a square flue for his boiler room. He has two horizontal boilers, each containing 96 tubes with a cross-section area of 3 sq. in. each. What must be the cross-section area of the chimney in order to equal the sum of all the cross-section areas of the tubes of the boilers?

9. Allowing 1000 shingles to 120 sq. ft., find how many thousand shingles will cover a pitched roof each side of which is 58 ft. long and 24 ft. wide.

10. A corner lot has 96 ft. front and is 180 ft. deep. Find the cost of laying a 4-foot stone walk on the front and side, at 18¢ per square foot.

11. Find in inches the depth of a rectangular tank, 9 ft. square at the base, that has the same capacity as a rectangular cistern 8 ft. square at the base and 6 ft. deep.

12. A man receives $3.60 for every 8 hr. of work, and works during 6 da. of a week the following number of hours: $7\frac{1}{2}$, 8, $7\frac{3}{4}$, 8, 8, $7\frac{1}{4}$. How much does he receive in all?

13. A certain kind of wheat produces 85% of its weight in flour. How many pounds of flour will be produced from 75 bu. of this wheat, allowing 60 lb. to the bushel?

14. If 15 burners, each consuming 6 cu. ft. of gas per hour, are used, on an average, 4 hr. per day for 365 days, what will be the gas bill at 80¢ per thousand?

15. Find to the nearest pound the weight of the air in a schoolroom $30' \times 16' \times 12'$, if water is 770 times as heavy as air and weighs 1000 oz. per cubic foot.

16. Bronze contains 9 parts of copper by weight to 2 parts of tin. How many pounds of tin in a bronze statue that weighs 2662 lb.?

17. A bell weighing 1250 lb. is composed of 77% copper, 21% tin, and the rest zinc. How many pounds of each metal does the bell contain?

18. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 150 lb. of flint.

19. If a certain gun metal is composed of 84% copper and the rest tin, how many pounds of tin will be needed in making 750 lb. of the gun metal?

20. A man, computing the cost of 54 articles, multiplied by mistake the cost of each by 45 instead of by 54, and obtained $121.50. What was the cost of the 54 articles?

21. Find the number of dry quarts (67.2 cu. in.) in a measure containing 1 cu. ft.

22. At what rate will $1000 yield $60 interest in 1 yr.? At what rate will $1000 yield $100 interest in 2 yr.? At what rate will $1000 yield $210 interest in 3 yr.?

23. What fraction of the year 1920 is the time from January 20 to March 15, including both of these days?

24. On seven successive days at noon the thermometer indicated respectively 66°, 72°, 61°, 68°, 73°, 62°, and 65°. What was the average for the week?

25. On seven successive days the barometer indicated respectively 29.22 in., 29.18 in., 29.65 in., 30.02 in., 30.12 in., 30.18 in., and 30.21 in. What was the average for the week?

26. A cubic inch of iron weighs $4\frac{4}{9}$ oz. Find the weight in pounds of an iron bar 1 in. square at the end and 1 yd. long; 1 in. square at the end and 15 ft. long.

27. Water is flowing at the rate of 2 cu. ft. in 3 min. into a rectangular tank, which is 6 ft. 8 in. long and 4 ft. 6 in. wide. How long will it take the water to fill the tank to a depth of 6 ft.?

Teachers will find it an excellent exercise to have a pupil read the problem silently, state what it means, state what is given, and then state what is required. When the problem allows, the pupil should be encouraged to tell in what situation such a problem might arise. Another pupil may then be called upon to plan the solution, and a third may be asked to solve it upon the blackboard. Short methods should always be commended.

208. Unitary Analysis. Examples like the following are easily solved by a particular kind of analysis called *unitary analysis.*

If 5 T. of coal cost \$27.50, what will 7 T. cost?

Since 5 T. cost \$27.50, 1 T. costs $\frac{1}{5}$ of \$27.50, or \$5.50.

Therefore 7 T. cost 7 times \$5.50, or \$38.50.

Notice that the problem was solved by finding the *unit,* the cost of *1 ton.* Hence the name "unitary analysis."

WRITTEN EXERCISE

1. If 3000 ft. of lumber cost \$96.60, what will 16,100 ft. cost at the same rate?

2. At \$460 per 100 bbl., how many barrels of flour can be bought for \$73.60? for \$782? for \$8556?

3. At \$5.60 a ton of 2000 lb., what will 2775 lb. of coal cost? 1575 lb.? 1850 lb.? 3225 lb.? 6450 lb.?

4. At \$12.50 per hundred pounds, what will $4\frac{1}{2}$ lb. of cheese cost? $7\frac{1}{4}$ lb.? 9 lb. 4 oz.? 12 lb. 12 oz.?

5. At \$5.20 a hundred pounds, how many pounds of meat can be bought for \$639.60? for \$587.60? for \$4004?

6. When the conditions of the air are such that sound travels 3696 ft. in 3.3 sec., how far does it travel in 7 sec.?

7. If 15 bales of cotton weigh 7305 lb., what will 4 such bales weigh? 7 bales? 16 bales? 25 bales?

8. If a tree 25 ft. high casts a shadow 20 ft. long, how long a shadow will a tree 30 ft. high cast at the same time?

9. If your schoolhouse casts a shadow 40 ft. long when a post 6 ft. high casts a shadow 8 ft. long, how high is the schoolhouse?

10. If a pole 6 ft. high casts a shadow 8.3 ft. long, how high is an adjacent tree that casts a shadow 74.7 ft. long at the same time?

209. Comparison of Numbers. A comparison of the numbers often leads to a short solution, as in the following examples:

1. If 3 tons of hay cost $36, how much will 6 tons cost?

Since $6 = 2 \times 3$, 6 tons will cost 2 × $36, or $72.

2. If 20 A. of land cost $1600, how much will 80 A. cost?

Since $80 = 4 \times 20$, 80 A. will cost 4 × $1600, or $6400.

ORAL EXERCISE

1. If 3 readers cost $1.50, how much will 9 readers cost?
2. If 4 horses cost $500, how much will 16 horses cost?
3. If 7 doz. eggs cost $2.10, how much will 21 doz. cost?
4. If 9 yd. of cloth cost $4.50, how much will 3 yd. cost?
5. If 15 bbl. of sugar cost $150, how much will 5 bbl. cost?
6. If you walk 2 mi. in 40 min., how long will it take you to walk 4 mi. at the same rate?

WRITTEN EXERCISE

1. If 4 M ft. (4000 board feet) of lumber cost $128, how much will 20 M ft. cost? How much will 60 M ft. cost?
2. If 7 bu. of apples cost $5.25, how much will 35 bu. cost?
3. If 5 sets of furniture cost $325, how much will 75 sets cost?
4. If a half-pound stick of solder is used to seal 30 cans of corn, how many pounds of solder are needed for 21,000 cans?
5. If $\frac{3}{8}$ of a farm is worth $2100, how much is $\frac{3}{4}$ of the farm worth at the same rate?
6. If $\frac{3}{10}$ of the value of a mill is $21,000, how much is $\frac{3}{5}$ of the mill worth? How much is $\frac{3}{20}$ of it worth?
7. If $2\frac{1}{2}$ bu. of apples cost $1.75, how much will 10 bu. cost?

210. Stating a Problem in One Step. It is often a good plan to indicate the solution of a problem in a single step, so as to shorten the work by canceling, as in the following example:

If 1 cu. ft. of water weighs $62\frac{1}{2}$ lb., and gold is 19.5 times as heavy as water, how much is 1 cu. ft. of gold worth at $18.60 an ounce avoirdupois?

1 cu. ft. of gold weighs $19.5 \times 62\frac{1}{2}$ lb., or $19.5 \times 62\frac{1}{2} \times 16$ oz.

Therefore the value of 1 cu. ft. of gold is

$$19\tfrac{1}{2} \times 62\tfrac{1}{2} \times 16 \times \$18.60.$$

But $19\frac{1}{2} = \frac{39}{2}$, and $62\frac{1}{2} = \frac{125}{2}$.

We therefore have $\dfrac{39 \times 125 \times 16 \times \$18.60}{2 \times 2}$.

We may now cancel and thus complete the solution more easily than by multiplying step by step.

WRITTEN EXERCISE

1. Finish the solution of the above problem.

2. When sound travels 1125 ft. per second, how many miles away is a lightning flash which is seen 13.2 sec. before the thunder is heard?

3. Soldiers marching quickstep take 120 paces a minute, averaging 28 in. each. At this rate, how many miles will a company of soldiers march in an hour?

4. If the circumference of a wheel is $3\frac{1}{7}$ times the diameter, and the drive wheel of a locomotive is 6 ft. in diameter, how many revolutions will it make in going from Albany to New York, 143 mi.?

5. What is the expense of covering a floor with plain matting 1 yd. wide, the room being 18 ft. by $21\frac{1}{2}$ ft., and the matting being worth 30¢ a yard, allowing 6 in. extra on each strip for turning under? Draw a plan of the floor.

6. If a piece of cast iron $10'' \times 8'' \times 2''$ weighs 41.6 lb., what is the weight of a piece $16'' \times 10'' \times 3''$?

7. If 475 horse power from several engines costs $13,490 a year, what will 60 horse power cost at the same rate?

8. A dealer bought 25 bu. of grass seed at $2.90 a bushel. He paid $1.20 for paper bags, and sold the seed in bags at 10¢ a quart. How much did he gain?

9. Two steamers started for Europe on the same route at the same time. After 4 days one had averaged 448.7 mi. a day, and the other 502.1 mi. a day. How far apart were they?

10. A box of arithmetics cost a dealer $53.94, including $1.48 freight. He sold the arithmetics for 50¢ each, thus gaining 7¢ on the list price of each book. How many did he buy?

11. A man bought two adjacent farms containing the same number of acres. For one he paid $95 per acre, and for the other $130 per acre. The two farms cost him $31,950. How many acres did he buy?

12. If it costs $702 to excavate a cellar $27' \times 39' \times 9'$, how much will it cost to excavate another cellar $30' \times 36' \times 12'$ at the same rate?

13. If 100 lb. of bituminous (soft) coal contain 76.05 lb. of carbon and 2.32 lb. of ash, how many pounds of each are there in 15 T.?

14. If 2000 lb. of anthracite (hard) coal contain 1828 lb. of carbon, how much carbon will 100 lb. contain? What is the per cent of carbon?

15. A gallon contains 231 cubic inches. Alcohol is 0.84 as heavy as water. Find the number of cubic inches of alcohol that will weigh as much as a gallon of water.

LITTLE EXAMINATIONS

I. **1.** \$275 is what per cent more than \$200?

2. \$200 is what per cent less than \$275?

3. If 17 cars cost \$13,600, how much will 9 cars cost?

4. How much is 72% of 350 lb.? of \$3500? of 35,000 lb.?

5. How much is 175% of \$2500? $\frac{1}{2}$% of \$2000?

II. **1.** 39,987 + 46,296.

2. 73,203 – 59,827.

3. $34\frac{1}{2}$ × \$42,346.

4. \$29.75 ÷ 25.

5. $\frac{7}{8}$ of 25 ft. 4 in.

6. CDXL = (?).

7. $32\frac{1}{2}$ ft. = (?) yd.

8. $18\frac{1}{2}$ gal. = (?) qt.

9. $4\frac{3}{4}$ bu. = (?) pk.

10. 66 ft. = (?) rd.

III. **1.** \$500 is what per cent of \$400?

2. \$500 is what per cent more than \$400? than \$250?

3. \$400 is what per cent less than \$500? than \$800?

4. If 40 mules cost \$5000, how much will 8 mules cost? How much will 15 mules cost?

5. At \$120 for $\frac{3}{4}$ A., how much will 1 A. of land cost? How much will 3 A. of land cost?

IV. **1.** $41\frac{2}{3}$ in. + $1\frac{1}{2}$ in.

2. $5\frac{3}{8}$ in. – $2\frac{3}{4}$ in.

3. $2\frac{3}{4}$ × $7\frac{1}{2}$ in.

4. $6\frac{7}{8}$ ft. ÷ $3\frac{7}{16}$ ft.

5. 3.75 ÷ $2\frac{1}{2}$.

6. 25% of \$1.60.

7. \$2.75 ÷ $1\frac{1}{4}$.

8. $\frac{2}{3}$ of $4\frac{1}{2}$ ft.

9. $\frac{7}{8}$ of 9 ft. 4 in.

10. 75% of 6 lb. 4 oz.

REVIEW AND DRILL

I. DRILL SECTION

Add the following:

1.	2.	3.	4.	5.
$27.56	$18.96	$41.63	$15.92	$17.26
32.75	26.45	18.99	17.86	18.32
41.07	33.87	17.46	33.40	15.88
9.89	91.82	33.81	29.69	27.40
10.07	5.50	40.00	8.72	82.60
12.50	17.	7.80	.50	82.74
16.75	8.96	16.15	3.00	81.68
8.92	14.73	27.75	27.60	84.12

6. In Ex. 1, add the first four numbers, then the second four, and then these two sums, thus checking the result of Ex. 1.

7. In Ex. 2, proceed as in Ex. 6, thus checking the result.

8. In Ex. 3, proceed as in Ex. 6, thus checking the result.

9. In Ex. 4, proceed as in Ex. 6, thus checking the result.

10. In Ex. 5, proceed as in Ex. 6, thus checking the result.

11. From the sum of $175.50 and $87.75 take the sum of $27.85 and $52.25.

12. Multiply $175.50 by 146, and divide the result by 73. What is the shortest way of obtaining the result?

13. Find the number of cubic inches in a box that is $8\frac{1}{4}$ in. long, $4\frac{3}{8}$ in. wide, and $3\frac{1}{2}$ in. deep. Use the shortest method.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. What is the best way of multiplying one mixed number by another?

2. If one writing pad costs a certain aliquot part of a dollar, what is the best way of finding the cost of a given number of these pads?

3. At a given price per yard, how do you find the number of yards of cloth, carpet, or linoleum that can be bought for a given amount of money?

4. If you know the price of pens per gross, how do you find the price per pen?

5. When a train is traveling a certain number of miles in a certain number of minutes, how do you find its rate per hour?

6. If you know what an automobile dealer pays for a car and the per cent above cost at which he sells it, how do you find the selling price?

7. If you know the length and width of this room in feet and inches, how do you find the area of the floor?

8. If you know the three dimensions of this room in feet and inches, how do you find the number of cubic feet in the room? Why should any one wish to find this result?

9. If you were asked by your mother to find how many yards of carpet were needed for your sitting room or stairs, how would you find it?

10. If you knew or were told the number of cubic inches in a bushel, how would you find, by first measuring the dimensions, the number of bushels of wheat that could be put into a bin?

III. INDUSTRIAL PROBLEMS

THE BOYS' WORKSHOP

1. The two cross sticks of this kite are 19 in. and 18 in. in length. They are joined at right angles in the middle of the shorter piece and 7 in. from the end of the longer one. Draw a plan of the kite to the scale $\frac{1}{4}$.

2. In your drawing in Ex. 1 measure the four sides of the kite and then compute their lengths in the real kite.

3. By dividing the kite mentioned in Ex. 1 into two triangles, compute its area.

4. Find the area of a rectangle if the dimensions are the lengths of the two cross sticks of the kite mentioned in Ex. 1.

5. The frame of another kite is made of five pieces of wood, the top one being 3 in. long, each of the other two short sides $8\frac{1}{4}$ in., and each of the two long sides $13\frac{1}{2}$ in. Find the perimeter.

The pupils should be encouraged to bring into class any problems relating to handwork of this kind in which they may be interested.

THE RAILWAY INDUSTRY

1. If railway ties are 8″ wide and are placed 18″ apart, how many ties are there to a mile?

2. Our railways use 90,000,000 new ties a year, averaging 8″ × 8′ 6″ × 6″. How many cubic feet of timber do they use?

3. The modern coal car usually has a capacity of 100,000 lb. At 35 cu. ft. to the ton of 2000 lb., what is the volume?

4. If a rail is 30′ long and weighs 120 lb. to the yard, and if a man can carry 160 lb., how many men will it take to carry a rail?

5. The standard gauge of railways in America and England is 4′ $8\frac{1}{2}$″. Express this in meters, as used in most European countries, the meter being 39.37 in.

6. The standard American and English gauge (see Ex. 5) is $\frac{1}{5}$ more than the gauge of a certain mountain road. What is the gauge of the latter?

7. If the cost of maintaining the single track of a certain railway averages $846 a mile, what is the cost of maintaining its 450 mi. of double track?

MAKING A LIVING

1. Mrs. Wiggs receives $1.25 a day for doing washing. How much does she make in a week of six working days if she spends 5¢ a day for car fare? How much does she make in 51 wk.?

2. Mr. Wiggs left her some life insurance which she invested so as to receive $45 a year. How much is this a month? If her month's rent is $9, what part of the rent will this pay?

3. John helps his mother by working before and after school. He receives $1.50 a week for delivering morning papers, and on Sunday mornings he earns on an average 85¢ more by selling extra papers. He averages 35¢ every evening by selling papers. If he also makes $1.10 a week by selling the *Saturday Evening Post*, what is his total weekly contribution to the family income?

4. A neighbor offers John's sister Jennie 10¢ an hour for wheeling the baby. If she works on the average 1½ hr. a day, missing only ten days, how much will she earn in a year?

5. Their brother Frank borrows $1.50 from John to pay for plowing and seed and $1.50 from Jennie to pay for a spade and hoe. If he raises $4.25 worth of cabbages, $3.25 worth of tomatoes, $6.15 worth of potatoes, and $7.75 worth of other vegetables, and then pays back the money he borrowed, how much is he able during the year to give his mother toward the family expenses?

6. They all think they can do better by keeping chickens. They buy six hens and a rooster for $5. Frank makes a coop out of a box that costs 15¢, and he spends 35¢ for nails, hinges, screws, and some nest eggs. What is the total investment?

7. Frank feeds each week ½ pk. of oats costing 40¢ a bushel, 7½ lb. of bran costing $1.30 per 100 lb., and the table scraps. If the hens lay on an average 15 eggs a week, and Frank sells the eggs at 36¢ a dozen, how much does he make in a week?

HOW TOM EARNED HIS SPENDING-MONEY

1. Tom's father offers him 35¢ a week for spending-money or a dozen hens with which to earn what he can. How much would 35¢ a week amount to in a year? If the hens lay 100 eggs apiece in a year and eggs average 28¢ a dozen, how much more would Tom receive by keeping hens?

2. Tom figured this out, but forgot to deduct the expense of feeding his hens. Suppose this to be $1 per year for each hen, and suppose that the hens averaged 125 eggs apiece, how much better would Tom's annual income be than 35¢ a week?

3. Tom chooses the hens. He finds that there are more table scraps than he had counted on, so that he needs each week only 1 pk. of corn costing 60¢ a bushel, $\frac{1}{2}$ pk. of oats costing 36¢ a bushel, and 5 lb. of bran costing $1.40 per 100 lb. What is the expense of the feed per year?

4. During the year the hens averaged $2\frac{1}{4}$ doz. eggs a week, and these he sold at an average price of 25¢ a dozen. What is his income per year? Deducting his expenses, does this leave him more than or less than 35¢ a week, and how much for the year?

5. Tom learns that by selling eggs for hatching purposes in the late winter he can get 75¢ for a setting of 15 eggs. If he sells 20 doz. at this price, how much does it add to his yearly income? What does this make his average income per week?

6. Dick keeps bees, getting 33 lb. of honey a year from each of four hives, and selling the honey for 18¢ a pound. Is he doing better or worse than Tom, and how much per week?

7. Tom decides to keep bees also. If he buys two hives of bees for $4.25 a hive, and needs in addition $1.80 worth of supplies, how many weeks will he have to save his income, as found in Ex. 5, in order to pay for the bees and supplies?

CHAPTER VIII

PRACTICAL MEASUREMENTS

211. Tables. Before beginning the study of practical measurements, we shall recall the principal tables which were given in Chapter III and which we shall use. They are as follows:

LENGTH

12 inches (in. or ″) = 1 foot (ft. or ′)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Surveyors use a *chain* 66 ft. long, composed of 100 *links* each 7.92 in. long.

AREA

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

We occasionally write sq.′ for square feet and sq.″ for square inches.

VOLUME

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

A cubic yard of earth is considered a load. A perch of stone or masonry is usually considered as $24\frac{3}{4}$ cu. ft., but it varies in different parts of the country.

212. Areas. You have already learned how to find the areas of several kinds of figures, so it is necessary at this time only to repeat the rules which we studied in the Intermediate Book.

The area of a rectangle equals the product of the base and height.

In all such cases abstract numbers are understood.

For example, if the base is 3 ft. and the height 2 ft., the area is found thus:

$$2 \times 3 \text{ sq. ft.} = 6 \text{ sq. ft.}$$

The area of a parallelogram equals the product of the base and the height.

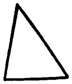
For example, if the base is 2.4 in. and the height 2 in., the area is 4.8 sq. in. That is,

$$2 \times 2.4 \text{ sq. in.} = 4.8 \text{ sq. in.}$$

The area of a triangle equals half the product of the base and the height.

For example, if the base is $3\frac{1}{2}$ in. and the height $2\frac{1}{2}$ in., the area is found thus:

$$\tfrac{1}{2} \text{ of } 2\tfrac{1}{2} \times 3\tfrac{1}{2} \text{ sq. in.} = 4\tfrac{3}{8} \text{ sq. in.}$$

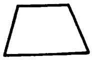


The actual work is as follows:

$$\frac{5 \times 7}{2 \times 2 \times 2} = \frac{35}{8} = 4\tfrac{3}{8}.$$

The area of a trapezoid equals half the product of the height and the sum of the parallel sides.

This brief form of stating the rule will now be understood. The longer form, used by beginners, says that *the area of a trapezoid equals half that of a rectangle with the same altitude and with a base equal to the sum of the two parallel sides.*



213. Volumes. We have learned that *the volume of a rectangular solid equals the product of its three dimensions.*

For example, if a box is 3 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. deep, the volume is found as follows:

$$3 \times 2 \times 1\tfrac{1}{2} \text{ cu. ft.} = 9 \text{ cu. ft.}$$

WRITTEN EXERCISE

1. How much will it cost, at $1.40 a yard, to cover the floor of a room 20 ft. 3 in. by 28 ft. 6 in. with 27-inch carpet, allowing 2 yd. for matching the pattern?

Similar practical problems should be secured by the pupils at home.

2. How many sheets for single beds can be made from a piece of sheeting $\frac{6}{4}$ wide and 72 yd. long, each sheet being $2\frac{1}{2}$ yd. long with a top hem $3\frac{1}{4}$ in. and a bottom hem $2\frac{1}{4}$ in., an allowance of $\frac{1}{4}$ in. for turning under being made for each hem?

In buying sheeting the width of the material is given in quarters of a yard. Sheeting $\frac{6}{4}$ wide means 6 quarters of a yard wide. Sheeting $\frac{6}{4}$ wide is used on single beds, $\frac{8}{4}$ wide on three-quarter beds, and $\frac{10}{4}$ wide on double beds.

3. A piece of sheeting of $\frac{8}{4}$ width contains 64 yd. How many sheets for three-quarter beds can be made from it, allowing the same measurements as in Ex. 2?

4. A piece of sheeting of $\frac{10}{4}$ width contains 72 yd. How many sheets for double beds can be made from it, allowing the same measurements as in Ex. 2?

The class should study the simple measurements needed in the household, including the measurement of materials used in food and clothing. The cost of furnishing a kitchen, laundry, dining room, living room, and bedroom, with the expense of renovating them, should also be considered.

Where electricity or gas is used, the meter should be read by the class, and the cost of lighting computed. The relative cost of different systems of heating and lighting in the school and home should be considered.

5. The ice chamber in an ice box is 16 in. by 24 in. by 18 in. Ice weighs 8% less than water, and 1 cu. ft. of water weighs $62\frac{1}{2}$ lb. If the ice comes in rectangular pieces weighing 10, 25, 50, 100, 150, and 200 lb., which one or which two of these pieces would come nearest to filling the chamber?

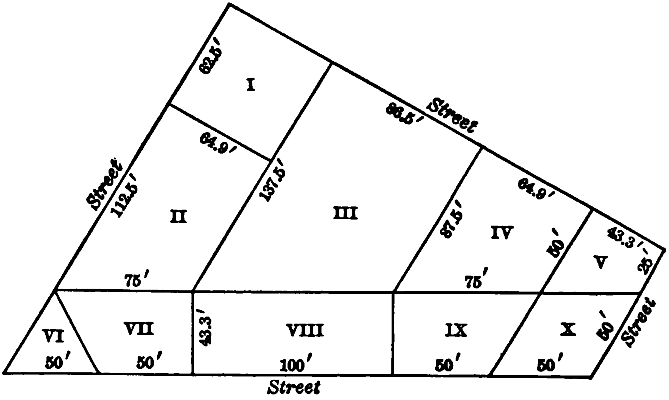
6. The bottom of an ordinary farm-wagon box is 3 ft. by 10 ft., and the box is from 24 in. to 26 in. deep. Find the contents in cubic feet and in cubic inches for each of these depths.

7. Such a wagon box 24 in. deep is estimated to contain 50 bu. of shelled corn. How many bushels of shelled corn will such a box 26 in. deep contain?

8. Certain farmers estimate that $2\frac{2}{5}$ cu. ft. of corn in the ear will produce 1 bu. of shelled corn. How many bushels of shelled corn in a crib 12 ft. by 20 ft. by 8 ft., filled with corn in the ear?

9. If a dirt car is 27 ft. long, 6 ft. wide, and $3\frac{1}{4}$ ft. deep, how many cubic yards will it carry?

10. How many cubic yards of earth must be removed in digging a tunnel 492 ft. long, 39 ft. wide, and 19 ft. 6 in. high?

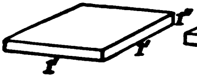


An irregular city block is divided into lots with measurements correct to one decimal place, as shown above; find the areas in square feet of the lots numbered as follows:

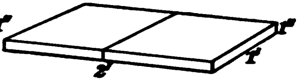
11. I. **13.** II. **15.** III. **17.** IV. **19.** V.

12. VI. **14.** VII. **16.** VIII. **18.** IX. **20.** X.

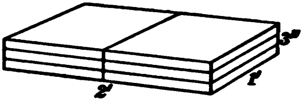
214. Board Measure. One *board foot* of lumber means a piece that is 1 sq. ft. on one surface and is 1 in. or less in thickness. In speaking of lumber, a board foot is often called simply one foot.



1 BOARD FOOT



2 BOARD FEET



6 BOARD FEET

For example, a board 2′ long, 1′ wide, and 1″ (or less) thick contains 2 board feet (usually written as 2 bd. ft.).

A plank 12′ long, 1′ wide, and 3″ thick contains 36 bd. ft.

A board 18′ long, 8″ wide, and 1″ (or less) thick contains $18 \times \frac{8}{12}$ bd. ft., or 12 bd. ft. A joist $12' \times 6'' \times 2\frac{1}{2}''$ contains $2\frac{1}{2} \times 12 \times \frac{1}{2}$ bd. ft., or 15 bd. ft.

A joist $12' \times 6'' \times 2\frac{1}{2}''$ means and is read "a joist 12 ft. by 6 in. by $2\frac{1}{2}$ in." In such cases always use cancellation; thus, since $2\frac{1}{2} = \frac{5}{2}$, we have

$$2\tfrac{1}{2} \times 12 \times \frac{1}{2} = \frac{5 \times \overset{3}{\cancel{12}}}{\cancel{2} \times \cancel{2}} = 15.$$

WRITTEN EXERCISE

Find the number of feet, board measure, in the following:

1. 32 planks 3 in. thick, 16 ft. long, and 8 in. wide.
2. 4 sticks of timber 16 ft. long, 1 ft. wide, and 8 in. thick.
3. 36 joists 3 in. by 4 in. and 16 ft. long.
4. 12 beams 10 in. by 12 in. and 14 ft. long.
5. 60 boards $\frac{7}{8}$ in. thick, 8 in. wide, and 16 ft. long.

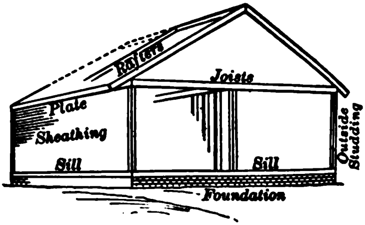
In such measurements, a board $\frac{7}{8}$ in. thick is considered to be 1 in. thick.

6. What is the cost of 30 spruce joists 2 in. by 6 in. and 14 ft. long, at $34 per M (that is, per 1000 bd. ft.)?

215. House Building. The house shown in the picture below is to be 27½′ × 34′. A builder estimates the cost of materials and labor. Part of the computation of cost and amount of materials is given in the following examples; the rest is left for the class. The specifications are taken from a recent government report.

Call a fraction of a foot a whole foot in any result.

The pupils should know, from the figure at the right, the meaning of the terms used.



WRITTEN EXERCISE

1. For sills the builder needs the following:

3 pieces, 6″ × 8″, 16′ long, 192 bd. ft.
4 pieces, 6″ × 8″, 14′ long, (?) bd. ft.
6 pieces, 6″ × 8″, 12′ long, (?) bd. ft.
1 piece, 2″ × 8″, 14′ long, 19 bd. ft.
1 piece, 2″ × 10″, 8′ long, (?) bd. ft.

2. The builder needs the following joists:

30 pieces, 2″ × 10″, 16′ long, (?) bd. ft.
18 pieces, 2″ × 8″, 12′ long, (?) bd. ft.
13 pieces, 2″ × 8″, 6′ long, (?) bd. ft.
27 pieces, 2″ × 8″, 16′ long, (?) bd. ft.
27 pieces, 2″ × 8″, 12′ long, (?) bd. ft.
27 pieces, 2″ × 6″, 16′ long, (?) bd. ft.

3. The builder also needs the following studding:

5 pieces, 4″ × 6″, 14′ long, (?) bd. ft.
100 pieces, 2″ × 4″, 14′ long, 934 bd. ft.
35 pieces, 2″ × 4″, 10′ long, 234 bd. ft.
25 pieces, 2″ × 4″, 18′ long, (?) bd. ft.
45 pieces, 2″ × 6″, 12′ long, (?) bd. ft.
50 pieces, 2″ × 4″, 8′ long, 267 bd. ft.
50 pieces, 2″ × 4″, 14′ long, 467 bd. ft.
40 pieces, 2″ × 6″, 18′ long, (?) bd. ft.

4. For the roof the builder needs 1050 bd. ft., and this, with the amounts in Exs. 1–3, should make a total of 8021 bd. ft.

5. How much will the lumber mentioned in Exs. 1–4 cost at $60 per M?

6. The builder also needs 2000 bd. ft. of sheathing at $65 per M; 1950 bd. ft. of flooring at $45; 125 bd. ft. of flooring for the porch at $65; 1500 shingles for the gables at $5.25 per 1000; 6500 shingles for the roof at $7. What will this material cost?

7. He also needs 28 foundation posts at 40¢ each; water tables, cornice, corner boards, etc., $35; porch columns, $7; 16 doors at an average cost of $5.75 each; 16 windows at an average cost of $5.50 each; hardware, $50; chimney and plastering, $200; painting, $148; tin work, $25.50; the work of 4 carpenters for 30 days at $4 a day each; excavating, $130; grading, $25. What is the total cost of these items?

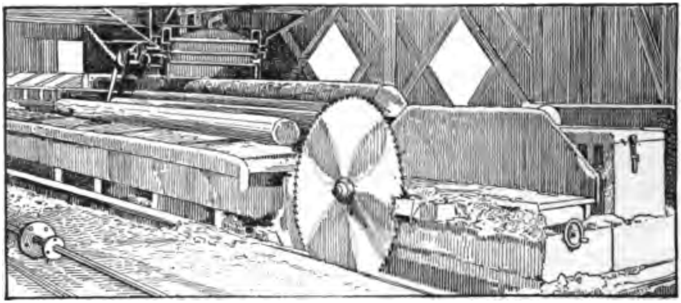
8. How much will the flooring for a house cost, taking the measurement of each floor as 28′ × 32′, and allowing for two floors, the lumber costing $40 per M?

One good way of treating the general topic of practical measurements is to work out problems in connection with building and equipping a house or the school building. If a building is being erected in the neighborhood it should be used for this work in order that the problems may be real.

216. Measuring Lumber in the Log. We could find by careful measurement almost the exact number of cubic inches in a log, but for practical purposes a rule known as Doyle's Rule gives the number of board feet with sufficient accuracy.

From the number of inches in the smallest diameter of the log, take 4, multiply the remainder by half itself, multiply this result by the number of feet in the length of the log, and divide by 8.

The rule need not be learned, but it may be used in solving the problems.



For example, find the number of feet of lumber in a log 16 ft. long, with its smallest diameter 18 in. Since 18 – 4 = 14, and ½ of 14 is 7, the work is as here shown. The result is 196 ft.

$$\boxed{\frac{14 \times 7 \times \overset{2}{\cancel{16}}}{\cancel{8}} = 196}$$

Here, as in all similar cases, cancellation should be employed.

WRITTEN EXERCISE

Determine the number of feet, board measure, of lumber in logs of the following lengths and smallest diameters:

1. 24′, 12″.
2. 20′, 14″.
3. 26′, 16″.
4. 18′, 18″.
5. 16′, 24″.
6. 20′, 22″.
7. 28′, 30″.
8. 26′, 18″.
9. 22′, 20″.

217. Plastering. The unit for estimating the cost of plastering is usually the square yard. The rule varies greatly in different parts of the country. Teachers should explain to the pupils the method in common use in their vicinity.

ORAL EXERCISE

1. If a room is 9 ft. high, what is the area of the four walls, not allowing for openings?

2. What is the area of the ceiling of a room 18 ft. by 20 ft.? How many square yards in the ceiling?

3. What will it cost to plaster 60 sq. yd. at 30¢ a square yard? at 33¢ a square yard? at 35¢ a square yard?

4. How many square yards of plastering are needed for the two sides and ceiling of a corridor 30 ft. long, 4 ft. wide, and 9 ft. high, not allowing for any openings?

WRITTEN EXERCISE

1. How many square yards of plastering are there in the walls and ceiling of a room 16 ft. by 20 ft., and 9 ft. high, allowing 14 sq. yd. for openings and baseboard?

2. How many square yards of plastering are there in the walls and ceiling of a hall 60 ft. long, 40 ft. wide, and 28 ft. high, allowing 36 sq. yd. for openings and baseboard?

3. How much will it cost to plaster a schoolroom 32 ft. long, 18 ft. wide, and 12 ft. high, at 32¢ a square yard, making an allowance of 30% for blackboards, baseboard, and openings?

4. On account of danger from falling plaster, it is desired to use pressed-steel ceilings in a schoolhouse. There are 742 sq. yd. of ceiling, and a contract is made for putting up steel ceilings at 10¢ a square foot. How much will the ceilings cost?

218. Carpeting. Carpet is usually either 27 in. or 1 yd. wide, and in estimating the amount needed for a room, allowance should in general be made for matching the pattern.

In allowing for waste in matching the pattern, no allowance need be made for loss on the first strip, since the other strips are cut to match that one. In these examples the strips are to run lengthwise of the room.

ORAL EXERCISE

1. How many strips of $\frac{3}{4}$-yard carpet are needed for a room 18 ft. wide, not allowing for matching the pattern?

2. A carpet has an 18-inch pattern. Will there be any loss in matching if the strips are 15 ft. long? if they are 14 ft. long?

WRITTEN EXERCISE

1. How many yards of carpet, $\frac{1}{2}$-yard pattern, are needed for a room 13 ft. 6 in. wide and 16 ft. long, the carpet being 27 in. wide? How much is wasted on each strip? Draw a plan.

Find how many yards of carpet are needed for the following, allowing for matching each strip, except the first, as indicated:

2. 18 ft. by 20 ft., 27-inch carpet, allowing 9 in.
3. 16 ft. by 18 ft., 27-inch carpet, allowing $\frac{1}{2}$ yd.
4. 14 ft. by 15 ft., 36-inch carpet, allowing $\frac{1}{4}$ yd.

Find the cost, at $1.20 a yard, of carpeting these rooms with 27-inch carpet, allowing 4 inches on each strip, except the first:

5. 15 ft. by 18 ft.
6. 14 ft. 3 in. by 18 ft.
7. 12 ft. by 13 ft. 6 in.
8. 17 ft. 6 in. by 18 ft.
9. $13\frac{1}{2}$ ft. by 16 ft. 3 in.
10. 4 ft. 6 in. by 23 ft. 8 in.
11. 15 ft. 9 in. by 25 ft. 6 in.
12. 22 ft. 3 in. by 28 ft. 6 in.

219. Papering. The standard width for estimating wall paper is 18 in., although it varies greatly. A single roll is considered to contain 8 yd. of paper. In estimating, paper hangers usually count 3 rolls to 100 sq. ft. Parts of rolls cannot be bought.

ORAL EXERCISE

1. What will the border for a room 16 ft. by 20 ft. cost, at 25¢ a yard?

2. Estimate the cost of the paper for a ceiling 15 ft. by 20 ft., at 40¢ a roll.

3. A room has nearly 500 sq. ft. of wall to be papered. Estimate the cost of the paper at 50¢ a roll.

4. A plain wall 14 ft. long and 9 ft. high above the baseboard is to be papered. How many rolls should you allow?

WRITTEN EXERCISE

1. The walls of a room are 8 ft. 6 in. above the baseboard, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, what will the paper cost at 40¢ a roll?

2. How many rolls of plain paper will it take to cover the four sides of a room 12 ft. high, 25 ft. long, and 20 ft. wide, the paper being 24 in. wide, allowing $1\frac{1}{2}$ rolls for windows and doors? At 35¢ a roll, what will it cost?

3. The walls of a room are to be papered with plain paper worth 30¢ a roll. They are 9 ft. high above the baseboard, and the total length of the four walls, allowing for windows and doors, is 60 ft. At the usual estimate, what will the paper cost?

The cost of papering rooms at home should be considered.

WRITTEN EXERCISE

1. The house described on page 46 was built on a lot 100 ft. wide and 200 ft. deep. The owner took a piece 40 ft. by 80 ft. for a vegetable garden, and one 60 ft. by 80 ft. for a fruit orchard. How many square feet were there in each of these two pieces?

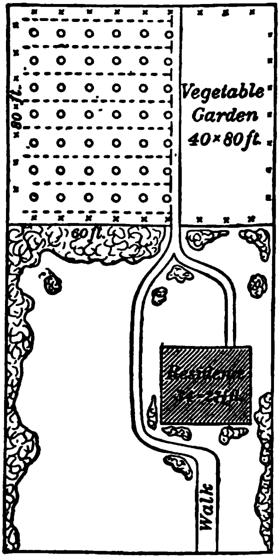
2. Express the area of the lot and of all three parts as fractions of an acre.

3. What was the area occupied by the house?

4. The excavation for the cellar was 5 ft. deep. How much earth was removed?

5. If the excavation for the cellar had been 6 ft. 4 in. deep, how much earth would have been removed?

6. If the owner fences the garden, the orchard, and the part on which his residence stands, how many feet of fencing will he need, not allowing for gates? how many rods of fencing? how many yards of fencing?

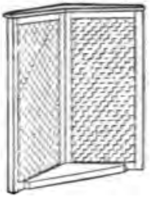


7. The owner planted a bed of asparagus 4 ft. wide and 36 ft. long; two beds of parsnips and two of beets, each $1\frac{1}{2}$ ft. wide and 36 ft. long; two beds of tomatoes and two of squash, each 3 ft. wide and 36 ft. long. How many square feet of area did he give to each of the beds?

8. He planted shrubs about the lawn. These, with the walks, occupied 2200 sq. ft. How many square feet were left for the lawn? See if you can improve this design.

DRAWING PLANS

1. A class in school made this lamp screen. It is $18\frac{3}{4}''$ high, and each wing is $9\frac{3}{4}''$ wide. Draw a plan of either wing of the screen to the scale $\frac{1}{2}$.



2. What is the total area of the side of the screen shown in the figure?

3. The woodwork is filled in with plaited ribbon, the spaces being $16\frac{1}{4}''$ by $7\frac{3}{4}''$. Draw a plan to the scale $\frac{1}{2}$, and find the sum of the two areas.

4. Jack made a swing for his sister's doll house. The crosspiece at the top is $7''$ long, $1''$ wide, and $\frac{1}{2}''$ thick. Make a drawing of the top of this crosspiece, first full size, and then to the scale $\frac{1}{2}$.

It may be necessary to remind the class again that drawing to the scale $\frac{1}{2}$ means that every inch is to be represented in the drawing by $\frac{1}{2}''$. All drawings should be made with a ruler and a sharp pencil and should look neat and clean.

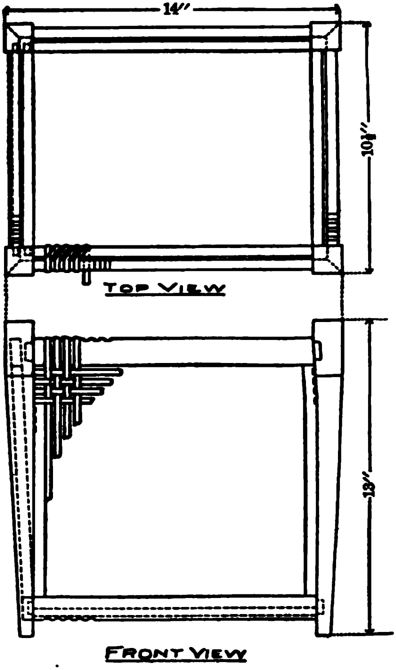
5. The seat in the swing is $3''$ long, $2''$ wide, and $\frac{1}{2}''$ thick. Make three full-size drawings, one showing the seat as seen from the top, one showing it as seen from the front, and one showing it as seen from the end.

6. Measure the top of your desk to the nearest inch. Make a drawing of the top, as seen from above, using the scale $\frac{1}{8}$.

7. Measure this page of the book. Make a drawing of the page, disregarding the print, using the scale $\frac{1}{4}$.

8. A room is $24'$ long and $16'$ wide. Draw a plan of the floor to the scale $1''$ to $8'$.

Pupils may be required to measure their living rooms at home and to draw a plan of the floor to some convenient scale.

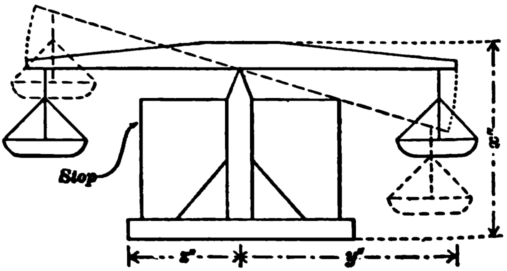


9. The boys in the class made a wastebasket, these being the working drawings for the top and the front. Measure the drawings and determine the scale that was used.

10. Taking the length and width as given, find the perimeter at the top, expressing it in feet and inches.

11. The height of the basket is how many inches more than the width? By what fraction of the width is the height greater than the width?

12. Measure the area to be filled in by wickerwork in the front of the basket. This is easily done because the scale was determined in Ex. 9, so you have only to measure the figure carefully, find the width and height, and then find the area.



13. The boys also made some wooden balances. These are the drawings, and they are made to the scale $\frac{1}{8}$. Determine the lengths marked x, y, and z.

Using a ruler and a sharp pencil, make drawings of the following to any convenient scale:

14. The front of your schoolroom, showing the blackboard and any doors or windows.

This will require measuring the schoolroom. In general, it is better to have the pupils take their own measurements, but if necessary the teacher may give the measurements to the class.

15. The floor plan of your schoolroom, showing the position and size of the platform, if there is one.

16. The school grounds, showing the location and size of the schoolhouse.

The measurements necessary for this drawing may take a whole class period or longer, but the time is profitably spent in this way. The pupils should first make a rough drawing of the plan, and should write upon it the necessary measures as they are taken, these measurements having been decided upon before the class goes outdoors.

17. Make a drawing, not to scale, of the front elevation of your schoolhouse, showing the position of the doors and windows.

The teacher should explain that the front elevation is the picture of the front of the building, not in perspective and without shading; that the side elevation is a similar picture of the side of the building; and that the plan means the ground plan. Such terms need not be strictly defined, but they should be used intelligently.

18. Draw a plan of the house in which you live and bring it to school to-morrow. Show both the first, or ground, floor, and the floor above it (the second floor).

19. Draw the side elevation of the schoolhouse.

20. Draw a plan of the first floor of the schoolhouse.

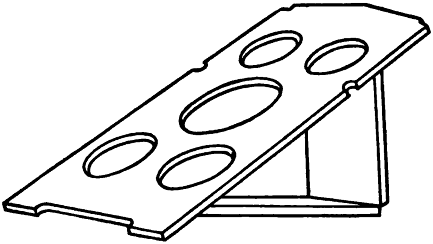
21. Let the boys draw a plan of a baseball field, showing the diamond, pitcher's box, and the position of the players.

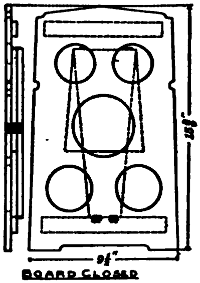
22. Let the boys draw a plan of a football field, showing the location of the goal posts, the two 25-yard lines, and the places for the kick-offs.

HANDWORK

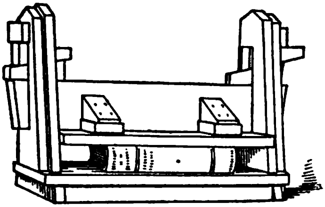
1. The boys made some bookracks. For the shelf they used a piece of wood 1′ 5″ long. How long a piece would they need for 7 shelves?

2. The boys of the class made this bean-bag board for the girls to use at recess. It was 1′ 3¾″ long. How long a piece of lumber would be needed to make the front of 2 such boards?





3. In making this bookpress the class found that 5 pieces of board 14″ long were needed. How many feet long was the board needed to make these pieces?



4. One piece 18⅜″ long and one 20¾″ long were needed for the bookpress. Express the sum of these lengths in feet and inches.

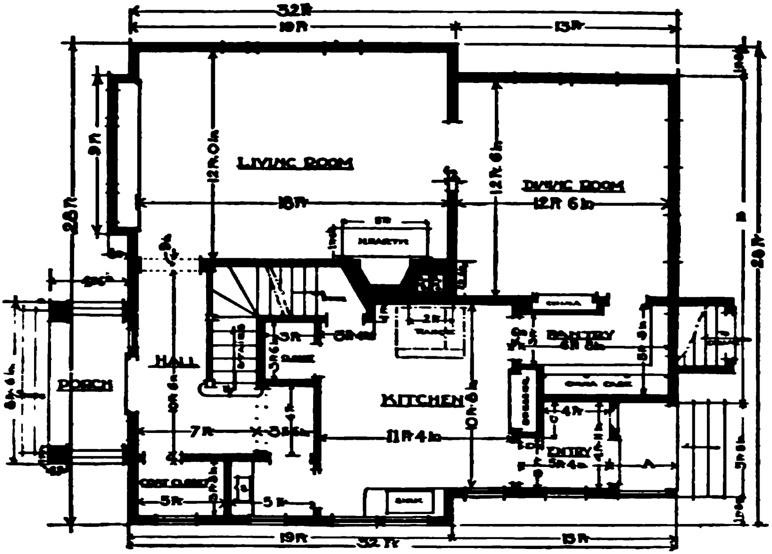
5. Each piece was 10¾″ wide. This is what part of a foot? Express the result both as a common fraction and as a decimal.

6. Using 10-inch board, ⅞″ thick, for the bookpress, what is the cost of 5 pieces 14″ long, 1 piece 18″ long, and one piece 20″ long, at $85 per M for best oak?

HOUSE PLANS

1. In this floor plan of a house, how many board feet of lumber, 1″ thick, are needed for the dining-room floor, allowing ⅕ extra for waste and joining?

2. In the same plan, how many square feet of floor space are there in the living room, not allowing for fireplace and hearth?



3. In the same plan, how many square feet of plaster must be allowed for the ceilings of the pantry and of the dining room?

4 In the same plan, how many cubic feet are there in the coat closet, the ceiling being 10 ft. above the floor?

5. In the same plan, how many square inches of tiling are needed for the hearth in the living room?

BUILDING A BARN

1. A farmer employs a carpenter to build a barn. The carpenter figures on the following as part of the materials: 4 pieces 2″ × 10″ × 18′; 8 pieces 2″ × 10″ × 10′; 8 pieces 2″ × 8″ × 24′; 36 pieces 2″ × 10″ × 14′; 60 pieces 2″ × 8″ × 22′; 12 pieces 2″ × 8″ × 20′; 10 pieces 2″ × 8″ × 18′; 30 pieces 2″ × 8″ × 16′; 220 pieces 2″ × 8″ × 12′; 20 pieces 2″ × 8″ × 10′; 40 pieces 2″ × 8″ × 8′; 22 pieces 2″ × 6″ × 24′; 8 pieces 2″ × 6″ × 12′; 5000 sq. ft. of siding 1″ thick; 22,000 shingles. Find the number of feet of lumber (board measure), its cost at $42 per M, and the cost of the shingles at $5.10 per M.

2. The carpenter also figured on needing 100 lb. of 4 d nails, 140 lb. of 8 d nails, and 650 lb. of spikes of various sizes. At 6¢ per pound, what was the cost of the nails and spikes?

A 4 d nail is called a fourpenny nail, an old name which has nothing to do with the cost. Specimens may profitably be brought to class.

3. The carpenter bought his windows ready made, needing ten at $2.60 each, and allowed $30 for foundation work. He also bought the necessary hardware, such as door hinges, latches, hangers, tracks for the large doors, and window fasteners, the bill amounting to $12.75. What was the cost of this material?

4. The carpenter figured his work at $3.75 a day, and the work of another man at $2.50 a day, both for a period of 37 days. How much was allowed for this labor?

5. The farmer decided to put in a concrete floor. The floor area was 1860 sq. ft. and the floor was 4 in. thick. How many cubic yards of concrete did he require? If 1 cu. yd. of the concrete required 1.3 bbl. of cement at $2.25 per barrel, how much would the cement for the floor cost?

In such cases figure as a whole barrel any fraction of a barrel in the total.

220. Ratio. The relation of one number to another number of the same kind, as expressed by division, is called the *ratio* of the first to the second. Without using the name, we studied about ratio in studying fractions.

The ratio of \$2 to \$4 is $\frac{2}{4}$, or, in its simplest form, $\frac{1}{2}$; the ratio of 6 ft. to 5 ft. is $\frac{6}{5}$; the ratio of 10 mi. to 2 mi. is $\frac{10}{2}$, or 5.

In the exercise below, when we speak of the ratio of E to D we mean the ratio of their number values; that is, of 1 to 2, the ratio being $\frac{1}{2}$. When we speak of the ratio of E to $2B$ we mean the ratio of 1 to 2 × 4, which is $\frac{1}{8}$.

ORAL EXERCISE

1. In this picture, what is the ratio of E to D? of E to C?

2. In the same picture, what is the ratio of E to B? of E to A?

Referring to the picture, state the following ratios:

3. E to $2B$ A to E D to C

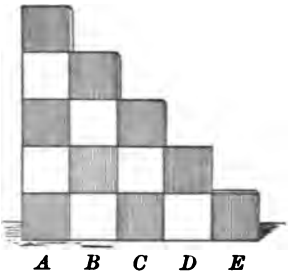
4. D to $2A$ C to B C to A

State also the following ratios:

5. $2E$ to D C to $3E$ $4E$ to A

6. $2D$ to B A to $2D$ $5E$ to A

7. What is the ratio of any two equal numbers?



8. In the picture, what is the ratio of A to $B+E$? of A to $B+C$? of B to $C+E$? of B to $C+D+E$?

The old method of solving business problems by ratio and proportion is no longer used to any extent. The subject of ratio has, however, a value of its own, and there is a special field in which proportion is peculiarly useful.

In this book we use the letter x to represent the number to be found, but teachers may prefer to use an interrogation point (?), vacant parentheses ( ), or an asterisk (*). The use of the equation is considered in the Supplement.

221. Writing a Ratio. A ratio may be written like a fraction, $\frac{2\text{ ft.}}{3\text{ ft.}}$, or with a sign of ratio (:), as 2 ft. : 3 ft.

We think of the ratio of 20 ft. to 4 ft. as $\frac{20\text{ ft.}}{4\text{ ft.}}$, or 20 ft. : 4 ft., or $\frac{20}{4}$, or 20 : 4, or simply 5. The word *ratio* is used for all.

222. Ratios always Abstract. Any number divided by a like number, as feet by feet, has an abstract quotient. Therefore,

A ratio is always abstract, and its terms may be written as abstract numbers.

Teachers should use the fractional form at first, because the pupils are already familiar with common fractions. Teachers should also recall the fact that all number is essentially abstract. In work in ratios it is better not to label with any denomination the numbers involved.

ORAL EXERCISE

Find the value of the missing number, designated by x, in each of the following ratios:

1. $\frac{x}{2} = 10.$
2. $\frac{x}{3} = 3.$
3. $\frac{x}{4} = 2.$
4. $\frac{x}{5} = 3.$
5. $\frac{x}{10} = 1.$
6. $\frac{x}{10} = \frac{1}{2}.$
7. $\frac{x}{7} = \frac{3}{7}.$
8. $\frac{x}{6} = \frac{2}{3}.$
9. $\frac{x}{5} = \frac{4}{5}.$
10. $\frac{2}{x} = \frac{1}{3}.$
11. $\frac{20}{x} = 0.4.$
12. $\frac{10}{x} = 1.$
13. What is the number whose ratio to 5 is 6?
14. What is the number to which 75 has the ratio 3?
15. What is the number whose ratio to $72\frac{7}{8}$ is 1?
16. The ratio of the height of a certain kitchen door to the width is $3\frac{1}{2}$. The door is 7 ft. high. How wide is it?
17. The ratio of the height of Fred's father to Fred's height is $\frac{3}{2}$. Fred's father is 6 ft. tall. How tall is Fred?

223. Separating a Number in a Given Ratio. Required to separate 25¢ in the ratio of 2 to 3.

With 2 parts in one group and 3 in the other, there are 5 parts in all.
Since one group has $\frac{2}{5}$, it has $\frac{2}{5}$ of 25¢, or 10¢.
The other group has $\frac{3}{5}$ of 25¢, or 15¢.
Check. 10¢ + 15¢ = 25¢; 10 : 15 = 2 : 3.

ORAL EXERCISE

1. Separate 9 in the ratio of 2 to 1.

2. What is the ratio of the number of boys in your class to the number of girls?

3. What is the ratio of the time you are in school to the whole 24 hours of the day?

4. What is the ratio of the length of summer, 3 months, to the whole year? to the remainder of the year?

5. There were 6 cloudy days in September, the rest being clear. What was the ratio of cloudy days to clear days?

6. In playing ringtoss, Will has a score of 15 and Mollie has one of 18. What is the ratio of Will's score to Mollie's?

WRITTEN EXERCISE

1. Divide $100 in the ratio of 4 : 1; of 7 : 3; of 1 : 9.

2. In a year having 165 cloudy days and 200 clear days, what is the ratio of cloudy days to clear days?

3. In November the ratio of clear days to cloudy days was 3 : 2. How many were there of each?

4. Four boys arrange a three weeks' camping trip. The first stays 18 da.; the second comes late and stays 12 da.; the third is in camp the last two weeks; the fourth stays the three weeks. The total expense is $32.50. What is the share of each?

224. Proportion. An expression of equality between two ratios is called a *proportion*.

For example, the ratio \$2 : \$3 equals the ratio 10 ft. : 15 ft. Therefore

$$\$2 : \$3 = 10 \text{ ft.} : 15 \text{ ft.}$$

is a proportion. It is read, "\$2 is to \$3 as 10 ft. is to 15 ft." It may, of course, be written simply $2 : 3 = 10 : 15$, or $\frac{2}{3} = \frac{10}{15}$.

225. Extremes and Means. The first and last terms of a proportion are called the *extremes;* the second and third terms are called the *means*.

In the proportion $$\frac{3}{7} = \frac{15}{35},$$

we see that $$7 \times 15 = 3 \times 35.$$

That is, in any proportion,

The product of the means equals the product of the extremes.

We also see that if $x : 21 = 35 : 105$,

or $$\frac{x}{21} = \frac{35}{105},$$

then $$x = \frac{21 \times 35}{105}.$$

Therefore, in any proportion,

The product of the means divided by one extreme equals the other extreme.

1. If $x : 7 = 13 : 21$, what is the value of x?

We have $$x = \frac{7 \times 13}{21} = 4\tfrac{1}{3}.$$

2. If $21.3 : 17.1 = 3 : x$, find the value of x.

We have $$x = \frac{\cancel{3} \times 17.1}{\underset{7.1}{\cancel{21.3}}} = 2\tfrac{29}{71}.$$

WRITTEN EXERCISE

Find the value of x in each of the following proportions:

1. $x : 51 = 12 : 17$.
2. $x : 17 = 54 : 51$.
3. $x : 22 = 41 : 11$.
4. $x : 250 = 35 : 50$.
5. $x : 27 = 36 : 243$.
6. $x : 1001 = 8 : 143$.
7. $24 : x = 3 : 20$.
8. $35 : x = 7 : 91$.
9. $111 : x = 37 : 38$.
10. $19 : x = 209 : 143$.
11. $x : 18.2 = 9.1 : 7.3$.
12. $x : 4.1 = 82.7 : 82$.
13. $x : 21.7 = 2.4 : 3.3$.
14. $x : 1.37 = 2.4 : 13.7$.
15. $x : 3.21 = 4.12 : 6.6$.
16. $x : 0.32 = 7.21 : 1.8$.
17. $3.17 : x = 4.28 : 3.2$.
18. $2.4 : x = 3.8 : 123$.
19. $8.21 : 3 = 4.105 : x$.
20. $3.9 : 4 = x : 3200$.

21. I am thinking of a number which has to 17 the ratio $35 : 119$. What is the number?

22. A miller uses 18 bu. of wheat for making 4 bbl. of flour. How many barrels of flour can he make from 207 bu. of wheat?

$$x : 4 = 207 : 18.$$

23. A certain circle has a circumference of 183.33 in. What is the circumference of a circle whose radius is to that of the first as $1.7 : 2.91$?

The ratio of two circumferences equals the ratio of their radii. That is, in this example, $x : 183.33 = 1.7 : 2.91$.

24. If 4 men can do a piece of work in 10 da., how many men would it take to do the work in 8 da.?

In all such problems the words "at the same rate" are evidently to be understood.

It will take more men to do the work in fewer days. Therefore x must be greater than 4, and the proportion is $x : 4 = 10 : 8$. Solving, $x = 5$. Therefore 5 men can do the work in 8 da.

25. How long will it take a train to go 148 mi., at the rate of 55½ mi. in 1 hr. 30 min.?

26. If 9 men can complete some work in 20 hr., how many less men can complete it in 16 hr. more time?

27. If a man can skate 75 yd. in 8⅕ sec., how long would it take him to skate 200 yd. if he could maintain this rate?

28. If a man can skate 220 yd. in 19 sec., how long would it take him to skate 5 mi. if he could maintain this rate?

29. If 750 lb. can be transported 100 mi. for a certain sum, how far should ½ T. be transported for the same sum?

30. If 6 men can complete some work in 15 hr., how many men must be added to complete it in 5 hr. less time?

31. If 10 men can complete some work in 30 hr., how many men must be added to complete it in 10 hr. less time?

32. If a crew is rowing at the average rate of 1 mi. in 5 min. 30 sec., how long will it take to row 1 mi. 1540 yd.?

33. If a man can swim 45 yd. in 23 sec., how long would it take him to swim 2 mi. if he could maintain this rate?

34. A prize of $60 was divided between two persons in the ratio of ⅔ to ⅚. How much was the share of each?

35. A map is drawn to the scale of 1 in. to 0.8 mi. How many acres of land are represented by a portion of the map 1 in. square? by a portion 3 in. square?

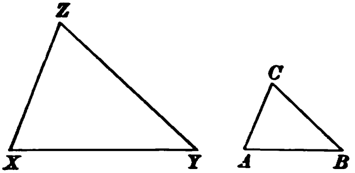
36. When a sum of money is divided equally among 7 persons, each receives $16.80. How much would each receive if the same sum were divided equally among 8 persons?

37. One man can do a piece of work in 10 da., and another can do the same work in 9 da. If the wages of the first are $3.60 a day, what should be the wages of the second?

226. Proportional Numbers. Numbers which form a proportion are called *proportional numbers* and are said to be *proportional*.

227. Similar Figures. Figures which are of exactly the same shape are called *similar figures* and are said to be *similar*.

For example, these two triangles are similar.



228. Proportional Lines. The lengths of corresponding lines in similar figures are proportional.

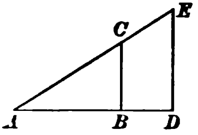
We express this more briefly by saying that the lines are proportional. For example, in the above triangles $XY:YZ=AB:BC$, where XY represents the numerical length of the line XY; in two circles the circumferences and radii are proportional, the circumference of the first being to that of the second as the radius of the first is to that of the second.

ORAL EXERCISE

1. In the above triangles, if XY is twice as long as AB, how does ZX compare in length with CA?

2. In the same triangles, state four proportions that exist among the sides AB, BC, XY, and YZ.

3. In this figure, state two proportions that exist among AB, AD, AC, and AE?



4. In this figure, if AB is $\frac{2}{3}$ of AD, what is the ratio of AC to AE?

5. In the same figure, if DE represents the height of a man 6 ft. tall, BC the height of a boy, DA the length of the shadow cast by the man, and BA the length of the shadow cast by the boy, show how to find the height of the boy by measuring the lengths of the shadows.

229. Measuring Heights and Distances. The height of a tree is often measured by proportion, as shown below.

The man holds a right triangle in which $AB = BC$. He holds AB level and walks toward the tree until he just sees the top along AC. Then because

$$AB = BC,$$

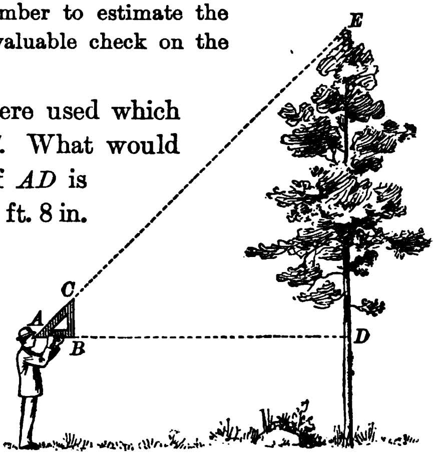
and

$$AB : BC = AD : DE,$$

the height *above* D will equal the distance AD.

WRITTEN EXERCISE

1. What is the height of the tree in the picture if the triangle is 5 ft. 4 in. from the ground, and AD is 23 ft. 8 in.?



In all such problems remember to estimate the result in advance. This is a valuable check on the accuracy of the work.

2. Suppose a triangle were used which had AB equal to twice BC. What would be the height of the tree if AD is 75 ft., and if the point D is 5 ft. 8 in. above the ground?

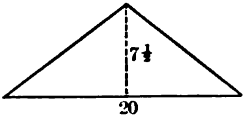
3. Suppose the triangle used has $AB = 2$ ft., $BC = 1$ ft. 6 in. What is the height of the tree if AD is equal to 4 rd., and if D is 5 ft. 6 in. above the ground? if AD is equal to 61 ft., and if D is 5 ft. 3 in. above the ground?

4. What is the height of a tree that casts a shadow 88 ft. long at the same time that a man 5 ft. 9 in. high casts a shadow 7 ft. 8 in. long?

5. A boy whose eye is 5 ft. above the ground sights over the top of a 10-foot pole and just sees the top of a tree. The boy is 6 ft. from the pole and 42 ft. from the tree. How high is the tree? Draw the picture.

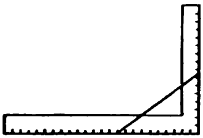
6. To find the height of a church spire a boy measured its shadow, and found it to be 152 ft. 3 in. At the same time he found that a post 3 ft. 6 in. high was casting a shadow 5 ft. 3 in. long. How high was the spire?

7. A carpenter wishes to construct a roof as here shown, the house being 20 ft. wide and the gable $7\frac{1}{2}$ ft. high. He wishes to know the length of slope. He draws a plan to the scale $\frac{1}{30}$. How many inches on the plan will represent the $7\frac{1}{2}$ ft., or "rise," as the carpenter calls it?



8. The carpenter found that the plan of Ex. 7 was not convenient, so he drew a right triangle representing half the plan, and represented by a line 10 in. long the 10 ft., or "run," as he calls it. Draw such a plan, representing the $7\frac{1}{2}$ ft. correctly, and measure the slope. What did the carpenter find to be the length of the slope of the roof?

9. But practically the carpenter would proceed in another way. He would take his square, and on the tongue, or short arm, he would take a point $7\frac{1}{2}$ in. from the corner; then on the blade, or long arm, he would take a point 10 in. from the corner. He would then measure the distance between these points, and the number of inches would be the same as the number of feet in the slope of the roof. Draw the figure to scale and write out the reason why this plan gives the correct result.



10. Two stockmen, A and B, maintain a common drinking trough and share the annual cost of repairs in proportion to the number of cattle owned by each. A has 1253 head of cattle and B has 2747. If the annual cost of repairs is $80, how much should each contribute?

11. If ¾ in. on a map represents a distance of 750 mi., how many miles will 2½ in. represent?

12. If a distance of 650 mi. is represented on a map by 1⅝ in., how many inches will represent a distance of 680 mi.?

13. If a tree casts a shadow 60 ft. long when a post 4 ft. high casts a shadow 4.8 ft. long, how high is the tree?

14. If a spire casts a shadow 200 ft. long when a post 4½ ft. high casts a shadow 5⅝ ft. long, how high is the spire?

15. In finding the height of the school building, one boy stood a yard measure upright and another boy lay down with his eye 4 in. from the ground where he could just see the top of the building in line with the top of the stick. The distance from his eye to the stick was 4 ft. 6 in., and the distance to the building was 45 ft. How high was the building?

16. If a building casts a shadow 75 ft. long when a post 6 ft. 2 in. high casts a shadow 12 ft. 6 in. long, how high is the building?

17. Edward and Henry wished to find the height of the school building. Edward suggested standing a yardstick upright, measuring its shadow, and at the same time measuring the shadow of the school. The yardstick shadow was 4 ft., and the shadow of the building was 36 ft. How high was the school building? Draw a picture.

It is very easy to measure the height of trees and buildings in this way, or by sighting from the ground across the top of a pole to the top of a building, and pupils will find such work interesting and profitable.

18. These boys find that this post is 3 ft. 6 in. high and casts a shadow 3 ft. 2 in. long. The shadow of the tree is 34 ft. 10 in. long at the same time. Required the height of the tree.

19. If a building casts a shadow 75 ft. long at the same time that a post 6 ft. 2 in. high casts a shadow 12 ft. 6 in. long, how high is the building?

20. If the railway fare for a journey of 75 mi. is $1.50, what will be the fare for 275 mi.?

Are you making use of short methods in your work whenever possible?

21. Two boys weighing respectively 100 lb. and 80 lb. sit 9 ft. apart on the ends of the plank of a see-saw. Not counting the weight of the plank, how far from the heavier boy must the stick on which the see-saw rests be placed so that they just balance?

22. How much pressure must you exert on the handles of a pair of shears just 3 in. from the *fulcrum*, or screw, to exert a pressure of 5 lb. at a point 5 in. from the fulcrum?

23. Three men pay $848 for some water power. The first uses 28 horse power, the second 36 horse power, and the third 42 horse power. How much should each pay? How much should each pay if the total amount were $1007?

LITTLE EXAMINATIONS

I. **1.** 17 rd. = (?) ft.

2. $\frac{3}{4}$ mi. = (?) rd.

3. 227 yd. = (?) ft.

4. $\frac{1}{4}$ sq. mi. = (?) A.

5. 320 A. = (?) sq. mi.

6. 216 sq. in. = (?) sq. ft.

7. 27 sq. yd. = (?) sq. ft.

8. 7 cd. = (?) cu. ft.

9. 864 cu. in. = (?) cu. ft.

10. 121 sq. yd. = (?) sq. rd.

II. *Express each of the following ratios in simplest form:*

1. $\frac{34}{51}$. **2.** $\frac{46}{115}$. **3.** $\frac{\frac{4}{5}}{6}$. **4.** $\frac{13}{3\frac{1}{4}}$. **5.** 58 : 145.

Find the missing term x in each of the following ratios:

6. $\frac{x}{4} = 2$. **7.** $\frac{x}{9} = \frac{1}{3}$. **8.** $\frac{10}{x} = \frac{5}{3}$. **9.** $\frac{x}{9} = 2$.

10. Two men rent a pasture for $132. One puts in 7 cattle and the other 15. How much rent should each pay?

III. *Express each of the following ratios in simplest form:*

1. $\frac{57}{76}$. **2.** $\frac{2}{\frac{3}{4}}$. **3.** $\frac{\frac{1}{2}}{\frac{7}{8}}$. **4.** $\frac{2\frac{1}{2}}{10}$. **5.** 93 : 124.

Find the missing term x in each of the following ratios:

6. $\frac{6}{x} = 2$. **7.** $\frac{5}{x} = \frac{1}{4}$. **8.** $\frac{7}{x} = \frac{1}{5}$. **9.** $\frac{7}{x} = \frac{1}{2}$.

10. If the ratio of a man's income to his expenses is 9 : 8 and his expenses are $2400 a year, how much is his income?

REVIEW AND DRILL

I. REVIEW SECTION

Add the following:

1.	2.	3.	4.
$20.07	20 ft. 7 in.	20 lb. 7 oz.	20 yd. 7 in.
8.09	8 9	8 9	8 9
7.11	7 11	7 11	7 11
16.10	16 10	16 10	16 10

The pupil should see that the figures in each set of examples are the same, but that the results differ. The pupil should explain the reason for this difference.

Subtract the following:

5.	6.	7.	8.
$30.01	30 ft. 1 in.	30 lb. 1 oz.	30 yd. 1 in.
16.09	16 9	16 9	16 9

Multiply the following:

9.	10.	11.	12.
$24.08	24 ft. 8 in.	24 lb. 8 oz.	24 yd. 8 in.
23½	23½	23½	23½

Divide the following, carrying each quotient to two decimal places:

13. $26.07 ÷ 9.

14. 26 lb. 7 oz. ÷ 9.

15. 26 ft. 7 in. ÷ 9.

16. 26 pk. 7 qt. ÷ 9.

17. 26 yd. 7 in. ÷ 9.

18. 26 mi. 9 rd. ÷ 9.

19. 26 sq. yd. 7 sq. in. ÷ 9.

20. 26 cu. ft. 7 cu. in. ÷ 9.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you know the width of a sidewalk in inches, how do you find the width in feet? in yards?

2. If you know the area of a table top in square inches, how do you find the area in square feet? in square yards?

3. If you know the number of feet in the length, width, and depth of a cellar, how do you find the number of cubic yards?

In computing excavations the cubic yard is the common unit.

4. If you know the number of rods in the length and width of a rectangular field, how do you find the number of acres?

5. How would you figure the cost of the lumber used in building a barn in your vicinity?

6. How would you figure the cost of a concrete walk in front of the schoolhouse?

7. Suppose that your school has a playground in the form of a rectangle, how would you find the area in acres?

8. If you know the weight of two volumes of a substance and one of the volumes, how do you state the proportion?

9. Knowing the number of men required to do a piece of work in a given number of days, how can you find how many men are required to do it in another given number of days?

10. How can you find the height of your school building by means of shadows and the height of a pole?

11. How can you find the height of a tree by sighting over the top of a pole of known length?

12. If you know the railway fare for a certain number of miles, how do you find the fare for a certain other number of miles?

III. INDUSTRIAL PROBLEMS

THE CONCRETE INDUSTRY

1. For ordinary work about the farm, as in silos, tanks, cisterns, and fence posts, concrete consists of half as much Portland cement as sand and half as much sand as crushed stone, all measured by bulk. This is called a 1 : 2 : 4 mixture. If 70 cu. ft. of ingredients are used, how many cubic feet are there of each?

2. In estimating the material needed for concrete, the contractors allow 0.058 bbl. of cement, 0.0163 cu. yd. of sand, and 0.0326 cu. yd. of stone or gravel to 1 cu. ft. of the finished concrete. Estimate the amount of each material for a concrete wall 1 ft. 3 in. thick, 6 ft. high, and 48 ft. long.

3. A farmer is building a silo, and he estimates that he needs 935 cu. ft. of concrete. Taking the amounts given in Ex. 2, how much of each kind of material does he need?

4. A man is connecting two barns by a concrete driveway. He finds that he needs 5 bags of Portland cement, $\frac{1}{2}$ cu. yd. of sand, and 1 cu. yd. of crushed stone or screened gravel for 60 sq. ft. of surface, and that these materials will cost $3.60. How much of each material will he need for a driveway 90 ft. long and 6 ft. wide, and how much will these materials cost?

5. A farmer wishes to put down a 6-inch concrete floor 24 ft. by 36 ft. He finds that he needs 20 cu. yd. of screened gravel at $1.10, 10 cu. yd. of sand at $1, and 28 bbl. of Portland cement at $2.50. How much does the material for the floor cost?

6. A contractor estimates the cost of a 5-inch concrete floor 32 ft. 6 in. by 48 ft. 6 in. as follows: 30 cu. yd. of screened gravel at $1.15, 15 cu. yd. of sand at $1.10, and 42 bbl. of Portland cement at $2.50. How much is his estimate for materials?

A LITTLE ABOUT MECHANICS

1. Tom bought a copy of a magazine treating of mechanics. He found some things that he could not understand. He asked his father what a foot pound was, and found that it was the work done in lifting a weight of one pound to a height of one foot. His father said: "Your weight is 108 lb. If you step up on the chair, which we will say is 2 ft. from the floor, you lift a weight of 108 lb. to a height of 2 ft., which takes as much work as to lift a weight of 1 lb. to a height of 2 × 108 ft., or 216 ft. Then you have done 216 foot pounds of work. How many foot pounds of work will you do in going upstairs?" It is 10 ft. upstairs. What is the answer?

2. Tom's father weighs 180 lb. How many foot pounds of work does he do when he goes up the stairs mentioned in Ex. 1?

3. "To find the work done in foot pounds, we do not always think of something as being lifted," said Tom's father; "the force may be used in pushing or pulling in any direction. When you pull your sled with a force of 1 lb., you use a foot pound of force when you pull it a foot." "Well," said Tom, "suppose I pull with a force of 8 lb. and draw my sled a quarter of a mile, how many foot pounds do I use?" "That is for you to find out," said his father. What is the answer?

4. Tom found something in his magazine about horse power and asked his father what that meant. His father told him that it was the force that equals 33,000 foot pounds per minute; that is, the force necessary to lift 33,000 lb. through a distance of 1 ft. in 1 min. Tom found something about the horse power necessary to raise a block of stone weighing 750 lb. to a height of 30 ft. in 5 min. What is the horse power necessary to do this?

Divide 5 × 30 × 750 by 33,000, always using cancellation in such cases.

CHAPTER IX

PERCENTAGE AND ITS APPLICATIONS

230. Per Cent. As you already know (§ 174), another name for hundredths is *per cent.*

Thus 0.01, or $\frac{1}{100}$, is the same as 1 per cent; 0.06 is the same as 6 per cent; $0.00\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{100}$, is the same as $\frac{1}{2}$ per cent.

That part of arithmetic which treats of per cent is called *percentage.* The word "percentage" is often used for per cent, as in speaking of the percentage of base hits in a ball game.

231. Symbol for Per Cent. We have found (§ 175) that the symbol for per cent is written thus: %.

We may read 0.06 either "6 hundredths" or "6 per cent." In the same way, we may think of 6% either as "6 per cent" or as "6 hundredths," although it is read "6 per cent."

The expression 800% means $\frac{800}{100}$ and equals the whole number 8; 225% means the mixed number 2.25, or $2\frac{1}{4}$; $\frac{1}{2}$% means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$, and is read either "$\frac{1}{2}$ per cent" or, quite commonly, "$\frac{1}{2}$ of 1%."

232. Relation to Fractions. Since 6% means $\frac{6}{100}$, which equals 0.06, or $\frac{3}{50}$, we see that we may express per cent as a decimal fraction or as a common fraction.

233. Per Cents as Common Fractions. We have learned that $62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$. Therefore,

To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and reduce this fraction to lowest terms.

ORAL EXERCISE

1. A man said that he gained 100% on some property that he sold. What does this mean?

2. A dealer said that he would sell some goods at 25% less than the marked price. Express this as a common fraction.

3. A boy beginning to play tennis won only 20% of the games played the first week. Express this as a common fraction.

Express as common fractions, whole numbers, or mixed numbers:

4. 50%.
5. 10%.
6. 30%.
7. 40%.
8. 60%.
9. 80%.
10. 75%.
11. 125%.
12. 150%.
13. 200%.
14. 300%.
15. 120%.

WRITTEN EXERCISE

1. A man said that he marked some goods $33\frac{1}{3}\%$ more than they cost him, and another said that he marked his goods $\frac{1}{3}$ more than they cost him. Was there any difference?

2. How much is $\frac{1}{25}$ of 75? 4% of 75? 4% of 80?

3. How much is $\frac{1}{4}$ of 640? 25% of 640? 25% of 888?

4. How much is $\frac{1}{2}$ of 1728? 50% of 1728? 50% of 5280?

5. How much is $\frac{3}{4}$ of 8440? 75% of 8440? 75% of 1672?

6. How much is $1\frac{1}{4} \times 864$? 125% of 864? 125% of 996?

7. Instead of multiplying 64 by $12\frac{1}{2}\%$, what fractional part of 64 may we take? Do this.

8. Instead of multiplying 720 by $37\frac{1}{2}\%$, what fractional part of 720 may we take? Do this.

9. Find $87\frac{1}{2}\%$ of 80 and 1600 in the shortest way.

10. Find $33\frac{1}{3}\%$ of 66 and 3672 in the shortest way.

234. Per Cents as Decimals. Since 25.5% and 0.255 have the same value, therefore,

To express as a decimal a number written with the per cent sign, omit the sign and move the decimal point two places to the left.

When we omit the per cent sign we must indicate the hundredths in some other way, as by moving the decimal point two places to the left.

Thus 1½% = 0.01½, or 0.015; 125% = 1.25; 0.6% = 0.006.

ORAL EXERCISE

1. If 2% of the pupils of this school are absent to-day, how many are absent out of every 100? out of every 50?

2. How much is $\frac{1}{100}$ of \$200? 1% of \$200? 1% of \$1700?

3. How much is 0.06 of $100? 6% of $100? 6% of $300?

Read as decimal fractions:

4. 25%.
5. 32%.
6. 17%.
7. 45%.
8. 150%.
9. 175%.
10. 225%.
11. 375%.

WRITTEN EXERCISE

1. Express ⅛% as a decimal; as a common fraction.

2. Express ⅞% as a decimal; as a common fraction.

3. Express 333⅓% as a decimal; as an improper fraction.

4. How much is 0.15 of $300? 15% of $300?

5. How much is 1¼ × $650? 1.25 × $650? 125% of $650?

6. How much is 2.50 × $460? 250% of $460?

Express as decimal fractions, whole numbers, or mixed decimals:

7. 50%.
8. 75%.
9. 12½%.
10. 66⅔%.
11. 0.5%.
12. 0.7%.
13. 200%.
14. 150%.
15. 125%.
16. 1000%.
17. 0.005%.
18. 0.065%.

235. Decimals as Per Cents. Since per cent means hundredths, to express a decimal as per cent we have to consider only how many hundredths the decimal represents.

1. Express 0.3 as per cent.

Since 0.3 = 0.30, we see that 0.3 is the same as 30 hundredths, or 30%.

2. Express 0.375 as per cent.

Here 0.375 = 37.5 hundredths, or $37\frac{1}{2}$ hundredths. Hence 0.375 is the same as 37.5%, or $37\frac{1}{2}$%.

3. Express 0.00125 as per cent.

Here $0.00125 = 0.00\frac{125}{1000} = 0.00\frac{1}{8}$, and therefore $0.00125 = \frac{1}{8}$%.

4. Express $6.5\frac{1}{2}$ as per cent.

Here $6.5\frac{1}{2} = \frac{655}{100}$, and therefore $6.5\frac{1}{2} = 655$%.

Therefore, *to express a decimal as per cent, write the per cent sign after the number of hundredths.*

ORAL EXERCISE

1. A pint, being 0.5 qt., is what per cent of a quart?

2. A peck, being 0.25 bu., is what per cent of a bushel?

3. A quart, being 0.125 pk., is what per cent of a bushel?

4. Express 0.007 of a mile as per cent of a mile.

5. What per cent of a dollar is 37¢? is 37 dimes?

WRITTEN EXERCISE

1. Express 1 gill as per cent of a quart; of a gallon.

2. Express 1 quart as per cent of a peck; of a bushel.

Express the following as per cents:

3. 0.8.

4. 5.2.

5. 0.37.

6. 0.62.

7. 0.225.

8. 0.450.

9. $3.333\frac{1}{3}$.

10. 0.00125.

236. Common Fractions as Per Cents. Since per cent means hundredths, to express a common fraction as per cent we have only to reduce it to hundredths.

For example, $\frac{4}{5} = \frac{80}{100}$, and this is the same as 0.80, or 80%. Likewise $\frac{2}{25} = \frac{8}{100} = 8\%$, and $2\frac{1}{2} = \frac{5}{2} = \frac{250}{100} = 250\%$.

Therefore, *to express a common fraction as a per cent, reduce it to hundredths, omit the denominator, and write the numerator followed by the per cent sign.*

ORAL EXERCISE

1. A cent is what per cent of $1? of a dime?

2. 5¢ is what per cent of 10¢? of 20¢? of $1? of 50¢?

3. 25¢ is what per cent of 50¢? of $1? of 75¢? of $1.25?

4. A foot is what per cent of a yard?

Express the following as per cents:

5. $\frac{1}{2}$.
6. $\frac{1}{4}$.
7. $\frac{1}{5}$.
8. $\frac{1}{10}$.
9. $\frac{3}{4}$.
10. $\frac{2}{5}$.
11. $\frac{3}{5}$.
12. $\frac{1}{20}$.
13. $\frac{1}{25}$.
14. $\frac{3}{50}$.

WRITTEN EXERCISE

1. An inch is what per cent of a foot? of a yard?

2. A pint is what per cent of a quart? of a gallon?

3. What per cent of a gallon is 3 qt.? 2 qt.? $2\frac{1}{2}$ qt.?

4. What per cent of a pound is 1 oz.? 2 oz.? $3\frac{1}{5}$ oz.?

5. What per cent of a pound is 7 oz.? $7\frac{1}{2}$ oz.? 11 oz.?

6. What per cent of a rod is 1 ft.? 1 yd.? $5\frac{1}{2}$ yd.?

7. If a man spends $\frac{5}{6}$ of his income, what per cent of his income does he spend?

8. If you have increased $\frac{1}{9}$ in weight during the past year, what per cent have you increased in weight?

237. Terms used in Percentage. The number of which a required per cent is to be taken is often called the *base.*

The number of hundredths of the base is called the *rate.*

For example, in 25% of $300, $300 is the base and 25% is the rate.

Sometimes 25 is called the *rate per cent,* 25% being called the *rate,* but these two terms are commonly used to mean the same thing.

The result found by taking a certain per cent of the base is called the *percentage.* Therefore,

The percentage is the product of the base and the rate.

The terms "base," "rate," and "percentage" are introduced in this review work because they are occasionally used. They are not needed in this book and may be omitted unless demanded by the course of study.

238. Important Per Cents. The following equivalents should be memorized:

$50\% = \frac{1}{2}$ $37\frac{1}{2}\% = \frac{3}{8}$ $16\frac{2}{3}\% = \frac{1}{6}$ $20\% = \frac{1}{5}$

$25\% = \frac{1}{4}$ $62\frac{1}{2}\% = \frac{5}{8}$ $33\frac{1}{3}\% = \frac{1}{3}$ $40\% = \frac{2}{5}$

$12\frac{1}{2}\% = \frac{1}{8}$ $87\frac{1}{2}\% = \frac{7}{8}$ $66\frac{2}{3}\% = \frac{2}{3}$ $60\% = \frac{3}{5}$

$6\frac{1}{4}\% = \frac{1}{16}$ $3\frac{1}{8}\% = \frac{1}{32}$ $83\frac{1}{3}\% = \frac{5}{6}$ $80\% = \frac{4}{5}$

To take $87\frac{1}{2}\%$ of 648 is, therefore, the same as to take $\frac{7}{8}$ of 648.

WRITTEN EXERCISE

Find the following:

1. 50% of $274.
2. 25% of $372.
3. $12\frac{1}{2}\%$ of $512.
4. $6\frac{1}{4}\%$ of $3376.
5. $37\frac{1}{2}\%$ of $17.76.
6. $62\frac{1}{2}\%$ of $11.92.
7. $87\frac{1}{2}\%$ of $34.48.
8. $33\frac{1}{3}\%$ of $77.16.
9. $66\frac{2}{3}\%$ of $80.07.
10. $83\frac{1}{3}\%$ of $71.10.
11. $112\frac{1}{2}\%$ of 72.
12. $137\frac{1}{2}\%$ of 96.
13. $133\frac{1}{3}\%$ of 81.
14. $166\frac{2}{3}\%$ of 84.

239. Finding a Required Per Cent of a Number. The more general case of finding per cents — that in which common fractions cannot be used to advantage — will now be considered.

For example, suppose a man is digging a canal, and finds that 9600 cu. yd. of earth must be carted away. If it is reported to him that $23\frac{1}{4}\%$ of the earth has been removed, how many cubic yards have been carted away?

Since $23\frac{1}{4}\% = 0.23\frac{1}{4}$, we multiply 9600 by $0.23\frac{1}{4}$. We might multiply by 0.2325, but this would be longer. The easiest way is simply to multiply 96 by $23\frac{1}{4}$.

$$\begin{array}{r} 9600 \\ \underline{0.23\frac{1}{4}} \\ 2400 \\ 288 \\ 192 \\ \hline 2232. \end{array}$$

Therefore, *to find a required per cent of a number, multiply the number by the given rate.*

WRITTEN EXERCISE

Find, by using decimals:

1. 22% of 75.
2. 35% of 86.
3. 27% of 32.
4. 41% of 78.
5. 62% of 93.
6. 26% of 3.4.
7. 43% of 6.7.
8. 39% of 5.3.
9. 28% of 0.7.
10. 35% of 0.63.
11. $2\frac{3}{4}\%$ of 8.4.
12. 0.5% of 5.7.
13. 1.9% of 6.8.
14. 235% of 742.
15. 622% of 0.72.

Find, by using common fractions:

16. 75% of 6448 yd.
17. $12\frac{1}{2}\%$ of $9616.
18. $6\frac{1}{4}\%$ of $24,328.
19. $33\frac{1}{3}\%$ of 9051 yd.
20. $66\frac{2}{3}\%$ of 7116.
21. $87\frac{1}{2}\%$ of 35,360.
22. $16\frac{2}{3}\%$ of $13,806.
23. $37\frac{1}{2}\%$ of $25,040.
24. $83\frac{1}{3}\%$ of $22,350.
25. 125% of 948 cu. ft.
26. $133\frac{1}{3}\%$ of 729 cu. yd.
27. $166\frac{2}{3}\%$ of 816 sq. ft.

28. A man borrowed $175 and had to pay 6% of that sum for the use of it for a year. How much did he pay at the end of the year, in addition to the sum borrowed?

29. If a manufacturer sells shoes at a profit of 15% and it costs him $2.33⅓ a pair to make them, how much is his profit on 1000 pairs?

30. If a shop manufactures 276 locomotives and sells 75% of them for $11,125 each and the rest for $9825 each, how much is received for all?

31. A farmer can have his land poorly plowed for $3.48 an acre, but it will cost him 66⅔% more to have a good job done. How much will it cost to have 60 acres well plowed?

32. How much butter fat is there in 375 lb. of milk brought to a creamery, the creamery test showing that 3.9% of the weight of the milk is butter fat?

33. A farmer takes 340 lb. of milk to a creamery and is paid 26¢ a pound for the butter fat. The tests show that the butter fat is 3.8% of the weight of the milk. Find the weight of the butter fat and the amount of money received.

34. One cow in a dairy gives 28 lb. of milk a day, which tests 3.1% of butter fat; another gives 24 lb., testing 3.7%. Butter fat being worth 23¢ a pound, which cow is the more profitable, and how much more?

35. A farmer has 75 trees on an acre of woodland, of which he decides to cut 60%. If wood is worth $5.75 a cord and he can cut 3 cords from 5 trees, how much will he receive for the wood?

36. A dressmaker bought 37½ yd. of chiffon velvet at $4.10 a yard, and received a reduction of 8% by paying promptly. She sold the velvet at $4.25 a yard. How much did she make on the transaction?

ORAL REVIEW

1. In a school of 300 pupils 9% were in the seventh grade. How many pupils were in that grade?

2. In a school of 200 pupils 11% were in the third grade and 14% were in the first grade. How many pupils were in each of these grades?

3. Of 300 pupils 5% have been tardy or absent this month. How many pupils have not been tardy or absent?

4. In a school of 300 pupils, if there should be an increase of $3\frac{1}{3}\%$ next year, how many pupils would there then be?

5. In a school of 250 pupils there were 10% less pupils four years ago. How many pupils were there then?

6. If $16\frac{2}{3}\%$ of the pupils in a school of 240 pupils were not in the school last year, how many new ones are there this year?

7. If 20% of the pupils in a school of 250 pupils are in the seventh and eighth grades, how many are in those grades?

8. If 25% of the pupils of a school of 300 pupils are in the first and second grades, how many are in those grades?

9. If 55% of the pupils in a school of 300 pupils are girls, how many girls are there in the school? how many boys?

10. How much must be added to 250 ft. to increase it 50%? to increase it 100%?

11. How much must be added to 240 lb. to make it 125% of the present weight?

12. A column of mercury in a thermometer is 6 in. high. If it increases $16\frac{2}{3}\%$ in height, how high is it then?

13. A rectangle 4 in. long and 3 in. wide is increased 50% in length and $33\frac{1}{3}\%$ in width. What was the original area, and what is the area now?

240. Three Important Problems of Percentage. There are three important problems in percentage. The first, and by far the most important one, is that of finding a required per cent of a number. This was considered in § 239. The second problem of importance is to find what per cent one number is of another, and this will be considered in § 241. The third problem of importance is to find the number of which a given number is a given per cent, and this will be considered in § 243.

The problems in §§ 241 and 243 depend upon this principle:

Given the product of two factors and one of the factors, the other factor may be found by dividing the product by the given factor.

That is, if we know the product of 2 and 5, which is 10, we can find the factor 2 by dividing 10 by 5, and we can find the factor 5 by dividing 10 by 2.

Teachers should not require the pupils to learn the three problems mentioned in § 240, for they will meet the two new ones in §§ 241 and 243. The principle should be clearly understood, however. Teachers who desire to introduce the equation in connection with §§ 241 and 243 may do so, consulting pages 529 and 530.

ORAL EXERCISE

The first number in each of the following examples is the product of two factors, and the second number is one of the factors. Find the other factor:

1. 48, 4.
2. 48, 12.
3. 72, 8.
4. 72, 9.
5. 70, 10.
6. 90, 9.
7. 90, 10.
8. 90, 3.
9. 200, 10.
10. 200, 20.
11. 8, 4.
12. $\frac{8}{10}$, 4.
13. 0.8, 4.
14. 0.80, 4.
15. 0.80, 8.
16. 6, $\frac{1}{4}$.
17. 7, $\frac{1}{2}$.
18. 7, 0.5.
19. 9, 0.5.
20. 8, 50%.

21. I am thinking of the number which, multiplied by 7, gives the product 56. What is the number?

241. Finding what Per Cent One Number is of Another. The second problem of importance in percentage mentioned in § 240 is to find what per cent one number is of another.

For example, if Fred was at bat 20 times in the baseball games of last June and made 6 base hits, what per cent of base hits did he make?

Here we have a certain per cent $\times 20 = 6$; that is, we have the product (6) and one factor (20) to find the other factor (a certain per cent). Therefore

$$6 \div 20 = 0.3 = 30\%, \text{ the per cent of hits.}$$

ORAL EXERCISE

1. If Fred was at bat 30 times and made 15 base hits, what per cent of base hits did he make?

2. If a baseball team wins 12 games out of 20, what per cent of games does it win?

3. If there are 40 pupils in a class and 4 of them are absent to-day, what per cent are absent?

4. On an automobile trip of 80 mi., what per cent has a man traveled when he has traveled 20 mi.?

Find what per cent the second number is of the first:

5. 2, 1.
6. 4, 1.
7. 4, 2.
8. 4, 3.
9. 8, 6.
10. 60, 30.
11. 70, 35.
12. 90, 45.
13. 60, 15.
14. 32, 8.
15. 64, 32.
16. 64, 16.
17. 64, 48.
18. 72, 54.
19. 36, 27.
20. 25, 5.
21. 25, 2.5.
22. 25, $12\frac{1}{2}$.
23. 25, 25.
24. 25, 50.

25. If a book has 350 pages, what per cent of the number of pages have you read when you have read it through page 35? through page 70? through page 50? through page 175?

242. Application to Written Exercises. You now understand the second important kind of problem in percentage. We shall consider once more the first kind of problem, and then the application of the second kind to written exercises.

1. An agent bought an automobile for $600 and sold it so as to gain 20% of the cost. How much did he gain?

Here we have to find 20% of $600, or 0.20 of $600. That is, we have two factors given, 0.20 and $600, to find the product. Therefore we have

$$0.20 \times \$600 = \$120.$$

This is the first kind of problem. We shall now consider the second kind.

2. An agent bought an automobile for $600 and sold it at a profit of $120. What per cent of the cost did he gain?

Here we have $120 equal to some per cent of $600.

That is, we have the product (120) of two factors, and one of the factors (600) given, to find the other factor. Therefore we have

$$\$120 \div \$600 = 0.20 = 20\%.$$

That is, the agent gained 20% of the cost.

WRITTEN EXERCISE

1. $130 is what per cent of $2600? of $3900?
2. $11.25 is what per cent of $375? of $56.25?
3. $10.12 is what per cent of $253? of $202.40?
4. $29.10 is what per cent of $465.60? of $174.60?
5. A foot is what per cent of a yard? of a rod?
6. A quart is what per cent of a gallon? of 25 gal.?
7. $\frac{2}{3}$ is what per cent of $\frac{1}{4}$? $\frac{3}{8}$ is what per cent of $\frac{1}{4}$?
8. 25% is what per cent of 75%? of 125%? of 250%?
9. $33\frac{1}{3}\%$ is what per cent of $66\frac{2}{3}\%$? of 1? of $133\frac{1}{3}\%$?
10. $16\frac{2}{3}\%$ is what per cent of $33\frac{1}{3}\%$? of $66\frac{2}{3}\%$? of 1?

11. If Ralph Jenkins is at bat 40 times and makes 12 base hits, what is his per cent of base hits?

12. If the seventh grade devotes 45 min. a day to arithmetic, and the sixth grade 42 min., what is the per cent of increase?

13. If a class had 24 examples to solve on Monday and 29 on Tuesday, what was the per cent of increase?

14. If you have read 78 pages in a book of 300 pages, what per cent of the pages have you read?

15. If the last chapter of a book is numbered XXXII, and you have finished reading Chapter XXIV, what per cent of the chapters have you read?

16. A man's income is $1650 a year, and he spends $693. What per cent of his income does he save?

17. In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys? What per cent are girls?

18. A man invests $6750 and gains $810. His gain is what per cent of the amount invested?

19. The purity of gold is measured in carats, or twenty-fourths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring?

20. What is the per cent of pure gold in a watch case that is 16 carats fine? in a chain that is 12 carats fine?

21. If 2 members of a class of 32 have not been either tardy or absent during the year, and 4 members have not been absent, what per cent have not been either tardy or absent? What per cent have not been absent?

22. A man's income is $2175 a year, and he saves $652.50. What per cent of his income does he save? What per cent does he spend? What is the sum of these two per cents?

243. Finding the Number of which a Given Number is a Required Per Cent. The third important problem in percentage mentioned in § 240 is to find the number of which a given number is a given per cent.

For example, if a baseball team lost 18 games, which is 45% of the number of games played, how many games were played?

Here we have 45% of the number of games = 18.

That is, we have the product (18) of two factors and one of the factors (0.45) given to find the other factor.

Therefore we have $18 \div 0.45 = 40.$

That is, there were 40 games played.

Check. 45% of $40 = 18.$

WRITTEN EXERCISE

1. If 20% of the inhabitants of a certain city are school children and there are 18,275 school children in the city, what is the total population?

2. If 23% of the inhabitants of a certain city voted at a certain election and there were 3956 ballots cast, what was the total population?

3. A player reached first base 29 times, or $33\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

4. A player reached home base 24 times, or $8\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

5. If you pay $37\frac{1}{2}\%$ of the expenses of a camping trip, your contribution being \$6.24, what are the expenses of the trip?

6. In a certain school there are 119 boys, which is 85% of the number of girls. How many girls are there in the school?

7. If a school is in session 50% of the days of a certain year, and is in session 183 da., is that year a leap year or not?

Find the numbers of which the following are the given per cents:

8. 64, 8%.

9. 72, 9%.

10. 88, 11%.

11. 85, 5%.

12. 84, 6%.

13. 91, 7%.

14. 64, 4%.

15. 111, 3%.

16. 112, 2%.

17. 97, $12\frac{1}{2}\%$.

18. 89, $3\frac{1}{3}\%$.

19. 62, $33\frac{1}{3}\%$.

20. 156, $37\frac{1}{2}\%$.

21. 125, $62\frac{1}{2}\%$.

22. 147, $87\frac{1}{2}\%$.

23. 42.06, $66\frac{2}{3}\%$.

24. 12.48, $16\frac{2}{3}\%$.

25. 57.8, 34%.

26. 90.2, 110%.

27. 65.72, 106%.

28. 622.6, 5.5%.

29. Of what number is 287 equal to 5%?

30. $3.15 is 15% of what amount? 20% of what amount?

31. $6.24 is 16% of what amount? 10% of what amount?

32. 12 ft. 5.6 in. is 22% of what length? 5% of what length?

33. 63 lb. 7 oz. is 35% of what weight? 2% of what weight?

34. If a farmer sold a horse for 80% of its cost and received $152, how much did the horse cost?

35. A poultry raiser set some eggs, and 15% failed to hatch. The number failing to hatch being 33, how many eggs did the poultry raiser set?

36. A man sells some lumber for $180, thereby gaining 20% on the cost. What per cent of the cost is the selling price? How much is the cost?

37. A man saves $675.20 a year, which is 32% of his income. How much is his income?

38. A school has 20% of its pupils in the sixth grade, which numbers 29. How many pupils are there in the school?

From the examples on pages 336 and 337 it will be seen that the third important problem of percentage is not nearly as practical as the first and second problems mentioned in § 240.

ORAL REVIEW

1. If a man lends \$250 and receives \$10 interest for a year, what is the rate?

2. If a man borrows \$300 and pays \$18 interest for a year, what is the rate?

3. If a salesman receives a commission of \$15 on a sale amounting to \$750, what is his rate of commission?

4. If a dairyman keeps 60 cattle this year and kept 48 last year, what is the increase per cent?

5. If the taxes of a certain village were \$8000 last year and \$7000 this year, what is the decrease per cent?

6. If the rainfall in a certain place was 7.2 in. in April and 6 in. in March, what was the per cent of increase?

7. If the seventh grade devotes 45 min. a day to arithmetic and the sixth grade 40 min., what is the per cent of increase?

8. If a dealer sold daily 150 qt. of milk last month and 180 qt. this month, what is the rate of increase?

9. If you have read 75 pages in a book of 300 pages, what per cent have you read?

10. If the last chapter of a book is numbered XXXV and you have finished reading Chapter XXVI, what per cent of the chapters have you read?

11. If you have read 60 pages of a book, and this is 25% of the book, how many pages are there in all? How many pages have you still to read?

12. A cork is pushed below the surface of a glass full of water, and the water that runs over is weighed. The cork is found to weigh 1.2 oz. and the water to weigh 5 oz. Cork is what per cent as heavy as an equal volume of water?

244. Miscellaneous Problems. We are now prepared to consider miscellaneous problems, and here you will find it necessary to decide upon the method of solution. You will find it best to ask yourself exactly what is given, and then to consider carefully whether you have to find a required per cent of a number (two factors given), or whether you have a product and one factor given, to find the other factor.

It will be noticed that only two principles are involved in the following exercise. The examples after Ex. 21 involve a variation of the principle mentioned on page 332, and may be omitted if desired. They are inserted for the use of those who may wish to carry the work into more practical fields.

WRITTEN EXERCISE

1. A cubic foot of water weighs $62\frac{1}{2}$ lb. From a tank containing 16 cu. ft. there is drawn off an amount of water that weighs 125 lb. What per cent of the water is drawn off?.

2. To a class of 32 pupils 8 more are admitted. What per cent of the original number are admitted? What per cent of the present number are admitted? If 10 pupils now leave the class, what per cent leave the class?

3. In an arithmetic class it is planned to solve 550 problems in a certain number of days. All but 44 problems have been solved. What per cent of the problems remain to be solved? What per cent of them have been solved?

4. If 45% of the 240 pupils that attend a certain school are boys, how many are girls?

5. A man purchased 530 head of cattle and sold 30% of them. How many head of cattle had he left?

6. If an automobile going at the rate of 14 mi. an hour increases its speed 37%, at what rate will it then be traveling?

7. A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than gold?

8. A bank averaged $1200 a day of new deposits in November, and 26% more a day in December. What were the average daily deposits in December?

9. A clerk's salary is $12 a week. If he spends 20% of it for rent, 10% for clothes, and 25% for other expenses, and saves the rest, how much does he save in a year of 50 weeks?

10. If a baker uses 639 lb. of flour in making bread and adds to the flour 213 lb. of liquid, the weight of the liquid is what per cent of the weight of the mixture?

11. If $3\frac{1}{2}$ teaspoons of sugar make a tablespoon, a teaspoon of sugar is what per cent of a tablespoon?

12. If there are 16 teaspoons to a cup, a teaspoon is what per cent of a cup?

13. A teaspoon of butter is $\frac{1}{4}$ of a tablespoon. Express a teaspoon of butter as per cent of a tablespoon.

14. The record of the champion team in baseball in a certain year was 52 games won, 14 lost. What per cent of the games did the champion team win that year?

15. One of the best records in baseball of recent years was when Brooklyn won the pennant with a total of 82 games won and 54 lost. What was the per cent of games won?

16. Two of the best averages for runs in baseball were 597 times at bat, 139 runs, and 543 times at bat, 145 runs. What was the per cent of runs in each case?

17. If the expenses of a certain village were $8250 last year and $8002.50 this year, what is the per cent of decrease?

18. If the rainfall in a certain place was 7.2 in. in April and $6\frac{1}{4}$ in. in March, what was the per cent of increase?

19. In 1801, of the 161,800,000 people who spoke European languages, 31,450,000 spoke French. Of the 585,000,000, in 1911, those who spoke French numbered 70,000,000. What per cent spoke French in each of these years?

20. A dressmaker bought a silk dress pattern for $45 less 20%. She sold it for $45. What per cent did she gain?

Notice that the gain is reckoned on the price she paid for the pattern.

21. Allowing 8 oz. of Paris green to 50 gal. of water in preparing a spray to destroy leaf-eating insects, and considering a gallon of water to weigh 8.4 lb., the weight of the Paris green is what per cent of the weight of the water?

22. A dealer sold pencils at 5¢ each, which gave him a profit of 25%. How much did they cost him a dozen?

23. A farmer bought some sheep and sold them for $252, which gave him a profit of 20%. How much did they cost him?

24. A dealer sells some goods for $31.05, thus making a profit of 15% on the cost. How much did the goods cost him?

25. A manufacturer sells a suit of clothes to a merchant at a profit of $12\frac{1}{2}$%. The merchant sells it to you for $12 and makes a profit of $33\frac{1}{3}$%. How much does the suit cost the merchant? How much does the suit cost the manufacturer?

It should be noticed that $133\frac{1}{3}$% of the cost to the merchant is $12.

26. An agent bought 100 doz. eggs, which he sold to a grocer at a profit of 20%. The grocer sold the eggs at retail for $22.44 and made a profit of 10%. How much did the eggs cost the grocer? How much did the eggs cost the agent?

It should be noticed that 110% of the cost to the grocer is $22.44.

27. A certain number when increased by 30% of itself becomes 650. What is the number?

Since once the number and 30% of the number are 130% of the number, 130% of the number is 650. Then how is the number found?

28. A certain number when decreased by 20% of itself becomes 400. What is the number?

Since once the number less 20% of the number is 80% of the number, 80% of the number is 400. Then how is the number found?

The following numbers are the given per cents more than what numbers, and also the given per cents less than what other numbers?

29. 144, 20%.
30. 150, 25%.
31. 91, 30%.
32. 99, $12\frac{1}{2}$%.
33. 350, $16\frac{2}{3}$%.
34. 550, $37\frac{1}{2}$%.
35. 7.28, $33\frac{1}{3}$%.
36. 15.84, 50%.
37. 390, $62\frac{1}{2}$%.
38. 49.5, $87\frac{1}{2}$%.
39. 36.9, 80%.
40. 27.3, $16\frac{2}{3}$%.

41. If an army has been increased 21% and now numbers 73,810, how many did it number before the increase?

42. A man sold some goods for $192.50, making a profit of 10%. How much did the goods cost him?

43. A dealer sold a horse for $181.50, making a profit of 10%. How much did the horse cost him?

44. A village has a population of 1853, an increase of 9% over ten years ago. What was the population ten years ago?

45. A village has a population of 2664, an increase of 11% over ten years ago. What was the population ten years ago?

46. If you weigh 78.1 lb. and have increased 42% in weight in the last five years, how much did you weigh five years ago?

47. If a library has increased in size $16\frac{2}{3}$% in the past five years and now has 8323 volumes, how many volumes had it before the increase?

245. Discount. If you have a catcher's mask that you have used during the season and wish to dispose of it so you can use the money toward buying a football, you will probably be glad to sell it for 25% less than it cost you. In that case you would sell the mask at a *discount* of 25%. You might speak of this as "a quarter off," or as "75¢ on a dollar," but the operation of finding the selling price would be to deduct 25%, or $\frac{1}{4}$, of the cost.

The price of goods as given in a printed list is called the *list price.* Manufacturers usually have price lists of this kind. In stores the goods are marked, and the marked price is what is given in the dealer's price list if he has one.

A reduction from the list price of an article, from the amount of a bill of goods, or from the amount of a debt is called *commercial discount.*

A regular reduction from the list price is called *trade discount.*

A reduction for payment within a fixed time is called a *time discount.*

A reduction for payment when the goods are delivered is called a *cash discount.*

Discounts are usually reckoned at some common fraction of the list price, or at some rate per cent of it.

The amount of the bill after the discount has been made is called the *net amount,* or the *net price.*

Read and study the solution of this example:

Find the net amount of a bill of $275 with a discount of 20%.

The amount of the bill = $275
20%, or $\frac{1}{5}$, of $275 = 55
The net amount = $220

Instead of speaking of "20% off," merchants often speak of "$\frac{1}{5}$ off." So they speak of "$\frac{1}{4}$ off," and "$\frac{1}{3}$ off," but it is not so convenient to use common fractions for such discounts as 12%.

ORAL EXERCISE

1. If some goods are marked $20, and 10% discount is allowed, what is the selling price?

2. If a dealer offers me an 80¢ book at a discount of 25%, this is how many cents below the regular price?

3. If a merchant buys $800 worth of goods and is allowed 10% discount in case he pays for them at once, how much does he gain by prompt payment?

Find the discounts on the following amounts at the rates specified:

4. $80, 10%.
5. $25, 10%.
6. $50, 20%.
7. $25, 20%.
8. $40, 25%.
9. $88, 25%.
10. $60, 50%.
11. $90, 50%.
12. $120, 10%.
13. $150, 20%.
14. $160, 25%.
15. $250, 50%.

WRITTEN EXERCISE

1. If goods marked $475 are sold to a dealer at a discount of 20%, how much do they cost him?

2. If a merchant marks a lot of suits at $24.75 each, and sells them at $\frac{1}{3}$ off, what is the net price of each?

Pupils should be asked to watch advertisements in the newspapers to see what discounts are offered. They should state the probable reasons for these discounts, and should bring to class the problems which they make.

Given the marked prices and rates of discount, find the net prices:

3. $17.50, 10%.
4. $16.50, 20%.
5. $27.75, 20%.
6. $64.40, 25%.
7. $86.60, 25%.
8. $48.60, 25%.
9. $72.30, $33\frac{1}{3}$%.
10. $38.70, $33\frac{1}{3}$%.
11. $43.50, $16\frac{2}{3}$%.
12. $86.40, $16\frac{2}{3}$%.
13. $73.60, $12\frac{1}{2}$%.
14. $27.70, 50%.
15. $35.00, 10%.
16. $34.75, 20%.
17. $65.25, 20%.

246. Several Discounts. In some kinds of business two or more discounts are allowed. For example, a dealer may buy $200 worth of hardware with discounts of 20%, 10% (20% and 10%). This means that 20% is first deducted from the list price, and then 10% is deducted from the remainder.

The list price is $200.
The list price less 20% is $160.
The $160 less 10% is $144, the *net price.*

ORAL EXERCISE

1. From $50 take 10%, and 10% from the remainder.

2. From $200 take 20%, and 1% from the remainder.

3. From $100 take 25%, and 10% from the remainder.

4. From $500 take 20%, and 25% from the remainder.

5. A man bought some paper at a list price of $80, with discounts of 25%, 10%. How much did he pay?

WRITTEN EXERCISE

Find the net prices of goods marked and discounted as follows:

1. $200, 30%, 20%.
2. $700, 35%, 5%.
3. $500, 25%, 15%.
4. $500, 12%, 2%.
5. $650, 10%, 10%.
6. $400, 30%, 10%.
7. $300, 15%, 10%.
8. $275, 10%, 10%.
9. $240, $12\frac{1}{2}$%, $33\frac{1}{3}$%.
10. $800, 25%, 25%.

11. What is the difference between a discount of 50% on $800, and the two discounts of 25%, 25%?

12. Is there any difference between a discount of 10%, 5%, and one of 5%, 10%, on $900? How is it on $400? on $200?

247. Sample Price List. The following is a price list of certain school supplies, with the discounts allowed to schools and dealers when the prices are not net:

Rulers,	$0.35 per doz., net
Composition books,	4.50 per gross, less 5%
Thumb tacks,	0.40 per 100, less 40%
Drawing pencils,	4.71 per gross, less 20%
Drawing paper, 9 × 12	1.10 per package, less $\frac{1}{10}$
Pens,	0.61 per gross, less $\frac{1}{4}$, 10%
Tubes of paste,	4.05 per gross, less 10%, 5%
Penholders,	3.20 per gross, less 12%, 5%
Drawing compasses,	1.65 per doz., less 10%, 5%

WRITTEN EXERCISE

1. How much must a school pay for $\frac{1}{2}$ gross of penholders?

2. A school wishes to buy 5 packages of drawing paper and 300 thumb tacks. How much will they cost?

3. How much will 8 gross of pens and $\frac{1}{2}$ gross of composition books cost? 2 gross of pens and 1 doz. rulers?

4. There are 40 pupils in a class, and each needs drawing compasses and a ruler. How much will they cost the school? How much would they cost if there were 60 pupils?

5. If a dealer sells pens at a cent apiece, how much does he gain per gross? If he sells penholders at 2¢ each, how much does he gain per gross?

6. A dealer buys a gross of rulers and a gross of drawing pencils. He sells both at 5¢ each. How much does he gain in all?

7. If a dealer buys a gross of tubes of paste for mounting pictures and sells the tubes at 5¢ each, how much does he gain on the purchase? how much on 3 gross?

248. Bill with Several Discounts. The following is a common form of a bill of goods with several discounts:

Burlington, Ia., May 13, 19...........

Mr. Geo. S. Miller, Cedar Rapids, Ia.

Bought of ROBERTS & COMPANY, Jewelers
1072 Passaic Avenue

Terms: *20%, 10%*

Feb.	*10*	*8 doz. spoons @ $14.75*	*118*	*00*		
		7 doz. plated forks @ $4.20	*29*	*40*		
					147	*40*
		Less 20%, 10%			*41*	*27*
					106	*13*

WRITTEN EXERCISE

Make out bills for the following:

1. 24 doz. files at $6.30. Discounts 30%, 20%.

2. 840 yd. silk at $1.50; 640 yd. velvet at $1.75. Discounts 25%, 10%.

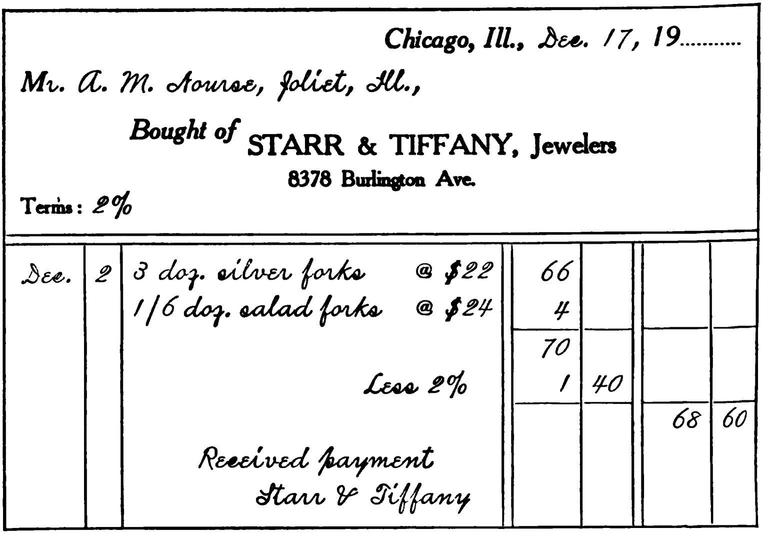
3. 8 doz. pairs hinges at $4.70; 36 doz. table knives at $8.60. Discounts 20%, 10%.

4. 24 doz. locks at $4.30; 8 doz. mortise locks at $4.75. Discounts 25%, 8%.

5. 460 yd. taffeta at 98¢; 8 gross pompons at $144; 4 doz. pieces braid at $21.60. Discounts 10%, 5%.

6. 680 yd. silk at $1.65; 840 yd. lawn at 23¢; 460 yd. taffeta at 96¢. Discounts 10%, 5%, 5%.

249. Receipted Bills. The following is a receipted bill for some goods purchased by a retail merchant from a wholesale dealer:



Chicago, Ill., Dec. 17, 19..........

Mr. A. M. Nourse, Joliet, Ill.,

Bought of STARR & TIFFANY, Jewelers
8378 Burlington Ave.

Terms: 2%

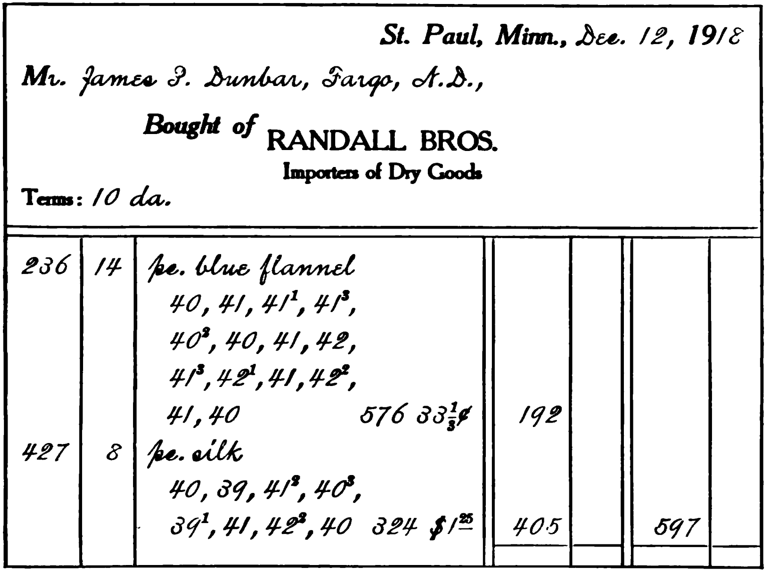
Dec.	2	3 doz. silver forks @ $22	66			
		1/6 doz. salad forks @ $24	4			
			70			
		Less 2%	1	40		
					68	60
		Received payment Starr & Tiffany				

In this case Mr. Nourse is the *debtor* (Dr.), since he is in debt for the amount. Starr & Tiffany are the *creditors* (Cr.), since they trust Mr. Nourse, or give him credit.

A receipt may also be written separately instead of appearing on a bill. Such a receipt should be dated, and should be in substantially this form: "Received from —— the sum of —— dollars for ——." The receipt should be signed by the creditor.

The subject of commercial discount is of great value because of its extensive use, not only in wholesale transactions but even in bargain sales. Pupils should understand some of the reasons for allowing discounts, for buying in large quantities, and for paying cash down or within a specified time, and they should become familiar with the ordinary deductions from list prices. It is well, for obvious reasons, to consider bills and receipts in connection with the study of this topic.

250. Invoice. A bill giving in detail a list of items and prices of goods sold is called an *invoice.* A sample invoice follows:



St. Paul, Minn., Dec. 12, 1918

Mr. James F. Dunbar, Fargo, N.D.,

Bought of RANDALL BROS.
Importers of Dry Goods

Terms: 10 da.

236	14	pc. blue flannel $40, 41, 41^1, 41^3,$ $40^2, 40, 41, 42,$ $41^3, 42^1, 41, 42^2,$ $41, 40$ 576 $33\frac{1}{3}$¢	192			
427	8	pc. silk $40, 39, 41^2, 40^3,$ $39^1, 41, 42^2, 40$ 324 $\$1^{\underline{25}}$	405		597	

In the above invoice, 41^1 means $41\frac{1}{4}$; 41^2 means $41\frac{2}{4}$, or $41\frac{1}{2}$; and 41^3 means $41\frac{3}{4}$. This is the customary way of indicating the number of yards in pieces of goods. The numbers 236 and 427 refer to the price list in which the goods are described. The 14 and 8 indicate the number of pieces (pc.). The number of yards of the first is 576, and of the second 324.

The expression "Terms: 10 da." means that Mr. Dunbar has 10 da. in which to pay for the goods. Such an invoice may or may not mention the discount allowed.

There is no essential difference between a bill and an invoice. The latter term is commonly used in wholesale transactions.

WRITTEN EXERCISE

Make out bills, or invoices, for the following:

1. 12½ doz. shirts @ $14.80, ⅔ doz. ties @ $9.75.

2. 16 pc. ribbon @ $1.25, 18 pc. @ $1.33⅓, 33 pc. @ $1.66⅔, 24 pc. @ $1.12½, 32 pc. @ $1.37½, 48 pc. @ $1.75.

3. 2 carloads coal, 21,700 lb., 24,200 lb., @ $4.66⅔ per ton; 3 carloads coal, 22,700 lb., 21,900 lb., 20,400 lb., @ $4.75 per ton. Terms: 3% for cash.

4. 1 gro. cans sardines @ $3.33⅓ per doz., 8 doz. cans shrimps @ $1.75 per doz., 9 doz. tins herrings @ $2.33⅓ per doz., 6¼ doz. cans lobster @ $2.88 per doz., 32 cans mackerel @ 16⅔¢ each. Terms: 4% for cash.

5. 8 doz. packages codfish @ $1.75 per doz., 9 doz. cans salmon @ $2.33⅓ per doz., 15 doz. cans caviar @ $3.33⅓ per doz., 18 doz. cans Yarmouth bloaters @ $2.33⅓ per doz., 24 doz. cans tongue @ $8.75 per doz. Terms: 3½% for cash.

6. 8 doz. jars meat extract @ $3.66⅔ per doz., 16 doz. cans chicken @ $3.12½ per doz., 24 doz. cans beef @ $2.33⅓ per doz., 18 doz. cans soup @ $3.33⅓ per doz., 9 doz. cans clam chowder @ $3.66⅔ per doz., 16 doz. cans clam juice @ $1.06¼ per. doz. Terms: 3%, 2%.

7. 5 armchairs @ $6.40 each, 37 kitchen chairs @ $1.12½ each, 8 kitchen tables @ $2.25 each, 3 bedroom sets @ $33.33⅓ per set, 16 dining tables @ $12.50 each, 9 sideboards @ $16.66⅔ each, 16 rockers @ $4.87½ each. Terms: 8%, 4%.

8. 20 pc. linen containing 39, 40^2, 38^3, 42, 41^1, 40^2, 40, 39, 40^1, 41^2, 43, 39^3, 40^1, 42^1, 40^2, 40^1, 41^3, 42, 39^1, 40 yd., @ 66⅔¢; 16 pc. silk containing 41^1, 42^1, 40, 39^2, 42, 40^3, 38, 41^1, 43, 41^3, 39, 40^2, 40^1, 41^1, 40, 39^2 yd., @ \$1.33⅓. Terms: 4%, 3%.

251. Interest. We have already learned that men often have to borrow money to carry on their business. For example, a merchant wishes to buy a lot of holiday goods. He feels sure that he can sell these for much more than he pays for them. It is therefore good business for him to borrow the money in the early part of the winter, buy the goods, sell them during the holidays, and pay back the money in January. But the bank from which he borrows should not only be paid the money borrowed, but be paid for the use of the money.

This payment for the use of the money is called *interest.*

If the merchant borrows $1000 and pays at the rate of 6% a year, he would pay $60 a year, or $5 a month.

ORAL EXERCISE

Find the interest on the given sums for 1 yr. at the given rates:

1. $100, 4%.
2. $100, 3%.
3. $300, 3%.
4. $250, 3%.
5. $250, 4%.
6. $600, 4%.
7. $400, 5%.
8. $600, 5%.
9. $800, 5%.
10. $400, 6%.
11. $700, 6%.
12. $900, 6%.
13. $100, 2½%.
14. $400, 2½%.
15. $400, 4½%.

16. What is the interest for 1 yr. on $200 at 6%? at 5%? at 4%? at 4½%? at 2%? at 2½%?

17. What is the interest for 1 yr. on $400 at 3%? on $300 at 4%? on $600 at 2%? on $200 at 6%?

18. What is the interest for 1 yr. on $2000 at 5%? at 6%? at 4%? at 4½%? at 3%? at 3½%? at 2½%?

19. What is the interest on $1000 at 6% for 1 yr.? for ½ yr.? for 6 mo.? for ¼ yr.? for 3 mo.? for ⅓ yr.? for 4 mo.?

20. What is the interest on $4000 at 6% for 1 yr. 6 mo.?

WRITTEN EXERCISE

Find the interest on the following at 6%:

1. $250 for 1 yr. 2 mo.

2. $375 for 1 yr. 3 mo.

3. $450 for 1 yr. 8 mo.

4. $575 for 1 yr. 6 mo.

5. $475 for 2 yr. 1 mo.

6. $650 for 2 yr. 6 mo.

7. $850 for 3 yr. 3 mo.

8. $750 for 3 yr. 6 mo.

9. A man borrowed $8000 in Boston at $4\frac{1}{2}\%$, and loaned it in Arkansas at 8%. How much did he gain in 3 yr.?

10. The day a certain boy is 14 yr. old his father invests $300 for him at 5%, the $300 and interest being due the day he is 21 yr. old. What is the total amount due at that time?

11. A real estate dealer buys 360 acres of farm land at $30 an acre, and after keeping it 7 mo. sells it at an advance of $3.75 an acre. The money being worth to him 5% a year, how much does he gain by the transaction?

12. A man having $26,750 invested in business has found that his profits average 18% a year. He is offered $35,000 for the business and he could invest the money at $4\frac{1}{4}\%$. If he sells out and retires, what is his annual loss in income?

13. In April a coal dealer borrowed $33,250 at 5%. With this he purchased his summer's supply of coal at $4.75 a ton. He sold the coal at $5.65 a ton, the buyers paying for the unloading and delivery, and he paid his debt in November after keeping the money 6 mo. How much did he gain?

14. I own a house which I rent at $25 a month. My taxes are $50 a year, my repairs $50, and my insurance $10. Not considering any change in value, would it be better for me to sell it for $5000 and invest the money at 4%, if my taxes would then be reduced to $10? Find the difference, if any, in income.

252. Interest for Years, Months, and Days. Suppose a man borrows from another man \$400 on September 10, 1916, at 6%. What will the interest amount to August 7, 1919? What would the interest amount to August 7, 1917?

yr.	mo.	da.	
1919	8	7	= second date
1916	9	10	= first date
2	10	27	= difference in time

Since 2 yr. = 720 da., and 10 mo. = 300 da., 2 yr. 10 mo. 27 da. = 1047 da. We therefore have

$$\frac{1047}{360}\text{ of }6\%\text{ of }\$400=\frac{\overset{349}{\cancel{1047}}\times\cancel{6}\times\$\overset{\cancel{4}}{\cancel{400}}}{\underset{\underset{\underset{5}{\cancel{15}}}{\cancel{90}}}{\cancel{360}}\times\cancel{100}}=\$\tfrac{349}{5}=\$69.80.$$

Hence the interest due August 7, 1919, is \$69.80.

To find the amount of interest August 7, 1917, we have

$$\frac{327}{360}\text{ of }6\%\text{ of }\$400=\frac{\overset{109}{\cancel{327}}\times\cancel{6}\times\$\overset{\cancel{4}}{\cancel{400}}}{\underset{\underset{\underset{5}{\cancel{15}}}{\cancel{90}}}{\cancel{360}}\times\cancel{100}}=\$\tfrac{109}{5}=\$21.80.$$

Hence the interest due August 7, 1917, is \$21.80.

WRITTEN EXERCISE

Find the interest on the following:

1. \$325 for 1 yr. 8 mo. 5 da. at 5%.
2. \$300 for 1 yr. 4 mo. 15 da. at 6%; at 5%.
3. \$150 for 1 yr. 3 mo. 20 da. at 5%; at $5\frac{1}{2}$%.
4. \$475 for 3 yr. 6 mo. 15 da. at 6%; at 5%; at 4%.
5. \$850 for 2 yr. 7 mo. 6 da. at 4%; at $4\frac{1}{2}$%; at $5\frac{1}{2}$%.
6. \$75, at 4%, from June 9, 1916, to June 9, 1918.

Find the interest on the following:

7. $172.60 for 3 yr. 2 mo. 10 da. at 4%.
8. $391.75 for 3 yr. 8 mo. 10 da. at 4%.
9. $235.50 for 3 yr. 1 mo. 15 da. at 6%.
10. $175.50 for 2 yr. 8 mo. 10 da. at 6%.
11. $142.80 for 1 yr. 3 mo. 15 da. at 5%.
12. $273.40 for 2 yr. 8 mo. 20 da. at 5%.
13. $172.60 for 2 yr. 5 mo. 6 da. at 4½%.
14. $182.60 for 4 yr. 8 mo. 15 da. at 5¼%.
15. $163.40 for 3 yr. 9 mo. 15 da. at 5½%.
16. $625.50 for 2 yr. 11 mo. 15 da. at 5¾%.
17. $475.50 for 3 yr. 10 mo. 15 da. at 4¼%.
18. $265.50 for 2 yr. 10 mo. 20 da. at 4¾%.
19. $3562.50 for 3 yr. 2 mo. 5 da. at 5½%.
20. $730, from August 17 to April 17, at 4%.
21. $1460, from May 23 to March 23, at 4%.
22. $1533, from February 1 to December 16, at 5½%.
23. $7300, from March 3 to October 3, at 5½%.
24. $3650, from May 23 to January 23, at 4¼%.
25. $4250, from April 7 to October 7, at 6%.
26. $2500, from May 9 to October 24, at 6%.
27. $1750, from January 21 to November 21, at 6%.
28. $1800, from February 17 to August 17, at 6%.
29. $3300, from March 5 to November 5, at 6%.
30. $2250, from May 4 to December 4, at 6%.
31. $4800, from March 2 to December 2, at 6%.
32. $1500, from May 6 to November 6, at 5%.
33. $3400, from June 5 to December 5, at 5%.

Find the interest on $275 at 5%, and also at 6%, for:

34. 3 yr. 6 mo. 3 yr. 9 mo. 4 yr. 7 mo.
35. 3 yr. 6 mo. 10 da. 3 yr. 9 mo. 15 da. 4 yr. 7 mo. 20 da.

Find the interest at 5%, and also at 6%, for 2 yr. 5 mo. 10 da. on the following sums:

36. $200 $2000 $20 $250 $2500
37. $400 $4000 $40 $475 $4750

Find the interest on $450 for 1 yr. 8 mo. 15 da. at these rates:

38. 4% 5% 6% 3½% 4½% 5½%

39. A man borrowed $250 on January 15, at 6%. How much was the interest on October 15?

40. A man borrowed $350 on January 27, at 6%. He paid the principal and interest on February 27 of the following year. How much did he pay?

41. A man bought 12 head of cattle on February 1, at $45 a head, promising to pay for them later, with interest at 6%. He paid the bill on the 16th of the following April. What was the amount of principal and interest?

42. A man borrowed $450 on March 10, at 6%, and $1200 on April 10, at 5%. He paid the principal and interest in each case on December 30 of the same year. How much did he pay?

43. A man borrowed $650 on May 1, at 5%, and $1500 on July 5, at 4½%. He paid the principal and interest in each case on December 16 of the same year. How much did he pay?

44. What is the amount of principal and interest on $750 borrowed February 10, at 6%, and $1800 borrowed April 15, at 5%, the payment in both cases being made on September 20 of the same year?

253. Partial Payments. If a dealer holds a *note*, or a written promise to pay money, against one of his customers, and it bears interest, and partial payments are made, the amount due on the day of settlement is found in the following way:

Add the interest to the principal whenever the payment, or the sum of the payments, equals or exceeds the interest.

Then deduct the payment or payments and continue as before.

This is the United States Rule for partial payments, the legal one in most of our states. Where other rules are legal, teachers should explain the law and require that the problems be solved in accordance with that law.

For example, a note for $1000, at 5%, is dated January 1, 1916. The following payments are indorsed (written across the back, as is the custom): July 1, 1916, $10; January 1, 1917, $40; July 1, 1917, $20; January 1, 1918, $130. How much is due July 1, 1918?

The interest due July 1, 1916, is $25, and the payment is $10. Hence the payment cannot be deducted. The reason is easily seen, for if we should take $1025 – $10 = $1015 as the new principal, we should be drawing interest on more than the $1000.

First principal	$1000
Interest for 1 yr.	50
First amount, January 1, 1917	$1050
1st and 2d payments	50
Second principal, January 1, 1917	$1000
Interest for 1 yr.	50
Second amount, January 1, 1918	$1050
3d and 4th payments	150
Third principal	$900
Interest for 6 mo.	22.50
Due July 1, 1918	$922.50

Therefore the amount due on the day of settlement is $922.50.

WRITTEN EXERCISE

1. A note for $375, at 5%, has a payment of $18.75 indorsed annually at the close of each year from its date, for three years. How much is due at the end of the fourth year?

2. A note for $500, at 6%, is dated January 1, 1916, and has the following payments indorsed: July 2, 1916, $100; December 1, 1916, $2; January 1, 1917, $98. How much is due July 1, 1917?

3. A note for $300, at 6%, is dated April 1, 1916, and has the following payments indorsed: September 1, 1916, $50; February 16, 1917, $75; January 1, 1918, $98. How much is due July 1, 1918?

4. A note for $1800, at 5%, is dated April 16, 1916, and has the following payments indorsed: July 2, 1916, $500; October 1, 1916, $250; February 1, 1917, $100; May 1, 1917, $375. How much is due September 5, 1917?

5. A note for $750, at 4%, is dated June 15, 1916, and has the following payments indorsed: March 6, 1917, $200; September 6, 1917, $150; January 17, 1918, $250. How much is due May 14, 1918?

6. A note for $600, at 5%, is dated July 2, 1916, and has the following payments indorsed: January 2, 1917, $50; February 11, 1917, $75; July 8, 1917, $200; September 10, 1917, $80; November 2, 1917, $50. How much is due January 2, 1918?

Teachers should remember that the subject of partial payments has lost most of its practical value in the last few years. Money is now usually borrowed at a bank for 30 da., 60 da., or 90 da., and if a payment is made, this is done at the end of these periods and a new note is then given. The subject is not obsolete however, and it affords such excellent drill in computing interest that we have included it in these Essentials. In communities where the so-called Merchants' Rule is still used although commonly illegal, the teacher may find it desirable to mention it at least.

LITTLE EXAMINATIONS

I. **1.** Express 0.1% as a decimal fraction.

2. Express 1.2 as per cent.

3. Express 2½ qt. as per cent of a gallon.

4. Express an inch as per cent of a yard.

5. Express 62½%, 87½%, and 83⅓% as common fractions.

II. *Find values of the following:*

1. 50% of 725.

2. 75% of 62½.

3. 37½% of $7.36.

4. 33⅓% of $19.56.

5. 87½% of $19.28.

6. 0.3% of 174 ft.

7. 0.06% of 48,000.

8. 125% of 2756.

9. 1.25% of 2756.

10. 300% of 42⅞.

III. **1.** $30 is what per cent of $360? of $3600? of $36,000?

2. 8 in. is what per cent of 12 in.? of 12 ft.? of 12 yd.?

3. A pint is what per cent of 1 qt.? of 1 gal.? of 6 gal.?

4. 56 ft. is 8% of what distance? 4% of what distance?

5. 7 ft. 6 in. is 10% of what distance? 8% of what distance?

IV. **1.** $60.80 is 5% less than what amount?

2. $102 is 15% less than what amount?

3. Find the interest on $275 for 6 mo. 8 da. at 6%.

4. List price, $450; discount 8%; find the net amount.

5. A bill of goods amounting to $725 is allowed a discount of 15%. Find the net amount.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write in words the numbers 400.075 and 0.475.

2. Write in Roman numerals the numbers 79 and 64.

3. Write in common figures the number seventeen million.

Add, without writing in columns:

4. \$2.20 + \$3.50 + \$1.75 + \$6.75 + \$9.30 + \$15.

5. 6 ft. 4 in. + 3 ft. 7 in. + 5 ft. 8 in. + 9 ft. 3 in. + 8 ft. 4 in.

Subtract the following:

6. \$497.63 – \$289.98.

7. \$601.71 – \$324.85.

8. \$510.00 – \$273.21.

9. \$812.27 – \$533.89.

Multiply or divide as indicated, carrying the quotients to four figures only:

10. 382 × \$296.50.

11. 427 × \$837.75.

12. $428.3 \div 29.4$.

13. $59.68 \div 3.74$.

14. 2.73×42.6.

15. $74.6 \div 2.13$.

Find the results of the following:

16. 16% of \$325.

17. $5\frac{1}{4}\%$ of \$650.

18. $3\frac{1}{2}\%$ of \$785.

Find the values of x in the following:

19. $x : 5 = 7 : 75$.

20. $5 : x = 75 : 7$.

21. $3 : 7\frac{1}{2} = x : 60$.

22. $4 : 9\frac{1}{4} = 16 : x$.

23. $1 : x = x : 1$.

24. $1 : x = x : 4$.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you buy a baseball bat and sell it at a certain rate below cost, how do you find the selling price?

2. If you buy a dog and sell him at a certain rate above cost, and know how much it has cost you to keep him while you owned him, how do you find whether you gained or lost, and the amount?

3. If you know the number of times a baseball player has been at bat during a season and the number of hits he has made, how do you find the percentage of hits?

4. If you know the number of times a player has been at bat during a season and the percentage of runs he has made, how do you find the number of runs?

5. If you know the number of pupils in a school and the number in Grade VII, how do you find the per cent that are in Grade VII?

6. If you know a man's income and the amount which he pays for house rent, how do you find the per cent of his income that is paid for house rent?

7. If you know the list price of some goods and the several rates of discount, what are you then able to find, and how do you find it?

8. A man borrowed a certain sum of money at a certain rate and for a certain length of time. The money was not paid when due, but some payments were made from time to time, each greater than the interest then due. Finally the money was paid. How do you find the amount due at the time of payment?

III. INDUSTRIAL PROBLEMS

BAKING BREAD

1. Harriet wishes to earn some money. She has learned to bake bread, and her mother suggests that Harriet should make their bread and receive what is saved. If flour is worth $6 per barrel (196 lb.) and $\frac{2}{3}$ lb. of flour makes a loaf of bread, and we allow $4.29 per barrel of flour for the cost of the other ingredients and of fuel, what will it cost per loaf to make the bread?

2. The family uses 3 loaves a day, and the baker's price is 5¢ a loaf. If Harriet bakes the bread, how much is saved every day, and how much does she earn in a year?

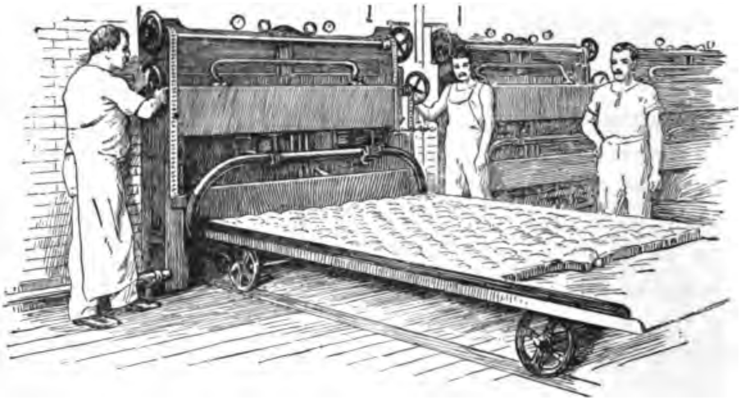
3. After a few weeks Harriet gets so that she makes much better bread than the baker, and Mrs. Cook, their neighbor, wishes to buy 12 loaves a week. The bread is so good that she is willing to pay 90¢ a week for the 12 loaves. How much are Harriet's profits per year on these sales?

4. Another neighbor wishes to buy 2 loaves a day, at the same rate per loaf that Mrs. Cook pays. If Harriet agrees to this, what is her yearly income from this source? What is now her total yearly income from baking bread?

5. Harriet wants to earn $2200 to pay her college expenses for four years when she is old enough to go, six years from now. How many loaves will she have to bake to save this amount?

6. Her father adds enough to her savings to put $300 in the bank at the end of each of the six years. If this money draws 4% interest, how much will she have at the end of 6 yr.?

7. If flour goes up to $6.25 a barrel, or $3\frac{1}{2}$¢ a pound if bought by the pound, how much will Harriet save on 196 lb. of flour if she buys it by the barrel?



8. This picture shows the interior of a large bakery. The bakery averages 9000 loaves of bread a day, six days a week, and sells its bread to dealers at 4¢ a loaf. The dealers, on an average, sell 93% of the bread they get from the bakery, at 5¢ a loaf, and send back the balance. How much do the dealers receive for the bread they sell?

9. Of the bread sent back, the bakery is able to sell 60% at an average price of 2¢ a loaf, the rest being ground up and sold for $2.50 for the lot. What are the average weekly receipts of the bakery from the bread? How much is this per loaf baked?

10. The bakery spends every week $1250 for flour, and $240 for various other materials; it pays its workmen $140; and the other charges are $25. What is the average cost per loaf of bread?

11. If the baker could sell all his bread at 4¢ a loaf, what would be his per cent of profit?

12. In making bread the baker uses $\frac{1}{3}$ as much liquid as flour. If he allows 1 pt. of liquid to a pound of flour, how many pints of liquid will he use with 20 bbl. of flour.

CHAPTER X

INDUSTRIAL APPLICATIONS

254. Industrial Applications of Arithmetic. You have now studied the most important operations of arithmetic, and are able to do any of the computation that you will meet in the ordinary problems of daily life. These important operations are the writing of numbers, addition, subtraction, multiplication, and division. These operations you have studied with whole numbers, with common fractions including ratios, with decimal fractions including per cents, and with compound numbers involving various measures.

You have also applied these operations to many problems, so that you know how each operation is used in a practical way. But up to this time you knew what operations were needed in most of the problems that you had to solve, because these problems came in chapters on certain subjects, such as addition, division, common fractions, or decimal fractions.

You are now ready to consider the simpler applications of arithmetic and to apply to each problem whatever operation may be needed for its solution. The leading applications are industrial; commercial, including banking, insurance, and a study of corporations; civic, including taxes and the expenses of the government; and scientific, including more difficult measurements. Problems in these various lines will be considered in the subsequent pages, particularly such problems as relate to the lives and interests of the people generally.

WOODWORKING OUTFIT

This price list may be used in solving the problems below:

School workbench .	$8.50	Plane	$1.70
Chisel, $\frac{1}{4}''$	.20	Vise for woodwork .	8.25
Chisel, $\frac{1}{2}''$	.25	Miter square . . .	.65
Try-square	.25	Steel square . . .	.75

1. What is the amount of a 10% discount on a workbench?

2. What is the cost of 5 quarter-inch chisels?

3. Find the cost of 3 half-inch chisels and a plane.

4. Find the cost of a vise, allowing 8% discount.

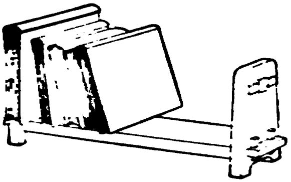
5. Find the cost of 2 miter squares and 3 steel squares.

6. Find the cost of 6 try-squares, allowing 6% discount.

7. A school buys 4 workbenches, 3 planes, and a vise for woodwork, and is allowed 12% discount. Find the net price.

INDUSTRY IN THE SCHOOL

1. The class made this bookrack. If the rack will hold 12 books 1½ in. thick, it will hold how many books ¾ in. thick?



2. The inside length of the shelf is 18 in., and each end piece is 1¼ in. thick, and the shelf projects 1¼ in. at each end. What is the total length?

3. The end pieces are 6½ in. high. The shelf is 4½ in. wide, and the greatest width of the end pieces is 5⅞ in. Taking the total length of the shelf as found in Ex. 2, what length of board 6 in. wide will be needed for the entire shelf, allowing ⅜ in. for waste in sawing and dressing?

4. The price of inch boards of this kind is $80 per M. This means, as we have already learned, that 1000 sq. ft. of boards 1 in. thick costs $80. At this rate, how much will boards 1¼ in. thick cost per M?

5. If a shelf requires a board 32 in. long and 6 in. wide, how many square feet will such a board cover?

6. If a shelf requires a board 32 in. long and 6 in. wide, what will be the cost of the lumber at $100 per M, if the board is 1 in. thick? if it is 1¼ in. thick?

7. If a shelf is 4½ in. wide and 18 in. long, the width is what per cent of the length?

8. If an end piece is 6½ in. high and 5⅞ in. wide, the width is what per cent of the height?

9. If an end piece is 1¼ in. thick and 6½ in. high, the thickness is what per cent of the height? The height is what per cent of the thickness?

HOUSEHOLD INDUSTRY

1. These girls are making 12 cups of cocoa in a saucepan. How large a pan must they use, estimating 2 cups to a pint and allowing an extra quart in the capacity of the pan?

2. If 12 cups of flour and 4 cups of liquid are used in making some bread, the liquid is what per cent of the mixture? If it should be a quarter of the mixture, is this mixture right?

3. If 1 cup of rice is cooked in 4 qt. of water and we allow 2 cups to a pint, the rice is what per cent of the total amount?

4. French salad dressing may be made with 6 tablespoons of oil and half as much vinegar, or a third as much vinegar. In each case the vinegar is what per cent of the dressing?

5. How large a kettle is needed for cooking 1 cup of rice in 4 qt. of boiling water, allowing 2 cups to a pint, and allowing $1\frac{1}{2}$ pt. extra in the capacity of the kettle?

6. In making ice cream the freezer should not be more than 75% full before freezing. How large a freezer will you need for $1\frac{1}{2}$ gal. of the liquid?

HOUSEHOLD ECONOMICS

1. If a woman can buy corn at 12¢ a can or $1.30 per dozen, how much does she save on 24 cans in buying by the dozen?

2. If a woman can buy soup at 16¢ a can or $1.80 per dozen, how much does she save on 36 cans in buying by the dozen?

3. A dealer sells coffee in half-pound packages at 17¢ a package or in 5-pound cans at $1.64 a can. If a woman wishes 5 lb., how much does she save in purchasing by the can?

4. A woman can buy Dutch cocoa in $\frac{1}{4}$-pound boxes at 22¢ a box or in 5-pound cans at $2.78 a can. If she wishes 5 lb., how much does she save in purchasing by the can?

5. A woman can buy maple sirup at 44¢ a quart or in gallon cans at $1.38 a can. If she wishes 4 qt., how much does she save in purchasing by the can?

6. A woman buys a pound each of mustard seed, allspice, and cayenne pepper, paying 60¢ for the three. The mustard seed costs 12¢ a pound, and the allspice costs the same as the pepper. How much does the allspice cost?

7. A woman bought a can each of cinnamon, nutmeg, and pepper, paying 39¢ for the three. The pepper cost 9¢, and the nutmeg cost twice as much as the cinnamon. How much did the cinnamon cost?

8. If flour costs $6.70 a barrel (196 lb.) or 4¢ a pound, how much does a family save in purchasing flour by the barrel if it requires 196 lb.?

9. If a family's ice bill is $1.50 a month, and ice costs 30¢ per 100 lb., how many pounds does the family use? If, by having a better ice box, only 0.9 as much ice is used, how many pounds are used?

HELPING AT THE MARKET

1. Henry helps his father at the market. They have been buying grapefruit, 64 to a box, at $4.50 a box, and selling them at 10¢ each, averaging two boxes a day. Henry notices that another dealer sells two boxes a day of larger grapefruit, 48 to a box, at 15¢ each. They can buy the larger fruit at $5.50 a box. Henry thinks it would be better to deal in the larger fruit. Is he right? What is the difference in income?

2. They find that they can buy a box of 360 lemons for $5.20, selling them at 22¢ a dozen. How much do they gain on a box? What per cent do they gain on the money invested? Is this more profitable, in per cent on the investment, than to buy a box of 196 oranges for $5.30, selling them at 40¢ a dozen?

3. Henry's father pays $8 a month for the rent of his stall in the market, $3.25 a month for light, and $2.25 a day for help, besides allowing Henry 10¢ an hour. Henry works 12 hr. a week. There being 313 market days in a year, how much are the expenses for a year?

SEWING AT HOME

1. If a sheet is $2\frac{1}{2}$ yd. long and $5\frac{1}{2}$ in. is allowed for hemming the ends, how many sheets can be cut from 54 yd. of the right width? Will there be any material left over?

2. In buying sheeting, the width of the material is given in quarters of a yard. If sheeting is $\frac{5}{4}$ (5 quarters) wide, what is the width in yards and a fraction? in inches?

3. How many sheets for single beds can be made from a piece of sheeting $\frac{6}{4}$ wide and 64 yd. long, each sheet being $2\frac{1}{2}$ yd. long, with a top hem $3\frac{1}{2}$ in. and a bottom hem 2 in., an additional allowance of $\frac{1}{4}$ in. being made for each hem?

4. A piece of sheeting of $\frac{8}{4}$ width contains 56 yd. How many sheets for three-quarter beds can be made from it, allowing the same measurements as in Ex. 3?

5. A piece of sheeting of $\frac{10}{4}$ width contains 48 yd. How many sheets for double beds can be made from it, allowing the same measurements as in Ex. 3?

6. Dish toweling can be bought for $5 per roll of 50 yd., or at the rate of 12¢ a yard. What per cent is saved in purchasing 50 yd. by the roll instead of by the yard?

7. Allowing 27 in. for each towel, how many dish towels can be made from a piece of toweling 39 yd. long?

8. A housekeeper wishes to make two curtains for each of four windows. The windows are 6 ft. 9 in. high, and the curtains are to be the same length. She wishes to put a hem 3 in. wide at the bottom and another $1\frac{1}{2}$ in. wide at the top, allowing $\frac{1}{4}$ in. for finishing each hem. She can buy a 20-yard piece of curtain material for $3, losing what she does not need, or she can purchase exactly the amount she needs at 16¢ a yard. Which is the more economical plan, and how much more economical?

PROBLEMS OF THE DRESSMAKER

1. A dressmaker bought 32 yd. of chiffon velvet at $4.10 a yard. The dealer allowed her a discount of 8%, 3%. She sold the velvet to a customer at $4.25 a yard. How much did she gain?

2. A dressmaker bought 50 yd. of silk at 95¢ a yard, less a discount of 10%, 2%. She sold half of it at $1.20 a yard and the remainder at $1.10 a yard. How much did she gain?

3. If it takes 2½ yd. of thread to stitch 1 yd. of cloth, how many spools of 50 yd. each will it take to stitch 40 yd. of cloth?

4. At 3⅝ yd. to a skirt, how many yards of ruffling will be needed for a dozen skirts?

5. How many quarter-inch tucks can be made in a piece of cloth 6¾ in. long, allowing ¼ in. between the tucks and ⅜ in. at each end of the cloth?

6. If a skirt requires 4⅜ yd. of lawn, what is the cost of the material for two skirts, the lawn costing 23¢ a yard, and the extras costing 11¢ per skirt?

7. A dressmaker used for a shirt waist for one of her customers 2½ yd. of Persian lawn costing 28¢ a yard, and 3¾ yd. of embroidery costing 20¢ a yard. What fraction of the cost of the embroidery was the cost of the lawn? Express this as a common fraction and also as a decimal.

8. If 3½ yd. of material 32 in. wide are required for a garment, how much material 42 in. wide will be required, if it cuts as economically?

9. If material 32 in. wide costs $1.50 a yard, what will be the cost of material of the same quality 42 in. wide, at the same rate per square yard?

PROBLEMS OF THE FARMER

1. A farmer asks his son to measure a field for him. The boy finds the field to be 20 rd. by 80 rd. How many acres does it contain?

2. In measuring the field the boy finds that on an average a strip 2 ft. wide, next to the fence inclosing the field mentioned in Ex. 1, is not cultivated, and that 1 ft. 6 in. of it could be cultivated as easily as the rest of the field. How many square feet of land go to waste?

3. The boy has a garden 60 ft. by 100 ft. which he must water on account of the dry weather. If a cubic foot of damp clay loam contains on an average 18 lb. of water, what is the weight in tons of water in the garden to the depth of a foot, when the soil contains as much water as damp clay loam?

4. The farmer wishes to fence in the garden mentioned in Ex. 3, using four strands of barbed wire. How many pounds of barbed wire will he need, allowing 1 lb. of wire to a rod?

5. On the farm is an artesian well producing $3\frac{1}{4}$ cu. ft. of water per minute. If a cubic foot of water weighs $62\frac{1}{2}$ lb., how long will it take the well to produce half enough water required for the garden as found in Ex. 3?

6. A farm wagon costing $75 depreciates in value about $6\frac{2}{3}$% a year if handled with reasonable care and kept in a shed when not in use. If left out in the yard it depreciates about twice as much. In three years what would be the loss by leaving it out, over the depreciation if it were carefully kept?

7. A farmer allows 350 cu. ft. of solidly stacked hay to the ton. How many tons in a mow containing 4375 cu. ft.?

8. If corn fodder loses 60% of its weight in curing, how much will 28 T. of green fodder weigh after it is cured?

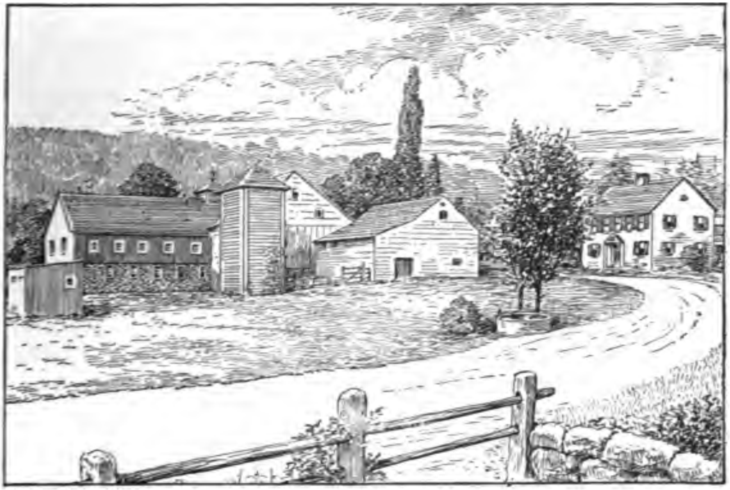
CORN AS BEEF AND PORK

1. On a farm where corn was selling at 30¢ a bushel it was decided to feed the corn to cattle. It was estimated that the increase in the value of the cattle, from the corn alone, was 40¢ for each bushel used for feed. What was the per cent of increase in the value of the corn by using it as feed?

2. On another farm corn was selling at 31¢ a bushel. Estimating that it was worth $62\frac{1}{2}$¢ when fed to cattle, find the per cent of increase in the value of the corn by using it as feed.

3. A record of the result of feeding corn to hogs was kept on several farms. On one farm, where corn was selling at 35¢ a bushel, it was found that the increase in the value of the hogs was equivalent to 79¢ per bushel of corn fed. What was the per cent of increase in the value of the corn by using it as feed?

4. On another farm the figures of Ex. 3 were 32¢ a bushel for corn when sold, and 64¢ a bushel when used as feed. What was the per cent of increase in the value of the corn?



PROBLEMS ABOUT A SILO

1. A farmer thinks of building a silo with inside measurements 12 ft. long, 10 ft. wide, 28 ft. high. Allowing 40 cu. ft. to a ton of ensilage, how many tons will this silo contain?

2. He decides, however, to build the silo in the more modern form, with a circular base. The rule for finding the volume of such a silo is to multiply the height by $\frac{22}{7}$ times the square of the radius of the base. If the radius of the base is 7 ft. and the height is 28 ft., how many cubic feet will the silo contain?

3. Allowing 40 cu. ft. to a ton of ensilage, how many tons can be placed in the silo mentioned in Ex. 2.

4. The concrete floor of the silo in Ex. 2 is 22 in. thick. By the rule given in Ex. 2, find the amount of concrete needed.

In places where silos are not used this page may be omitted. The teacher should explain that the allowance in Exs. 1 and 3 is only approximate. The more ensilage is put in the more it is pressed down.

THE FARMER'S INCOME

1. James's father owns two farms of the same size and value. One he runs himself and the other he lets on shares. James asked his father if it was worth while to learn any more, and his father said: "Let us see what brains do on our property. Here is the total income of the farm we let on shares, and there are my books for last year. Now see the result." This is what James found:

	Home farm	*Farm rented*
Dairy products	$248.60	$93.75
Wool	21.75	10.43
Eggs and poultry	232.60	80.04
Domestic animals	595.75	288.17
Crops	975.30	507.22

Find the total receipts on each farm.

2. James's father told him that the tenant had very little schooling as a boy and that he did not read the agricultural papers, and he asked James to find the per cent of increase of each of their items over the tenant's. How much was this?

3. He told James that the expenses of the two places showed poor management by the tenant, and showed him these figures:

	Home farm	*Farm rented*
Labor	$175.45	$102.43
Fertilizers	86.75	10.00
Feed	52.60	47.13
Maintaining buildings	22.60	49.72
Maintaining equipment . . .	18.75	39.78
Taxes and miscellaneous . . .	85.70	83.03

Find the total expenses, the difference in each pair of items, the difference in the total expenses, and the net gain of each farm.

PROBLEMS OF THE PAY ROLL

1. A mason in a certain city receives 70¢ an hour, and a plasterer 62½¢ an hour. How much will 8 masons and 4 plasterers receive for 8 hr. of work?

2. The following is last week's pay roll of a manufacturer:

PAY ROLL *For the week ending* * 19 *

No.	Name	No. of Hours per Day						Total Time	Wages per Hour	Total Wages	
		M.	T.	W.	T.	F.	S.				
1	J. P. Driscoll	8	8	8	6½	8	4	42½	50¢	$21	25
2	R. L. Bennett	8	7½	8	8	8	4	*	50¢	*	*
3	P. F. Purvis	7	7¼	8	7½	8	4	*	50¢	*	*
4	B. J. Mead	8	7	6	8	7½	3	*	55¢	*	*
5	R. K. King	8	8	7	7¾	8	3½	*	55¢	*	*
6	M. L. Drake	8	6	8	0	7	3¾	*	27½¢	*	*
		*	*	*	*	*	*	*		*	*

Fill out the spaces marked with an asterisk (*).

Make out pay rolls (inserting names), when the numbers, the hours per day, and the wages per hour are as follows:

3. No. 1: 8, 8, 8, 8, 7½, 6, 62½¢; No. 2: 8, 8, 8, 7, 6, 8, 57½¢; No. 3: 8, 7½, 8, 8, 8, 6, 48¼¢; No. 4: 8, 8, 8, 8, 7½, 8, 66⅔¢; No. 5: 8, 7, 8, 8, 8, 6½, 72¢.

4. No. 1: 8, 7, 6, 8, 8, 8, 67½¢; No. 2: 8, 7, 6, 8¾, 8¾, 8¾, 60¢; No. 3: 8, 6, 6, 8, 8, 7¾, 62¢; No. 4: 7, 8, 7¾, 6½, 8, 8, 57¢; No. 5: 8, 6, 7, 7½, 6¼, 8, 48¢.

5. No. 1: 8, 8, 8, 8, 8, 8, 62½¢; No. 2: 8, 8, 7¾, 7½, 7¾, 7¾, 60¢; No. 3: 8, 8, 8, 7¾, 7¾, 7¾, 58¢; No. 4: 7½, 7¼, 7½, 8, 7¾, 7¾, 62¢; No. 5: 8, 8, 8, 8, 8, 8, 63⅓¢.

OWNING A HOUSE

1. John's and Helen's father rents the house in which they live. He pays as rent $22 a month. How much does this amount to in a year?

2. Their father has saved $3000, which he has invested in 5% bonds. How much would he lose in interest on his money if he bought the house and lot for this amount?

3. He asks John and Helen to figure out whether it would pay to own a house instead of renting one. First they put down as loss the interest on $3000 at 5%. Then they put down 2% of $3000 for yearly repairs, $48.50 for taxes, and $6.50 for fire insurance. So far, what is the total annual expense of owning their home? How does this compare with their annual rent?

4. John and Helen conclude that there is no great advantage in owning a house, except for the pleasure, and they cannot measure this in money. But they know that a neighbor bought a house for $2400 and sold it for $3120. He gained what per cent on his investment? If their father could gain the same per cent, how much would he gain on the cost of the house?

5. But people do not always gain on such a transaction. Their father might have to sell again for only $2700. In this case what would be his per cent of loss?

6. John's and Helen's father finally decides to risk the purchase. But he finds that he has underestimated the repairs, which average $75 a year for the next four years. His taxes, however, come only to $35 a year, and his insurance costs $15 for a four-year policy. At the end of four years he sells the property for $3600. Meantime he has lost the interest on his $3000 at 5%, but has saved his rent of $22 a month. What per cent on $3000 does his investment net him per year?

HEATING THE HOUSE

1. Maude's father put a steam-heating plant in his house last winter. In a season it consumes 15 tons of chestnut coal at $7 per ton. How much does the coal cost?

2. Of this coal, 15% is lost in ashes, the rest being turned into "heat units," as the dealers say; that is, 85% is really used for heating. How many pounds of coal are lost in ashes? How many pounds are turned into heat units?

3. The house has 10 rooms and was heated for 212 da. What was the average cost per day of heating the house? What was the average cost per room per day?

4. Maude's father paid $325 for putting in the steam-heating plant. If it consumes, on an average, 15 tons of coal at $7 per ton each year, and lasts 10 yr., we have to consider the loss of interest at 6% on $325, the final loss of the $325 through the loss of the heating plant, the cost of the coal, and the loss of interest on the money paid for the coal. Taking this last item as $20, in round numbers, find the average annual cost of heating the house.

5. Maude's uncle, Mr. Preston, thought it would be better for him to put in a hot-water heater costing $500. This consumes 10 tons of coal per year at $7 per ton. Considering only the first cost of the two heating plants, and the cost of the coal consumed, which, if either, is the cheaper plant at the end of 5 yr.? How much cheaper?

Problems concerning the cost of heating and lighting the home and the school building should be made up from actual data. If possible, some boy in the class should measure the amount of coal put into a furnace during the day, and the amount of ashes taken out, so as to give the class a practical knowledge of the amount of coal used in heating a house, and the per cent of coal that actually goes into heat.

LITTLE EXAMINATIONS

I. 1. A dealer sold some fruit for 12½% less than it cost, thereby losing $140. What was his selling price?

2. How much will 360 ft. of iron pipe cost if listed at $1.50 a foot, a discount of 40% and 10% being allowed?

3. How much will 4 gross of jars of mixed pickles cost at $6 a dozen, a discount of 20% and 10% being allowed?

4. A man's salary of $2100 a year was increased 10% and then it was decreased 10%. How much was it then?

5. If the ratio of cream to the rest of the milk of a certain quality is 1: 5, how many pounds of cream in 750 lb. of milk?

II. 1. A triangular gable to a house has a base 36 ft. and an altitude 12 ft. 8 in. Find the area of the gable.

2. If 72 reams of paper are used in printing 3800 books, 84 sheets to a book, how many reams will be needed for 9800 books, each requiring 57 sheets.

3. If the ratio of butter fat to the rest of the milk of a certain quality is 1: 24, how many pounds of butter fat in 1050 lb. of milk of that quality?

4. If the ratio of solid matter to the rest of the milk of a certain quality is 33: 217, how many pounds of solid matter in 375 lb. of milk of that quality?

5. A hardware dealer bought a shipment of stoves. After deducting a discount of 33⅓% and 25%, and paying $5 cartage and $19.50 freight, the stoves cost him $504.50. What was the list price of the stoves?

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write in ordinary numerals the numbers five hundred five thousandths, and five hundred and five thousandths.

2. How many years from MDCCCLXXVI to MDCCCCXX, or MCMXX, as it is also written?

Add, and also subtract:

3.	**4.**	**5.**	**6.**
\$427.32	\$932.01	\$870.23	\$900.01
198.56	687.56	592.68	48.75

Multiply the following:

7.	**8.**	**9.**	**10.**
\$423.78	\$573.81	\$609.98	23 ft. 8 in.
65	74	127	15

Divide the following, in turn:

11. 864 by 16, 32, and $\frac{4}{5}$.

12. 9.72 by 3.6, 0.36, and 36.

13. 58 ft. 8 in. by 8.

14. 64 lb. 2 oz. by 9.

Express as decimal fractions and also as per cents:

15. $\frac{1}{2}$.
16. $\frac{1}{4}$.
17. $\frac{3}{4}$.
18. $\frac{3}{8}$.
19. $\frac{5}{8}$.
20. $\frac{7}{8}$.
21. $\frac{2}{3}$.
22. $\frac{3}{5}$.
23. $\frac{5}{6}$.
24. $\frac{7}{16}$.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. A man buys a certain number of pecks of chestnuts. How do you find how many bushels he buys?

2. Given the length of a sheet and the amount to be added for hemming, how do you find the number of sheets that can be made from a given number of yards of sheeting of the right width?

3. If you know the number of yards of lawn needed to make a skirt and the price per yard, how do you find the cost of the lawn for a given number of skirts?

4. If you know the number of cups of flour needed for a certain number of loaves of bread, how do you find the number of cups of flour needed for a certain other number of loaves?

5. If you know how much water flows through a pipe in a minute, how do you find the amount of water that will flow through the pipe in a whole day, at the same rate?

6. If a boy wishes a piece of board to make a bookshelf, how do you find the cost of the board, knowing the size of the shelf and the cost of the lumber per M?

7. If you know how much per foot an iron rod expands in length when heated from the ordinary temperature to red heat, how do you find the amount of expansion of an iron rod a given number of inches in length, when heated to red heat?

8. If a salesman sells on a certain salary plus a certain commission, how do you find the amount of his income for a year?

9. If you know the number of hours per day that a man works, for each day of the week, and his rate of wages per hour, how do you find the amount due him at the end of the week?

III. INDUSTRIAL APPLICATIONS

THE DAIRY INDUSTRY

1. If the cream from this model dairy contains 25.9% butter fat, what is the weight of butter fat in 970 lb. of cream?

2. If a cow's average daily ration is 28 lb. of ensilage, 12 lb. of clover hay, $6\frac{1}{2}$ lb. of bran, and $3\frac{1}{2}$ lb. of corn meal, what per cent of the total is each kind of food?

3. If each cow averages 4400 lb. of milk per year and the milk contains 4.3% butter fat, how many pounds of butter fat will 32 cows produce in a year?

4. If a cow in a year eats $3\frac{1}{4}$ T. of hay at $12 a ton, and 900 lb. of ground feed at 85¢ per C, and if $6 per cow is allowed for pasturage, how much will it cost to keep 32 cows for 1 yr.?

5. Allowing 4 ft. 3 in. by 9 ft. 6 in. of floor space for a stall and 4 ft. 6 in. by 3 ft. extra for a manger, how much floor space will be required for 32 cows?

In rural schools there should be special computations suited to farm practice, on such subjects as rations, the cost of labor, and the income from cows.

THE COTTON INDUSTRY

1. A mill bought 10 bales of cotton weighing respectively 489 lb., 513 lb., 478 lb., 492 lb., 521 lb., 498 lb., 503 lb., 496 lb., 503 lb., and 517 lb. If the mill bought on the basis of 500 lb. of cotton to the bale and paid 14¢ a pound, did it gain or lose by the variation in weight, and how much?

2. A mill bought 34 bales of Egyptian raw cotton, averaging 800 lb. each. At 30¢ a pound, how much did the shipment cost?

3. A New Jersey manufacturer agreed to take $12\frac{1}{2}\%$ of a cargo of cotton shipped from New Orleans to New York. The cargo consisted of 4864 bales averaging 500 lb. each, and the manufacturer paid 14¢ per pound for the cotton. How many bales did he buy, and how much did they cost him?

4. A certain cotton mill used in one day 4840 lb. of Egyptian cotton valued at 31¢ a pound, 12,674 lb. of upland cotton valued at 13¢ a pound, and 1247 lb. of sea-island cotton valued at 62¢ a pound. Find the total value of the cotton used that day.

5. A pound skein of No. 1 cotton yarn contains 840 yd. How many yards are there in 840 lb. of cotton yarn?

6. A pound skein of No. 3 cotton yarn contains 2520 yd. How many yards are there in 2520 such skeins?

7. A certain manufacturer finds that in cleaning baled cotton he has to allow 30% for waste. If he uses 48 bales of 500 lb. each, how many pounds of waste are there?

8. The teacher took the class to a mill one day to see cotton thread made. The foreman told them that the frames for spinning a certain size of cotton thread consist of three sides with 104 spindles to each side, and that the room they were visiting had 72 such frames. He then asked them to find the number of spindles in the room. How many were there?

CHAPTER XI

BANKS AND BANKING

255. Bank Account Essential. One thing that is essential at some time to any one who hopes to succeed is a bank account. A reliable person may "open an account," as it is called, as soon as he begins to save even a small amount.

256. Savings Bank. To deposit money a person goes to a bank, says that he wishes to open an account, and leaves his money with the officer in charge. The officer gives him a book in which the amount is written, and the depositor writes his name in a book or on a card, for identification. When he wishes to draw out money, he takes his book to the bank, signs a receipt or a check for the amount he desires, and receives the money.

WRITTEN EXERCISE

1. How much does 25¢ saved each day amount to in a year?

2. How much will 15¢ saved each day, 310 days to the year, amount to in 10 years, not counting interest?

3. If a boy, beginning when he is 16 yr. old, saves 25¢ a day for 300 da. a year, and deposits it in a savings bank, how much will he have when he is 21 yr. old, not counting interest?

4. If a father gives his daughter $1 when she is 1 yr. old, $2 on her next birthday, and so on until she is 21 yr. old, depositing it for her in a savings bank, how much will she have when she is 21 yr. old, not counting interest?

5. A man saves on an average 25¢ a day the first year he works on a certain farm, and 20% more the second year, depositing it in a savings bank. If he works 306 days each year, how much does he save in the two years?

6. A farmer's savings-bank deposits average $25 a month during a certain year, and 97% as much during the next year. The year following he deposited 110% as much as he did the second year. How much did he deposit in the savings bank during the three years?

7. If a man saved $1 a week during the year in which he was 21 years old, and increased his savings each year by 10% on the amount of the preceding year, and worked 50 weeks a year, how much would he save during the year in which he was 29 years old?

8. A merchant saves $375 the first year he is in business. The second year he increases his savings one third. The third year they are only 85% as much as the second year. The fourth year they increase 30% over the third year. How much does he save in the four years?

9. A man works on a salary of $15 a week for 50 weeks in a year. His expenses are $15 a month for house rent, 60% as much for clothing, 300% as much for food as for clothing, and 20% as much for other necessary expenses as for food. How much of his salary can he reserve for the savings bank?

10. A clerk in a store had a salary of $12 a week two years ago, and a commission of 2% on the goods he sold. That year he worked 48 weeks and sold $2350 worth of goods. Last year his salary was increased 25%, his commission remaining the same. He worked 49 weeks and sold $2750 worth of goods. How much was his income increased?

257. Interest at Savings Banks. Savings banks usually pay interest every six months or every three months. This interest is added to the principal, and the total amount draws interest.

258. Compound Interest. When interest as it becomes due is added to the principal and the total amount then draws interest, the investor is said to receive *compound interest* on his money.

Compound interest is no longer allowed on notes; but if one collects his interest when it is due and at once reinvests it, of course he practically has the advantage of compound interest. The method of finding compound interest is substantially the same as that used in simple interest.

For example, required the amount of $2000 in 2 yr., deposited in a savings bank that pays 4% annually, the interest being compounded semiannually. How much is the compound interest?

```
$2000.    = first principal
   .02
 $40.     = int. first 6 mo.
 2000.
$2040.    = amt. after 6 mo.
   .02
  $40.80  = int. second 6 mo.
 2040.
$2080.80  = amt. after 1 yr.
      .02
  $41.62  = int. third 6 mo.
 2080.80
$2122.42  = amt. after 1½ yr.
      .02
  $42.45  = int. fourth 6 mo.
 2122.42
$2164.87  = amt. after 2 yr.
 2000.
 $164.87  = int. for 2 yr.
```

Simple interest for the same time is $160, or $4.87 less than the compound interest.

The compound interest in this example was found exactly, but savings banks pay interest only on the dollars and not on the cents. In this example the compound interest would be $164.84 instead of $164.87. The saving to the bank in the work of computation is considerable.

259. Savings Bank Account. Usually savings banks pay interest semiannually, adding this to the principal on deposit. This is, therefore, a case of compound interest. The following is a specimen account at a bank paying 4% interest, payable semiannually, on the smallest balance on deposit in an interest period.

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1917									
July	1	$600	50					$600	50
July	20	75						675	50
Sept.	6					$120		555	50
Dec.	7	60						615	50
Dec.	20					65		550	50
1918									
Jan.	1			$11				561	50
May	9	200						761	50
July	1			11	22			772	72

Here the smallest balance during the first interest period was $550.50. Interest is computed on the dollars only, the cents being neglected. At 2% the interest on $550 is $11. In the second period the smallest balance is $561.50, the interest being 2% of $561, or $11.22.

Some banks allow interest from the first of each month; others from the first of each quarter; others, as above, from the first of each half year. The interest is computed on the smallest balance on hand between this day and the next interest day, and is usually added every half year.

In the above example, if the interest period had been 3 mo. and the interest had been credited every 6 mo., the interest would have been:

First quarter, 1% of $555 = $5.55.
Second quarter, 1% of $550 = 5.50. Credit $11.05.
Third quarter, 1% of $561 = 5.61.
Fourth quarter, 1% of $561 = 5.61. Credit $11.22.

In this case the difference in interest would have been only five cents.

WRITTEN EXERCISE

At simple interest, and also at interest compounded annually, find the amount of principal and interest:

1. $2500, 4 yr., 5%.

2. $4000, 3 yr., 6%.

3. $3500, 3 yr., 4%.

4. $2250, 4 yr., 3%.

5. $2750, 4 yr., 3%.

6. $1750, 3 yr., 3½%.

7. $625.50, 4 yr., 3½%.

8. $10,000, 6 yr., 4%.

9. $425.50, 4 yr., 4½%.

10. $275.50, 3 yr., 4½%.

Find the amount of principal and interest, the interest being compounded semiannually:

11. $200, 3 yr., 4%.

12. $300, 2 yr., 4%.

13. $650, 2 yr., 6%.

14. $700, 3 yr., 3%.

15. $800, 3 yr., 4%.

16. $500, 2 yr., 4%.

17. $1000, 2 yr., 4½%.

18. $1000, 3 yr., 4%.

19. $2000, 2 yr., 3½%.

20. $3000, 2 yr., 2½%.

21. Which brings the more interest in 2 yr. on $1250, 4% in a savings bank, compounded semiannually (the money being deposited at the beginning of the year), or 4½% simple interest?

22. If a man on January 1 deposited $2000 in a savings bank, and left it for 2 yr., at 4%, the interest being compounded semiannually, how much less interest would he receive than by lending it at 5% simple interest?

23. If a man deposits $200 in a savings bank on January 1, $300 on February 1, $100 on May 1, $400 on August 1, and $350 on November 1, and the rules of the bank allow 4% interest on all these sums, compounding it on July 1 and on January 1, how much will the man have to his credit on the January 1 following these deposits?

260. Postal Savings Bank. The United States government conducts a savings bank in connection with the post office. Although all savings banks are very carefully regulated and inspected by the state governments, there are many persons who prefer a smaller rate of income than ordinary savings banks pay, so long as they know that our government guarantees the payment of their money when they wish to use it.

Postal savings banks allow any person of the age of 10 yr. or over to deposit money in amounts of not less than $1, but no fractions of a dollar are accepted for deposit. No one can have a total balance of more than $500, exclusive of accumulated interest. Deposits are made at the larger post offices, and postal savings certificates are given to the depositor. Interest is paid by the government at the rate of 2% a year, beginning on the first day of the month following the one in which the deposit is made. Interest is not paid for any fraction of a year. A person may exchange his deposits in sums of $20 or multiples of $20 for government bonds bearing $2\frac{1}{2}$%.

The following exercise refers to postal savings banks.

ORAL EXERCISE

Find the interest for 1 yr. on the following deposits:

1. $20. 2. $50. 3. $100. 4. $200. 5. $400.

Find the interest for 2 yr. on the following deposits:

6. $10. 7. $40. 8. $200. 9. $300. 10. $500.

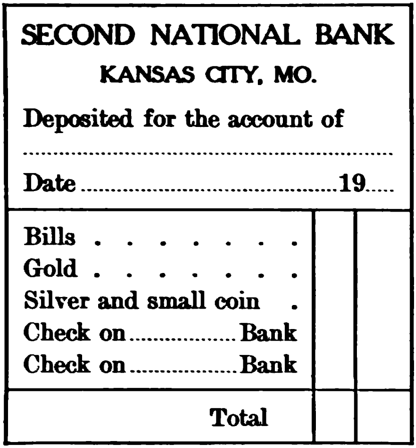
Find the interest received for 1 yr. by a holder of $2\frac{1}{2}$% bonds of:

11. $100. 12. $200. 13. $300. 14. $400. 15. $500.

16. How much does a depositor increase his annual income by transferring $400 of deposits into a government bond?

261. Bank of Deposit. When a man has money enough ahead to pay his bills by checks, he will find it convenient to have an account with a bank such as merchants commonly use, sometimes called a *bank of deposit*.

Such banks do not pay interest on small accounts, the deposit being a matter of convenience and safety. If a man wishes to open an account he sometimes has to give references, for banks do not wish to do business with unreliable persons.


SECOND NATIONAL BANK
KANSAS CITY, MO.
Deposited for the account of
....
Date ....19.....

Bills		
Gold		
Silver and small coin .		
Check on .... Bank		
Check on .... Bank		
Total		

262. Deposit Slip. When a man makes a deposit of money or checks in a bank, he fills out a *deposit slip*, as shown above, and leaves it at the bank.

ORAL EXERCISE

Give the sum of each of the following lists of deposits:

1.	2.	3.	4.	5.
$15.50	$12.75	$23.80	$12.	$62.50
3.75	2.25	12.70	13.75	2.50
1.	5.	10.	1.25	3.

6.	7.	8.	9.	10.
$10.	$22.50	$45.	$16.75	$75.50
5.75	2.50	5.75	10.	4.50
1.25	5.	1.25	3.25	20.
2.	3.	5.	7.	10.

WRITTEN EXERCISE

Fill out deposit slips for the following deposits, inserting the name of the depositor and of some bank in your vicinity:

1. Bills, $254; silver, $40; checks on First National Bank, $87.50; Traders Bank, $127.50.

2. Bills, etc., $423.57; gold, $275; silver, $135.75; check on Cotton Exchange Bank, $342.60.

3. Bills, etc., $135.50; checks on Chemical National Bank of New York, $325; Third National Bank, $63.73.

4. Bills, etc., $1726.45; gold, $125; silver, $100; checks on Traders Bank, $335.50; Second National Bank, $175.40.

5. Bills, etc., $262.75; checks on Garfield National Bank, $96.50, $200; Jefferson and Lee National Bank, $325, $46.50.

6. Gold, $50; checks on First National Bank, $27.62, $41.75, $32.80; Lincoln Trust Company, $37.42, $21.85.

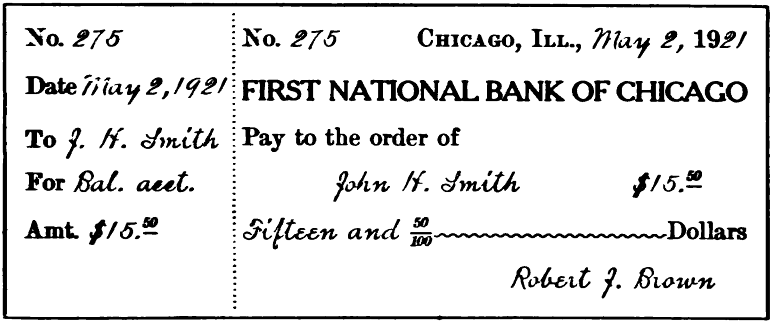
7. Bills, etc., $146.73; silver, $275; checks on Miners National Bank, $43.50; Rocky Mountain National Bank, $250.

8. Bills, $145; silver, $350; gold, $480; checks on Merchants National Bank, $255; Farmers Trust Company, $162.50; Second National Bank, $275.50.

9. A merchant deposited $175.80 in cash to-day, a check for 25% of a debt of $176 due him, and a check in payment for 18¾ yd. of velvet at $2.16 a yard less 33⅓% on account of a bargain sale. Make out a deposit slip.

10. A jeweler received cash for the following: 3 doz. forks @ $16.50, 4½ doz. teaspoons @ $15, a watch @ $32.75, 2 clocks @ $6.75. He also received a check on the First National Bank for the following: 3 doz. dessert spoons @ $17.75, 9 nutcrackers @ $9 a dozen. Make out a deposit slip.

263. Check Book. A *check book* containing *checks* and *stubs* is given the depositor when he opens an account at the bank.



Stub	*Check*
No. 275	No. 275 CHICAGO, ILL., May 2, 1921
Date May 2, 1921	FIRST NATIONAL BANK OF CHICAGO
To J. H. Smith	Pay to the order of
For Bal. acct.	John H. Smith $15.50
Amt. $15.50	Fifteen and $\frac{50}{100}$ Dollars
	Robert J. Brown

264. Payee. The person to whom a check is payable is called the *payee.* A check is usually made payable to the order of the payee, in which case the payee must *indorse* it, that is, he must write his name across the back; or to "Self," in which case the drawer alone can collect it; or to the payee or bearer, when any one can collect it; or to "Cash," when any one can collect it.

ORAL EXERCISE

Given the following deposits and checks, tell the balances:

1.

Deposits	*Checks*
$25	$15
70	27

2.

Deposits	*Checks*
$37	$19
41	32

3.

Deposits	*Checks*
$48	$20
27	15

4.

Deposits	*Checks*
$62	$37
48	41

5.

Deposits	*Checks*
$75	$62
25	17

6.

Deposits	*Checks*
$50	$65
40	20

WRITTEN EXERCISE

1. If your deposits in a bank have been $14.75, $32, $15, $25, $50, $75, and $1.75, and you have drawn checks for $9.50, $18.75, and $30, what is your balance?

2. A man earning $15 a week deposits $12 every Saturday, and each Monday gives a check for $6 for his board. What will be his balance in 6 months (26 weeks)?

3. If your deposits in a bank have been $32.75, $49.63, $28, $14.90, $25, $10, $30, $42.75, and $71.25, and you have drawn checks for $5.75, $13.50, $32, $4.50, $36.50, and $52.80, what is your balance?

4. A farmer having $275.60 in the bank deposits during the next six months $23.50, $17.42, $75, $32.40, and the proceeds of the sale of 7 cows @ $37.50 less 5%. He gives checks for $17.92, $8.50, $72.80, $9.50, $6.75, $13.90, and $26.50. What is his balance?

5. A merchant's deposits this week have been $127.42, $135, $72.50, $265.75, $327.40, and $182.50, and he has drawn checks for $236.50, $15.75, $27.90, $241.60, and $29.50. He has also paid by check a bill of $176.70 less 10%, another of $12 less 4%, and another of $75.25 less 10%, 3%. At the beginning of the week he had $475.80 in the bank. What is his balance at the end of the week?

6. A merchant's deposits for this week have been $261.50, $392.75, $62.40, $112.30, $98.76, and $115, and he has drawn checks for $42.80, $174.95, $162.30, $81.50, $15, $27.40, and $37.50. He has also given a check for $125 plus 6 months' interest at 5%, and one in payment of a bill for $248.50 less 8%, 5%. At the beginning of the week he had $692.80 in the bank. What is his balance at the end of the week?

365. Promissory Note. A paper signed by a borrower, agreeing to repay a specified sum of money on demand or at a specified time, is called a *promissory note*, or simply a *note*.

366. Face of Note. The sum borrowed is called the *principal*, or, if a note is given, the *face of the note*.

367. Amount of Note. The sum of the principal (or face) and the interest is called the *amount* of the note.

A note should state the date, face, rate, person to whom payable, and date of maturity (date when it is due to be paid), and should state the fact that it has been given for value received.

The following is a common form of a time note:

$75.^{00} TOPEKA, KANSAS, *February 7*, 19*17*

Six months after date, for value received, I promise to pay

to .... *John Williamson* .... or order,

Seventy-five and $\frac{00}{100}$ ~~~~~~~~~~~~ Dollars,

with interest at *5%*.

Frank Francis

The following is a common form of a demand note:

$50.^{00} AUSTIN, TEXAS, *May 2*, 19*19*

On demand, for value received, I promise to pay to

.... *Robert J. Jones* .... or order,

Fifty and $\frac{00}{100}$ ~~~~~~~~~~~~ Dollars,

with interest.

James P. Poole

268. Indorsing a Note. If the payee wishes to sell the note, he must, when it is payable to himself or *order*, indorse it.

A note is *indorsed* by the payee writing his name across the back. The indorser must pay the note if the maker does not.

If the payee wishes to make the note payable to the order of Frank R. Roberts, he should write across the back, "Pay to the order of Frank R. Roberts," and sign his name.

A note payable to John Williamson or *bearer* may be sold without indorsement. Such notes are not common.

WRITTEN EXERCISE

1. Compute the amount of the first note on page 145.

2. Write a promissory note, signed by X, payable to Y, for $25, due in 1 yr., at 6%. Find the amount.

3. Thomas H. Holmes borrows $350, at 6%, for 1 yr., from A. B. Seel. He gives a note payable to Mr. Seel or order. Mr. Seel sells the note to B. C. Dean. Make out the note, indorse it, and find the amount due at the end of the year.

Make out and indorse, payable to the order of the buyer, the following notes, and find the amount due at the end of the time stated:

	Maker	*Payee*	*Buyer*	*Face*	*Rate*	*Time*
4.	J. R. Jones	A. N. Cole	P. R. Carr	$375	6%	6 mo.
5.	M. L. King	A. R. Doe	E. F. Dun	$750	5%	2 yr.
6.	A. K. Iles	S. M. Roe	E. L. Cree	$75	6%	10 mo.
7.	B. S. Hill	O. N. Olds	B. R. Hall	$250	5%	6 mo.
8.	O. M. Coe	D. E. Bell	C. N. King	$125	6%	1 yr.
9.	A. R. Carr	A. J. Burr	G. F. Ray	$250	5½%	4 mo.
10.	F. E. True	A. B. Bain	G. F. Coe	$350	4½%	3 mo.
11.	J. B. Kaye	E. P. Fell	P. R. Call	$275	6%	8 mo.

269. Loan. When a person deposits money in a bank he really lends to the bank. The bank lends the money to some one else at a higher rate of interest than it pays the depositor. When a person takes a note for money, this note represents a *loan*. When a lender wishes excellent security for a loan of some size, he may take a *bond* instead of a note. A bond is a longer document than a note and is more formal, being acknowledged before an officer of the law (usually before a *notary public*), and being made more secure by a *mortgage*.

It should be remembered that all the forms of investment we have studied are in reality different ways of lending money, the income being the amount received for the use of the money lent. Show how this is true of interest. In addition to these indirect methods of lending money, people frequently lend money as a matter of accommodation, or for interest received, usually at a higher rate than can be obtained at banks.

WRITTEN EXERCISE

Find the income on the following, whether in the form of real estate, bonds and mortgages, or notes:

1. $750, 2 yr., 9%.
2. $875, 3 yr., 8%.
3. $4750, 2 yr., 8½%.
4. $5500, 3 yr., 9½%.
5. $6000, 4 yr., 7%.
6. $7500, 3 yr., 9%.
7. $8000, 5 yr., 8½%.
8. $6000, 3 yr., 2½%.

The above examples represent investments in real estate. In most states bonds and notes do not in general pay more than 6%.

9. $1250, 4 yr., 6%.
10. $2250, 2 yr., 5%.
11. $1875, 2 yr., 4%.
12. $6200, 3 yr., 5%.
13. $3300, 2 yr., 3%.
14. $4500, 2 yr., 3½ %.
15. $750, 2 yr. 6 mo., 3%.
16. $500, 1 yr. 6 mo., 3%.
17. $1500, 1 yr. 3 mo., 6%.
18. $2500, 2 yr. 6 mo., 5%.

270. Borrowing Money. If Fred lends a baseball bat to Frank, it is evident that Frank borrows the bat from Fred. Lending by one person means borrowing by another.

Business houses usually have need of large sums of money at certain times of the year, and it is better for them to borrow it for two or three months than to keep such an amount on hand for a whole year. Borrowing money is, therefore, often good business policy.

Business houses usually borrow money at banks. The banks get their money from depositors, who find it safer and more convenient to keep their money in a bank than to lock it up in their homes or their offices. The banks get paid for their work by charging borrowers more than they pay depositors.

Ask the pupils to inquire about the rates of interest charged by local banks.

WRITTEN EXERCISE

1. Find a bank's net annual gain if it has $1,250,000 on deposit, pays its depositors 3%, and lends $950,000 of its deposits at an average of 5¼%.

2. Find a bank's net annual gain if it has $2,750,000 on deposit, pays its depositors on an average 1⅛%, and lends $2,175,000 at an average of 5⅜%.

3. If a bank lends $150,000 at 5% and pays its depositors 2%, how much does it gain in 3 mo.?

4. If a bank lends $275,000 at an average of 5¼% and pays its depositors 2½%, how much does it gain in a year?

5. A bank has many expenses (*overhead charges*, as they are called). If the bank pays 2% for its money, lends it at 4½%, and has overhead charges of 0.7% on a certain loan of $250,000, how much is its gain in 2 mo.?

271. Note Payable at a Bank. If John Brown wishes to borrow some money from a bank, and if Richard Roberts will indorse a note for him, and if the bank is willing to lend the money on this security, a note similar to the following is then made out:

> LOUISVILLE, KY., *May 15*, 19*18*
>
> *Two months* after date, *I* promise to pay to the order of
>
> ....*Richard Roberts*................................*$1000.00*
>
> *One thousand*~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~Dollars
>
> at **The First National Bank, Louisville, Ky.**
>
> Value received
>
> Residence *7030 Beacon Street* *John Brown*
>
> Due *July 15, 1918*

The original idea was that Mr. Brown borrowed of Mr. Roberts, and Mr. Roberts sold the note to the bank, thus getting the money to lend to Mr. Brown. But now the note means simply that Mr. Roberts agrees to pay if Mr. Brown does not. Banks often take notes payable directly to the cashier.

Banks often lend money without an indorser if the borrower will deposit property of enough value to make them secure.

272. Usual Time for Bank Notes. Notes are usually made for 30 da., 60 da., or 90 da., and are then renewed if the borrower desires to keep the money for a longer time. On account of the great increase in banking facilities, money is more commonly borrowed from banks than was formerly the case. Hence the most common interest periods are now 30 da., 60 da., or 90 da.

A note payable 2 mo. after May 15 is due July 15, but a note payable 60 da. after May 15 is due July 14.

273. Bank Discount. When a man borrows from a bank he pays the interest in advance. Interest is not mentioned in the note, because it has already been paid.

Interest paid in advance on a note is called *discount.*

Teachers should call attention to the fact that the same word is used for bank discount and for commercial (trade) discount and should explain that the process is the same in both cases, that is, finding some per cent of a number. The teacher should ascertain the local custom in discounting notes.

In all work at present, call 30 days a month.

274. Proceeds. The face of a note less the discount is called the *proceeds.*

What are the discount and the proceeds on a note for $200 for 6 mo. at 5%?

The discount on $200 for 1 yr. at 5% is 5% of $200, or $10.
The discount for 6 mo. is ½ of $10, or $5.
The proceeds are $200 – $5, or $195.

WRITTEN EXERCISE

Find the discounts and the proceeds on the following:

1. $75, 1 mo., 6%.
2. $90, 30 da., 6%.
3. $250, 3 mo., 5%.
4. $275, 3 mo., 5%.
5. $425, 4 mo., 5%.
6. $300, 90 da., 6%.
7. $375, 6 mo., 6%.
8. $400, 90 da., 6%.
9. $150, 60 da., 6%.
10. $400, 1 mo., 5½%.
11. $200, 60 da., 6%.
12. $175, 2 mo., 4½%.
13. $300, 4 mo., 5½%.
14. $500, 2 mo., 3½%.
15. $2975, 60 da., 5%.
16. $4250, 90 da., 6%.
17. $1250, 30 da., 6%.
18. $2500, 60 da., 6%.
19. $1500, 90 da., 6%.
20. $1250, 90 da., 5%.
21. $1750, 90 da., 6%.
22. $2475, 30 da., 5%.

Find the discounts and the proceeds on the following:

23. \$875, 30 da., 6%.

24. \$975, 90 da., 5%.

25. \$425, 30 da., 6%.

26. \$2568, 60 da., 5%.

27. \$1250, 60 da., 6%.

28. \$1500, 90 da., 5%.

29. \$2565, 90 da., 5%.

30. \$3250, 90 da., 5½%.

31. \$4250, 30 da., 4½%.

32. \$325.50, 30 da., 6%.

33. \$427.50, 60 da., 5%.

34. \$237.50, 30 da., 5%.

35. Make out a 60-day note for \$750, dated to-day, payable to John Doe's order at some bank. Discount it at 5%.

36. Make out a 30-day note for \$675, dated to-day, payable to Richard Roe's order at some bank. Discount it at 6%.

37. Make out a 90-day note for \$1250, dated to-day, payable to James Jameson's order at some bank of which you know. Discount it at 6%.

38. A man's bank account shows deposits of \$37.50, \$82.50, \$75, \$100, \$50, \$300, \$40, \$125, and \$500; checks drawn, \$13.75, \$62, \$5, \$125.50, and \$82.75. He needs \$2000 to start him in business, and wishes to keep about \$500 in the bank. How much money should he borrow, to the nearest \$100?

39. If he makes out a note for the amount found in Ex. 38 for 90 da. at 6%, how much discount must he pay? What are the proceeds? What are the proceeds for 60 da.?

40. A merchant has to pay a debt of \$1275 less 10%. He has in the bank \$672.80, and has \$127.50 in cash in his safe. He wishes to leave about \$300 in the bank and about \$65 in his safe. How much must he borrow, to the nearest \$50? Discount the note for 30 da. at 6%.

Teachers should from time to time call attention to the importance of estimating the answer in advance.

275. Discounting a Note. If a dealer buys some goods for the fall trade but does not wish to pay for them until after the holidays, he often buys them on credit, giving his note. The manufacturer may need the money at once, in which case he will indorse the note and sell it to a bank for the sum less the discount.

For example, if you give a manufacturer your note for $500, dated September 1 and due on January 1, with interest at 5%, and he needs the money and discounts the note at a bank on September 16 at 6%, what are the proceeds?

Face of the note	$500.
Interest for 4 mo. at 5%	8.33
Amount due at maturity	$508.33
Discount for 107 da. at 6%	9.07
Proceeds	$499.26

Banks usually compute the discount for the exact number of days. In some parts of the country both the first day and the last day are included in the discount period, thus making it 108 da. in the above example.

WRITTEN EXERCISE

Find the discounts on the following notes:

1. $500, 30 da., 6%.
2. $400, 60 da., 6%.
3. $600, 90 da., 6%.
4. $750, 30 da., 6%.
5. $875, 60 da., 6%.
6. $925, 90 da., 6%.
7. $350, 30 da., 6%.
8. $475, 60 da., 6%.
9. $525, 90 da., 6%.
10. $750, 93 da., 6%.
11. $1250, 30 da., 6%.
12. $3750, 60 da., 6%.
13. $7500, 90 da., 6%.
14. $12,500, 30 da., 5½%.
15. $17,500, 60 da., 5½%.
16. $22,500, 90 da., 5½%.
17. $35,000, 30 da., 5%.
18. $75,000, 30 da., 5%.
19. $125,000, 10 da., 6%.
20. $250,000, 10 da., 5%.

Find the discounts and the proceeds on the following notes:

	Face	*Date*	*Due*	*Interest*	*Date of Discount*	*Rate of Discount*
21.	$300	July 1	July 31	4%	July 16	6%
22.	$800	Sept. 1	Nov. 1	5%	Sept. 11	6%
23.	$550	Oct. 1	Jan. 1	4%	Dec. 2	6%
24.	$325	July 1	Dec. 1	6%	Nov. 1	5%
25.	$1000	Nov. 1	Dec. 1	6%	Nov. 1	6%
26.	$1250	Aug. 15	Jan. 1	5%	Sept. 1	5%

The notes in Exs. 27–52 draw no interest.

27. $3750, 60 da., 5%.

28. $4225, 60 da., 5%.

29. $9500, 60 da., 6%.

30. $7500, 60 da., 5%.

31. $4225, 90 da., 5%.

32. $5575, 90 da., 6%.

33. $7500, 30 da., 5%.

34. $4750, 30 da., 6%.

35. $42.50, 30 da., 6%.

36. $27.60, 30 da., 6%.

37. $35.50, 30 da., 6%.

38. $28.75, 30 da., 6%.

39. $125.50, 60 da., 6%.

40. $275.25, 60 da., 6%.

41. $375.50, 90 da., 6%.

42. $450.75, 90 da., 6%.

43. $457.50, 30 da., 5%.

44. $296.50, 30 da., 5%.

45. $287.60, 90 da., 5%.

46. $375.40, 90 da., 5%.

47. $2750, 93 da., 4½%.

48. $3275, 93 da., 4½%.

49. $4760, 63 da., 5½%.

50. $2745, 63 da., 5½%.

51. The discount, at 6%, on a note for 90 da. is $18.75, and on another note for 30 da. the discount is $32.50. What is the face of each note?

52. A dealer buys $1750 worth of goods, giving his note on October 4, for 90 da., at 5%. On October 10 the holder of the note discounts it at 6%. Find the proceeds.

276. Draft. A written order by which one person directs another to pay a certain sum of money is called a *draft.*

A check is therefore a kind of draft. Usually the word *check* is used for an order on a bank, signed by any person except a representative of another bank; *draft* is used for an order other than a check. A draft may therefore be made by an individual on a company, by one bank on another, by a company on a private individual or firm, and so on.

277. Commercial Draft. A draft made to secure the payment of a debt is called a *commercial draft.*

The following is a form of commercial draft payable at sight:

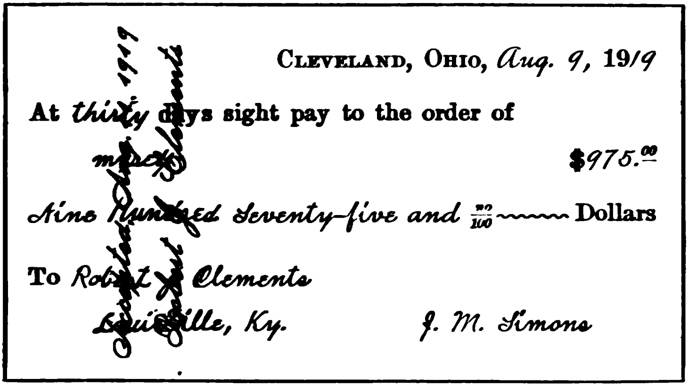
DAYTON, OHIO, *Feb. 15,* 19*20*
At sight pay to the order of
The First National Bank of Dayton *$735.80*
Seven hundred thirty-five and $\frac{80}{100}$ ~~~~~~~~~~ Dollars
To *The National Cash Register Co.*
James X. Wye *Per J. H. P.*
Cleveland, Ohio

278. Parties to a Draft. Here the National Cash Register Company is the *drawer*; Mr. James X. Wye is the *drawee*; The First National Bank is the *payee.*

The Register Company deposits this draft with the First National Bank. This bank sends it to some bank in Cleveland. The Cleveland bank sends to Mr. Wye for the money, and, having collected it, sends the money, or its equivalent in a check, to the Dayton bank. The Dayton bank then notifies the company that the draft is paid, and the amount, less some slight commission, — the *proceeds* of the draft, — is added to the company's account in the First National Bank.

279. Time Draft. A draft that is payable some time after being presented to the payee is called a *time draft.*

The following is a common form of time draft:



CLEVELAND, OHIO, *Aug. 9,* 19*19*

At *thirty* days sight pay to the order of

myself $*975.00*

Nine Hundred Seventy-five and 00/100 ~~~~ Dollars

To *Robert J. Clements*

Louisville, Ky. *J. M. Simons*

Accepted Aug. 11, 1919

Robert J. Clements

In this case J. M. Simons has sold Robert J. Clements $975 worth of goods on 30 days' credit. When he sends the bill he also sends this draft. Mr. Clements writes "Accepted, Aug. 11, 1919, Robert J. Clements" across the face and returns it to Mr. Simons. This becomes thereby a promise to pay $975 "at 30 days sight," that is, on Sept. 10, or 30 days after the acceptance on Aug. 11. Mr. Simons may now discount this at a bank.

This is one way of borrowing money at a bank, replacing the older method by a promissory note. A general name for such evidences of debt as can be sold to banks is *negotiable paper.*

WRITTEN EXERCISE

1. Brown & Co. draw on J. H. Brownson for $750. The banks charge 0.1% for collection. What are the proceeds?

2. The Electric Company draws on Mr. X for $550. The banks charge 0.1% for collection. What are the proceeds?

3. The Arithmetic Publishing Company draws on R. T. Dow for $150. The banks charge 0.2% for collection. What are the proceeds? Write the draft.

4. Robertson Bros. draw on J. P. Shipley for $37.50. The bank's charges for collection are 10¢. This is what per cent of the face?

5. M. D. St. John collects a $150 debt through the bank, the proceeds being $149.70. What is the bank's rate for collecting this debt?

6. S. L. James of Hoboken draws on L. D. Richards of Cedar Rapids, Iowa, for $75. The bank charges $\frac{1}{5}$% for collecting. What are the proceeds?

7. A. B. Stanley owes M. S. Stanton for 2 doz. suits @ $113.50; 1½ doz. suits @ $130; 7½ doz. pairs trousers @ $40. Stanton draws on Stanley for the money, and the bank charges him 50¢ for collecting. What rate is this?

8. Suppose William Bentley of Winnipeg owes you $250, and you wish to draw upon him for this amount. Write the draft, payable to the order of some bank near your home. If the bank charges $\frac{1}{10}$% for collecting, what are the proceeds?

9. R. H. Dudley owes Cayley & Co. for 4 dressers @ $14, and 6 washstands @ $3.50, all less 15%, and a bedroom set at $34.55 net. They draw on him for the amount, the bank charging 10¢ for collecting. What is the net amount received by Cayley & Co.?

10. R. J. Doane of Montreal owes A. D. Kane of Pittsburgh for 10 steel girders @ $50, and tells the latter to draw upon him for the amount. Mr. Kane keeps his account at the Iron Exchange Bank. Make out a draft for Mr. Kane. What is the bank's charge for collecting, at 0.1½%?

11. A draft dated Aug. 8, at 30 days sight, for $975, is accepted on Aug. 10, and is discounted on Aug. 16 at 6%. What are the proceeds?

First find when the draft is payable, then discount it at 6% for the number of days between Aug. 16 and the date of payment.

12. A draft dated Sept. 1, at 30 days sight, for $350, is accepted on Sept. 4, and is discounted on Sept. 7 at 6%. What are the proceeds?

13. A draft dated April 2, at 60 days sight, for $725 is accepted on April 7, and is discounted on April 12 at 6%. What are the proceeds?

14. A draft dated Oct. 15, at 60 days sight, for $1225.75, is accepted on Oct. 19, and is discounted on Oct. 25 at 6%. What are the proceeds?

15. A draft dated Feb. 7, at 90 days sight, for $1675.50, is accepted on Feb. 14, and is discounted on Feb. 22 at 5%. What are the proceeds?

16. A manufacturer sells goods listed at $725.50, discounted at 10% and 5%, and draws on the buyer at 60 days sight. The buyer accepts the draft, and it is discounted 10 days later at 6%. Write the draft and find the proceeds.

17. A manufacturer sells goods listed at $1672.25, discounted at 8% and 3%, and draws on the buyer at 60 days sight. The buyer accepts the draft, and it is discounted 5 days later at 5%. Write the draft and find the proceeds.

18. A wholesale house sells goods listed at $2737.50, discounted at 15%, 8%, and 3%, and draws on the buyer at 90 days sight. The buyer accepts the draft, and it is discounted 30 days later at 5%. Write the draft and find the proceeds.

280. Paying Bills at a Distance. If you owe money to a merchant living in another place, from whom you have ordered a base ball, tennis racket, book, or other article, you may send the money by a registered letter. You will be more likely, however, to pay the bill by a money order. A man who owed a large amount would be apt to pay it by a check or a draft.

281. Exchange. The payment of money by means of checks, money orders, or drafts is called *exchange.*

282. Paying by Check. If a man sends his check for the amount owed, the creditor will deposit it in the bank where he keeps his account. This bank will send it to the debtor's bank for collection, and may charge a small amount for the trouble.

283. Paying by Money Order. A money order may be purchased at the post office, or from an express company, or it may, at considerable expense, be telegraphed. Postal or express orders may be sent to the creditor, who can then obtain the money at his post office or express office. The extra cost of postal money orders is as follows:

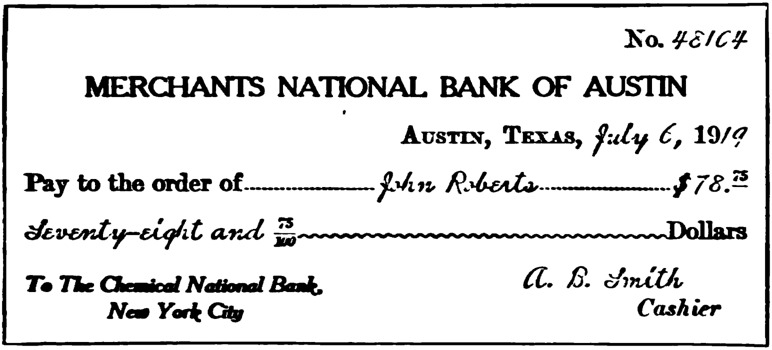
For sums not exceeding $2.50, 3¢; from $2.51 to $5, 5¢; $5.01 to $10, 8¢; $10.01 to $20, 10¢; $20.01 to $30, 12¢; $30.01 to $40, 15¢; $40.01 to $50, 18¢; $50.01 to $60, 20¢; $60.01 to $75, 25¢; $75.01 to $100, 30¢.

ORAL EXERCISE

Referring to the above list, state the cost of money orders for:

1. $31.50.
2. $19.90.
3. $52.75.
4. $86.50.
5. $92.30.
6. $69.95.
7. $16.30.
8. $90.50.
9. $75.50.
10. $60.
11. $80.
12. $100.

284. Paying by Bank Draft. One of the common methods of paying a debt due in another place is by means of a bank draft.



No. 48164

MERCHANTS NATIONAL BANK OF AUSTIN

AUSTIN, TEXAS, July 6, 1919

Pay to the order of John Roberts $\$78.^{75}$

Seventy-eight and $\frac{75}{100}$ Dollars

To The Chemical National Bank,
New York City

A. B. Smith
Cashier

A draft is the same as a check, except that it is made by the cashier of some bank and is drawn on another bank.

Banks usually charge a slight premium on the face of the draft. Thus, a \$250 draft at 0.1% premium would cost \$250, with the premium at 0.1% of \$250, or \$250.25 in all.

If John Roberts, who purchased the above draft, owed Robert Jones the money, he could indorse it thus:

Pay to the order of Robert Jones
John Roberts

It might have been made payable directly to the order of Robert Jones in the first place, but this is not the custom.

Drafts on large money centers are usually cashed for customers of the bank without any discount.

How much would the above draft cost at 0.1% premium?

0.1% of \$78.75 = \$0.08, to the nearest cent.

The bank, however, would probably charge 10¢ to make a convenient amount. Therefore the draft would cost \$78.85.

WRITTEN EXERCISE

1. How much will a draft for $300 cost at $\frac{1}{5}$% premium?

2. How much will a draft for $3200 cost at 0.1% premium? a draft for $2500 at $\frac{1}{20}$% premium?

3. How much will a postal money order for $37.50 cost? for $62.75? for $14.30? for $86.50? for $75.40?

4. How much will a Camden merchant pay for a draft on Chicago for $4000 at 40¢ premium per $1000?

5. When the government charges 30¢ per $100 for a money order, what per cent premium does it charge?

6. A draft cost a merchant $2752.75, including 0.1% premium. What was the face of the draft? What was the premium?

7. A draft cost a merchant $3751.50, including the premium of 40¢ per $1000. What was the face of the draft?

8. If a man owes bills of $250, $150, $100, and $350 to different jobbers from whom he buys goods, how much will drafts for these amounts cost at 0.1% premium?

9. Which is cheaper for you if you owe $75 for some goods, to send a money order, or a draft for which you have to pay 15¢ premium? how much cheaper?

10. It costs 10¢, besides the regular postage, to register a letter. If the letter is not delivered, the government will pay the loss, up to $50. If you owe $22, is it cheaper for you to send it by registered letter or by money order? how much cheaper?

11. If you owe $100 to a manufacturer at a distance, from whom you have bought goods, how will you make the payment? Tell why you will make it in that way, and how much it will probably cost. The premium on a draft may be taken at the common rate of 0.1%.

285. Rate of Exchange. A money order is always sold at a slight advance over its face, and usually a bank draft costs more than its face. In each case the rate of the variation from the face is called the *rate of exchange.*

286. Premium. If the exchange is added to the face, exchange is *at a premium,* as we have already seen.

287. Par. If there is no exchange, exchange is *at par.*

288. Discount. If the exchange is deducted from the face, exchange is *at a discount.*

For small sums, say for $500 or less, New York, Chicago, or Philadelphia exchange usually sells at a premium of about 0.1%. This is to pay the bank for its trouble and for the expense of shipping the money when its balance at these cities gets low. Banks usually buy such drafts at their face value, thus making no charge for cashing them.

For large sums the rate of exchange varies. If the Chicago banks owe the New York banks $2,000,000, they must send that amount by express, an expensive proceeding. If a man in Chicago at that time wished to buy a draft on New York for $30,000, they would charge him more than usual because they would have to express that much more to New York; but if a man in New York wished to buy a draft on Chicago, he might buy it for less than $30,000 because the bank would get its money at once, and the risk and expense of transmitting it would be saved.

The premium or discount is usually quoted as a certain per cent of the face of the draft, but sometimes as so much on $1000, a fraction of $1000 counting as $1000. In the latter case the quotation of $\frac{1}{4}\%$ premium is the same as that of $2.50 premium.

The explanation of the *Clearing House*, a place where bank officials of a city meet daily to exchange drafts and checks and to pay balances due one another, is too technical for most classes, and if given at all, this should be done orally.

WRITTEN EXERCISE

Find the cost of the following drafts:

1. $3756.70, at par.
2. $750, 0.2% premium.
3. $2450, 0.1% premium.
4. $3500, 0.1% discount.
5. $6750, $\frac{1}{20}$% premium.
6. $17,500, 0.2% discount.

7. What is the cost of a draft on San Francisco for $5200 at $\frac{3}{4}$% discount? Write the draft.

8. What must be paid in Rahway for a draft on Chicago for $3400, exchange being at $\frac{1}{4}$% premium?

9. A draft for $4800 was bought for $4794. Was exchange at a premium or a discount? What was the rate?

10. When a Boston draft for $35,000 can be bought in New Orleans for $34,930, is exchange at a premium, at par, or at a discount? What is the rate?

11. If J. R. Glover draws on J. B. Thornton for $250, and the banks charge 0.1% for collection, what are the net proceeds that Glover will receive? Write the draft.

12. My agent in Toronto sells a house for me for $2500. He charges 2% commission, and the bank charges $2.50 premium for a draft for any amount between $2000 and $2500. What sum does he remit to me?

13. A telegraphic money order costs twice the rate for a ten-word message, plus 1% premium on the face. A ten-word message from Trenton to Kansas City costs 50¢. How much will a telegraphic money order for $375 cost?

14. Mr. Edgcomb of Denver owes $3243.24 to Mr. Nourse in that city. Mr. Nourse has gone to New York on business and asks that the money be sent, less the cost of exchange. Exchange being at 0.1% premium, what is the face of the draft?

REVIEW PROBLEMS

1. Find the amount of principal and interest of $500, compounded semiannually, at the rate of 4%, for 5 yr.

2. A man deposits $750 in a savings bank on July 1, 1918. His further deposits are as follows: Aug. 1, 1918, $30; Sept. 15, 1918, $25; Dec. 31, 1918, $100; June 15, 1919, $50. He draws out money as follows: Oct. 12, 1918, $40; June 20, 1919, $75; Aug. 2, 1919, $[illegible]; Aug. 15, 1919, $30; Sept. 12, 1919, $25. The savings bank pays 1% every three months on the smallest balance for the preceding quarter, on Jan. 1, April 1, July 1, and Oct. 1. Find his balance on Jan. 1, 1920.

3. Make out a deposit slip for the following deposits, naming some bank in your vicinity: bills, $575; gold, $35; silver, $72.25; checks, $48.25, $72.50, $61.30, $227.52, and $86.95. If your balance was $326.42 before, how much was it after the deposit?

4. A merchant's balance on the first of the month was $1725.60. He has since deposited $37.50, $296.75, $21.30, $572.32, and $217.65. He has given checks for $128.92, $27.36, $18.92, $78.76, and $273.96. How much is his present balance?

5. Make out a 90-day note for $1250, dated to-day, payable to R. P. Foster, or order, at some bank in your vicinity. Discount it at 6%. If with the proceeds you buy 10 shares of stock and pay $108.50 a share for them, how much money will you have left?

6. Which is the greater, the proceeds of a note for $750 for one year, drawing interest at 6%, and discounted on the day of its date at 5%, or the amount of a note for $700 for one year at 6%? How much greater?

In this review, or earlier in this chapter, pupils should be encouraged to tell what plans they have for saving money, and in what form of investment they would put their money if it amounted to a considerable sum. This may be tactfully done so as to encourage no boasting or rivalry.

LITTLE EXAMINATIONS

I. *Find the cost of postal money orders for:*

1. $2.75. **2.** $5.50. **3.** $10.75. **4.** $35.

Refer to the rates given on page 158. Each answer should include the amount sent and the charge made by the post office.

Find the cost of these drafts at the premiums stated:

5. $3750, 0.1%. **6.** $4800, 0.2%. **7.** $725, $\frac{1}{4}$%.

Find the cost of these drafts at the premiums per $1000 stated:

8. $1500, $1.50. **9.** $3750, $1.60. **10.** $25,750, $1.80.

II. *Find the cost of these drafts at the discounts stated:*

1. $7500, 0.1%. **2.** $4500, $\frac{1}{5}$%. **3.** $25,000, $\frac{1}{20}$%.

4. Find the discount on a note for $725 for 60 da. @ 6%.

5. Make out a check for $2.75 and a bank draft for $50.

6. Make out a promissory note for $17.50.

7. Make out a deposit slip for four items deposited.

8. Make out a commercial draft for $75.

9. What is the cost of a draft for $1750 at par?

10. Find the cost of a draft for $1800 at 0.1% premium.

III. *Find the discounts and the proceeds on the following:*

1. $350, 30 days, 6%. **4.** $375, 60 days, 6%.

2. $625, 60 days, 5%. **5.** $4275, 60 days, 5%.

3. $800, 60 days, 6%. **6.** $3450, 90 days, 5%.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

	I	II	III	IV
a	$275	0.1% premium	$217.50	0.3% premium
b	$450	0.2% discount	$316.75	$\frac{1}{20}$% premium
c	$1250	0.1% discount	$425.25	$\frac{1}{20}$% discount
d	$1500	0.2% premium	$675.75	0.4% discount
e	$1875	0.3% discount	$842.36	0.4% premium

1. For each sum in columns I and III above find, in turn, the premium or the discount at each of the rates in II and IV.

This gives 100 examples. The teacher should select as many as necessary.

	I	II	III	IV	V	VI
a	$\frac{1}{32}$	0.23	41.3	1.276	$6\frac{1}{4}$	12,250
b	$\frac{1}{16}$	0.32	62.1	2.007	$7\frac{3}{5}$	13,175
c	$\frac{1}{8}$	0.41	67.9	3.982	$8\frac{2}{3}$	16,022
d	$\frac{5}{32}$	0.68	71.8	4.871	$9\frac{1}{6}$	19,685

In each of the six columns above perform the following operations, carrying each decimal fraction to two decimal places:

2. $a + b$.

3. $b - a$.

4. $a + d$.

5. $a + c$.

6. $d - a$.

7. $c \div a$.

8. $c \times a$.

9. $b \times d$.

10. $b + c$.

11. $c - b$.

12. $c + d$.

13. $d - c$.

14. $a \times b$.

15. $b \div a$.

16. $a \times d$.

17. $d \div a$.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. How do you ascertain the interest due on the first day of the year on an account in a savings bank?

2. Which pays the better interest at the same rate, if the money is left undisturbed for five years, a promissory note or a deposit in a savings bank? Why?

3. How do you fill out a deposit slip? After entering the items, what operation do you perform? How do you make sure that the result is correct?

4. How do you find the discount on a promissory note?

5. If a note drawing a certain rate of interest is discounted on the day it is made, at the same rate, are the proceeds greater than the face, or equal to it, or less? Why is this?

6. How can a manufacturer discount a claim against a purchaser, the claim not being yet due? How is the discount found?

7. If you know the proceeds and the rate of discount, how do you find the face of a note?

8. If you know the face of a note and the proceeds, how do you find the discount?

9. If you know the face of a note and the proceeds, how do you find the rate of discount?

10. If you change money from a postal savings-bank to a government bond, how much higher rate of interest do you get?

11. If a dealer buys some produce and agrees to pay for it in 90 days, with interest at 6%, and the seller takes a note and at once discounts it at 5%, will he receive more than the selling price, or the same amount, or less? Why?

III. INDUSTRIAL PROBLEMS

MONEY IN FERTILIZERS

1. George's big brother has come home from the agricultural college and his father decides that on their farm they ought to try more modern methods of feeding their plants. His brother tells him that they need nitrogen, phosphoric acid, and potash, and that if these are put in the soil by means of fertilizers the plants grow better. George's brother tried applying $2.16 worth of fertilizer to one acre of a field, and the increase in the yield of corn was 8.6 bu. Corn being worth 40¢ a bushel, what was the value of the increase? The value of the increase was how much more than the fertilizer cost?

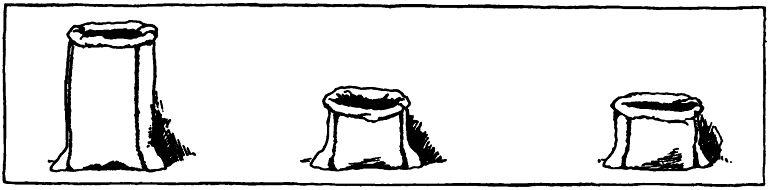
2. George's brother tried another kind of fertilizer on another acre of corn, paying $3.46 for this kind. He increased the yield 7.2 bu. If corn was worth 40¢ a bushel at that time, was this experiment profitable or not? Find the gain or loss.

3. George's brother knew of another fertilizer, said to be excellent for wheat, and put $5.19 worth on an acre. The increase in yield was 8.4 bu., and wheat was then selling on the farm at 80¢ a bushel. What was the gain or loss in this case?

4. George's brother explained to him that a complete fertilizer is one containing all three of the foods that a plant needs. These are nitrogen, phosphoric acid, and potash. A formula written 2–8–2 means that this fertilizer contains 2% nitrogen, 8% phosphoric acid, and 2% potash. How many pounds are there of each of these foods in a ton of 2–8–2 fertilizer?

In trade, the order of the formula given in Ex. 4 is often stated thus: nitrogen, potash, phosphoric acid instead of nitrogen, phosphoric acid, potash. In this book the order recommended by experiment stations and given in Ex. 4 is followed. Pupils should inquire as to local customs.

5. George's brother showed the picture given below, representing different yields of wheat, and asked him to find how many pounds of nitrogen, phosphoric acid, and potash there are in 300 lb. (the amount used per acre) of the fertilizer that gave the 35-bushel yield. How many pounds are there of each?

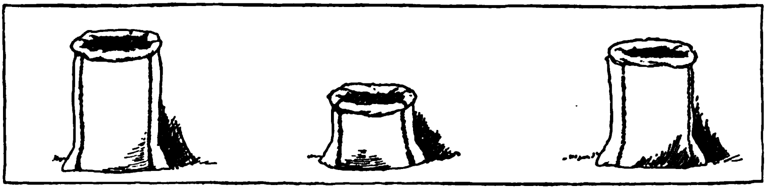


4-8-4 FORMULA
35 BU. PER ACRE

UNFERTILIZED
15 BU. PER ACRE

4-0-4 FORMULA
17 BU. PER ACRE

6. In the same picture, what is omitted in the 17-bushel yield? How many pounds of each of the other important plant foods are there in 300 lb. of this fertilizer? How many are there in a ton?



4-8-0 FORMULA
33 BU. PER ACRE

UNFERTILIZED
15 BU. PER ACRE

0-8-4 FORMULA
30 BU. PER ACRE

7. In the above picture, which also represents different yields of wheat, how many pounds of each of the three plant foods are there in 300 lb. of fertilizer?

Teachers in rural schools should talk with practical farmers on this subject and get the pupils to do so. Different localities have different soils, and hence the question of fertilizers is to some extent a local one. Agricultural colleges are glad to give farmers expert advice as to the best fertilizers to use for various crops in various kinds of soil. In rural schools the pupils should be encouraged to make inquiries as to the success of commercial fertilizers.

CHAPTER XII

CIVIC AND BUSINESS RELATIONS

289. Meaning of Taxes. Some boys in school wanted to organize a baseball team. They found that they could rent a piece of land for a ball field for $4 a month, and decided that they needed $5.50 at once for balls and bats. Each boy bought his own suit, but they had to raise $9.50 for the first month. They decided that ten boys should be allowed to join, and that each should pay his share, which was 95¢ for the first month. This 95¢ was a *tax* upon each member of the club.

A class wished to buy a picture for the schoolroom. There were 30 pupils in the class, and the picture cost $3.90. Each pupil agreed to pay his share. A committee was appointed, and each member of the class was taxed 13¢. Here the *total tax* on the class was $3.90, the tax of each pupil was 13¢, the committee *levied a tax* of 13¢ on each, and a *collector* received the money.

A village needed $8000 for building a new schoolhouse, and its citizens voted to raise the amount by a tax. The sum was so large that it would not be fair to make each citizen pay the same amount, so each one was required to pay according to the value of his property. In this way those who had more property were taxed more than those who had little property. A man who owned $10,000 worth of property was required to pay twice as much as one who owned $5000 worth, and half as much as one who owned $20,000 worth. This is the rule of taxation usually followed by states, cities, villages, and counties.

290. Expenses of a City, County, or Village. It costs a large amount of money to run a city, county, or village. In a city the streets have to be cleaned, lighted, and kept in repair; the policemen have to be paid; the fire department has to be kept up; the city buildings have to be cared for; the city officers have to be paid; the schools have to be supported; and there are many other expenses. In the county the roads have to be kept in good condition, bridges have to be built and kept in repair, and the officers have to be paid.

For example, suppose that all the expenses of a certain city amount to about $470,000. The city may decide to attempt to raise by tax $480,000, so as to have a little margin. If the property of the citizens is valued at $60,000,000, each dollar of property ought to pay $480,000 ÷ 60,000,000, or $0.008; that is, every dollar should pay 8 mills. In this case 8 mills is called the *tax rate.*

291. State and Local Taxes. State and local taxes are usually a certain per cent levied on the land, money, and other property of individuals, business concerns, and corporations.

292. Assessors. The property to be taxed is valued by officers called *assessors.* The value placed by the assessors upon property for taxation is called the *assessed valuation.*

293. Tax Rate. Upon the assessed valuation a certain *tax rate* is fixed. The words *tax rate* are often used to designate the *number of mills* of tax on each dollar of valuation. Thus, a tax of $5\frac{1}{4}$ mills means $5\frac{1}{4}$ mills on a dollar.

Male citizens over 21 years of age pay a *poll* (head) *tax* in many places. This tax may be levied by the state, the county, or the local taxing body, or by all three, and the amount of the poll tax varies, but it is usually between $1 and $3.

294. Finding the Tax Rate. If a village with an assessed valuation of \$3,200,000 must raise \$16,800, what is the tax rate?

$$\$16{,}800 \div 3{,}200{,}000 = \$0.005\tfrac{1}{4}.$$

$$\begin{array}{r} \$0.005\frac{1}{4} \\ 32\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\overline{)\$0.168\cancel{0}\cancel{0}} \\ \underline{160} \\ 8 \\ \frac{8}{32}=\frac{1}{4} \end{array}$$

That is, for every dollar of property that a person living in the village owns, he must pay 5¼ mills of tax. Although, as we have seen, the mill is not coined, it is sometimes used for such purposes as this.

295. Finding the Tax. After the tax rate has been determined for the village, city, county, or other political division, the tax of each taxpayer can be found.

If a man's property is assessed at \$12,000, for example, and the tax rate is 5¼ mills, he must pay 5¼ mills on every dollar; that is, he must pay 12,000 × 5¼ mills; or, what is the same thing, he must pay 0.005¼ × \$12,000, which is \$63.

The method of collecting the tax varies so much in different parts of the country that the teacher should have the pupils inquire at home and bring the information to school.

WRITTEN EXERCISE

1. If the tax rate is 8 mills on \$1, what are the taxes of a man whose property is assessed \$5000?

2. If the tax rate is \$2.18 on \$100, what are the taxes of a man whose property is assessed \$8000?

Find the taxes paid on the following, at the rates specified:

3. \$8500, 7½ mills on \$1.
4. \$9250, 7.4 mills on \$1.
5. \$16,500, \$1.46 on \$100.
6. \$13,800, \$0.45 on \$100.

7. The assessed valuation of a certain city is \$75,000,000 and the total tax levied is \$675,000. What is the tax rate?

WRITTEN EXERCISE

Find the taxes at $5\frac{1}{2}$ mills on the following amounts:

1. $6500.
2. $8750.
3. $4875.
4. $7675.
5. $12,000.
6. $16,500.
7. $24,400.
8. $23,500.
9. $31,100.
10. $23,250.
11. $18,875.
12. $45,250.

Find the taxes at 8 mills on the following amounts:

13. $4500.
14. $6500.
15. $8000.
16. $4600.
17. $7000.
18. $4550.
19. $8750.
20. $9750.
21. $6750.
22. $8500.
23. $12,000.
24. $25,000.
25. $32,500.
26. $38,500.
27. $45,250.
28. $75,750.

29. The tax rate is 9.6 mills. What is the tax on property assessed at $6800?

30. The tax rate is 12 mills. What is the tax on property assessed at $22,800?

31. The tax rate is $1.23 on $100. What is the tax on property assessed at $12,500?

32. If a man's tax is $195 and the rate is 13 mills on a dollar, find the assessed valuation of his property. What is the assessed valuation of the property of a man whose tax is $234?

33. If the state tax is 1.1 mill, the county tax 3.6 mills, and the town tax 4.2 mills, what tax must a man pay on $8000?

Local problems of this kind should be brought to class.

34. A man died, leaving an estate valued at $23,750, of which $22,000 was willed to his children and the rest to his nephew. Under the laws of his state there is no inheritance tax to be paid by children, but 4% must be paid by the nephew. What inheritance tax must be paid by the nephew?

The details of the inheritance tax varies in different states.

296. Expenses of the United States Government. The expenses of the United States government vary from year to year, but they average about $1,900,000 a day, or about $700,000,000 a year. Some items of our income and expenditures are as follows:

Income

Customs (duties on imported goods)	$300,000,000
Internal revenue (tobacco, etc.)	380,000,000
Income tax and miscellaneous	80,000,000

Expenditures

War Department	$174,000,000
Navy Department	140,000,000
Pensions	173,000,000
Indians	20,000,000
Salaries, diplomatic service, etc.	194,000,000

WRITTEN EXERCISE

1. When our income was $724,000,000, our customs receipts were $319,000,000. Customs were what per cent of the income?

2. Taking our expenditures as $701,000,000 a year, what per cent of this goes to the War Department, as above stated? to the Navy Department? to pensions?

3. In a certain year the government received an income of $155,000,000 from taxes on spirits, $43\frac{1}{5}\%$ as much from taxes on tobacco, and $4\frac{1}{2}\%$ less on fermented liquors than on tobacco. Find the total receipts from these sources.

4. In a certain year the internal revenue receipts amounted to $344,000,000. The largest amount paid by any state was $16\frac{2}{3}\%$ of this total, which was paid by Illinois. What was this amount?

297. Tariff. The United States collects a large part of its income by a tax on goods brought into the country. This income is called *customs revenue*, *tariff*, or *duty*.

298. Customhouse. Customs revenue is collected at customhouses. These are situated at *ports of entry*.

299. Classes of Goods. Goods imported may be on the *free list*, and not subject to duty, as raw silk; subject to *ad valorem* (on the value) duty, a certain per cent on the value at the place of purchase, as books, on which the duty is 15% ad valorem; subject to *specific* duty, a certain amount per bushel etc., as apples, on which the duty is 10¢ per bushel of 50 lb.; subject to both *ad valorem* and *specific* duty, as perfumery, on which the duty is 40¢ per pound and 15% ad valorem.

ORAL EXERCISE

State the duty on the following:

1. 700 lb. of firecrackers, duty 6¢ per pound.
2. 1 T. of Roman candles, duty 10¢ per pound.
3. $100 worth of beads, duty 35%.
4. $800 worth of buttons, duty 25%.
5. $200 worth of dolls, duty 35%.
6. $700 worth of furs, duty 40%.
7. $500 worth of fans, duty 50%.
8. $2000 worth of jewelry, duty 50%.
9. $1250 worth of pearls, duty 20%.
10. 250 doz. pairs of gloves, duty $1 per doz.
11. 7000 pineapples, duty $5 per thousand.
12. 6000 lb. of cork bark, duty 4¢ per pound.

WRITTEN EXERCISE

1. How much is the duty on $4350 worth of needles at 20%? Is this specific duty or ad valorem duty?

2. How much is the duty on 3500 bu. of barley at 15¢ per bushel? Is this specific duty or ad valorem duty?

3. Find the duty on 2750 lb. of hops at 16¢ a pound.

4. How much is the duty on $7565 worth of horses at 10%?

5. How much is the duty on 9750 bu. of oats at 6¢ a bushel?

6. A dealer imports 375 lb. of maple sugar from Canada. How much is the duty at 3¢ per pound?

7. A grocery house imports $1250 worth of cheese from Canada. How much is the duty at 20%?

8. A dealer imports 2750 lb. of butter. How much is the duty at 2½¢ per pound?

9. The duty on honey is 10¢ per gallon. If a dealer imports 975 gal., how much is the duty?

10. A dealer imported 475 T. of straw, paying duty at the rate of 50¢ per ton. How much duty was paid?

11. How much is the duty on 15 sets of Thackeray's works at £7 (7 English pounds) a set, allowing $4.87 to the pound, the duty being 15%?

12. How much is the duty on 2 doz. sets of Scott's works at £4 10s. (4 pounds 10 shillings, or £4½) a set, allowing $4.87 to the pound, the duty being 15%?

13. How much is the duty on 2000 yd. of carpet, invoiced at $1.50 a yard, at 50%?

14. The duty on cigars is $4.50 per pound, plus 25% ad valorem. If the consignment weighs 275 lb. and is worth $825, how much is the total duty?

300. Income Tax. The United States secures part of the money needed to run the government by means of a tax on incomes. Every citizen of the United States, whether residing at home or abroad, and every person residing in the United States, is required to pay to the government 1% of his income each year, $3000 of the income being exempt from taxation. In the case of a husband and wife living together, their combined income is taxed only on the amount that exceeds $4000. Persons whose incomes exceed $20,000 per year are taxed more heavily; there is, for example, an extra tax of 1% on incomes in excess of $20,000 and not exceeding $50,000, and this extra tax increases in rate as the income becomes larger.

The minor details and other exemptions of this tax need not be considered in school.

For example, a married man has an income of $5000 a year. How much is his income tax?

He has an exemption of $4000, as a married man.
He pays 1% on the excess.
Therefore his income tax is 1% of $1000, or $10.

$$\begin{array}{r} \$5000 \\ 4000 \\ \hline \$1000 \\ 0.01 \\ \hline \$10 \end{array}$$

The income in this case means the total income of husband and wife.

WRITTEN EXERCISE

1. An unmarried man has an income of $3500 a year. What is his income tax?

2. A married man has an income of $3500 a year, and another has an income of $7500 a year. Find the income tax of each.

3. A married man has an income of $25,000. What is his regular income tax of 1%? He pays an extra 1% on the excess of his income above $20,000. How much is his total tax?

4. In Ex. 3, what would be his total income tax if the man were not married?

301. Taxing the Manufacturer. Not only do persons pay taxes to the government, but manufacturing establishments have to do the same. These establishments are usually taxed by cities and states on the value of their property, and by the government on their income and through duties on imports.

WRITTEN EXERCISE

1. A manufacturer needs some crucible plate steel. He decides to buy $12,650 worth in England. The duty is 12% ad valorem. How much does the shipment cost including the duty, but not including the freight?

2. A manufacturer needed a supply of galvanized iron wire. He placed the order in Germany for $5467 worth. How much duty must he pay, the rate being 15%?

3. In a large factory a time detector records, by electricity, the places visited by the watchman in his rounds, and the hours. The factory purchased a detector in Germany for $286, and paid 15% duty on it and $7.20 transportation charges. How much did it cost the factory?

4. A clothing manufacturer is taxed on his manufacturing plant. This is assessed at $75,000, and the tax rate is 6½ mills. How much is his tax?

5. The income tax applies also to the entire net income of every business carried on by a corporation in the United States. If the net taxable income of a certain corporation is $17,500, what is the amount of the tax?

6. A manufacturing establishment wishes to import some lithographic advertising matter from Germany. The quality of material desired is subject to a duty of 7¢ per pound, and the amount desired weighs 748 lb. What is the amount of the duty?

302. Nature of Insurance. If your house should burn, the loss would be very great to you and your family. It would come suddenly, without any chance of providing for it, and a long time might be needed to save the money for a new house. A sudden calamity like this means a great loss to those who bear it. On this account people sometimes join together and contribute a small sum each year, and if the house of any one of them burns, they pay him enough to build a new house from the money they have collected. Thus no one of them suffers any great loss, and every one feels sure that if a fire comes he will be protected as far as possible.

This shows the general nature of insurance. Usually, however, instead of a number of people joining together and paying a little each year, they pay annually a small sum (a *premium*) to an insurance company, which agrees to pay for some specified loss. This company goes to the trouble of getting many people to join for protection and collects their contributions, charging a certain amount for its work.

303. Insurance. A written agreement to compensate any one for some specified loss is called *insurance.*

304. Policy. The written agreement of an insurance company to pay a certain amount in case of loss is called a *policy.*

305. Face of the Policy. The amount specified to be paid in case of loss is called the *face of the policy.*

The teacher should encourage the pupils to make inquiry as to the insurance of houses and of various forms of property in the vicinity, and to report to the class, avoiding all matters of a purely personal nature. In particular, the value of life and accident insurance, of fire insurance, and, in some places, of tornado insurance and plate-glass insurance should be mentioned. In states where there is a workmen's compensation act, it should be explained that this forms a kind of insurance, and the leading features should be ascertained, particularly with reference to the industries of the locality.

306. Premium. The cost of insurance is called the *premium*. The *rate of premium* in fire insurance is usually stated as a certain sum for each $100 of the face for a certain period of years, generally from 1 yr. to 5 yr.

What is the premium for insuring a store for $4000 against loss by fire, at $1.20 a year?

40 × $1.20 = $48, the premium.

ORAL EXERCISE

1. What is the premium on a policy for $650 at 80¢?

2. What is the premium on a policy for $3000 at 60¢?

3. A man paid $50 for insuring a house, the rate being $2.50. What was the face of the policy?

State the premiums on the following policies at the rates specified:

4. $1000, 50¢.
5. $2000, 80¢.
6. $1250, $1.
7. $4250, $2.
8. $1750, $1.
9. $3750, $2.
10. $725, $2.
11. $825, $2.
12. $3500, $1.20.
13. $6250, $2.25.
14. $10,000, $0.80.
15. $50,000, $1.50.

State the faces of the policies, given the following premiums and rates:

16. $25, $2.50.
17. $17.50, $1.
18. $40, $2.
19. $80, $1.
20. $13.75, $1.
21. $18.50, 50¢.

State the rates, given the following faces of policies and premiums:

22. $5000, $25.
23. $6000, $60.
24. $5000, $50.
25. $4000, $80.
26. $7500, $150.
27. $6500, $130.

28. A building worth $8000 is insured for $\frac{3}{4}$ of its value at $1. What is the premium?

WRITTEN EXERCISE

1. At $0.95, what is the premium on a $2500 policy?

2. At $1.10, what is the premium on a $2800 policy?

3. At $1.15, what is the premium on a $3750 policy?

4. A building worth $12,000 is insured for $\frac{2}{3}$ of its value at $1.10. What is the premium?

5. If you insure the contents of your store for $6000, what is the premium at $1.25?

6. The premium for insuring some property at $1.50 is $52.50. What is the face of the policy?

7. A man insured his factory, valued at $135,000, for $\frac{4}{5}$ of its value, at $1.90. What was the premium?

8. A dealer insured his stock of goods, valued at $14,000, for $\frac{7}{8}$ of its value, at $1.65. What was the premium?

9. If a three-year policy for $1750 costs $27, what is the rate of premium for the 3 years? What is the rate per year?

10. If a three-year policy for $3000 costs $36, what is the rate of premium for the 3 years? What is the rate per year?

11. A merchant insures his stock for $\frac{3}{4}$ of its value, at $1.75. The premium is $131.25. What is the value of the stock?

12. A manufacturer insures his factory for $\frac{5}{8}$ of its value, at $2. The premium is $210. What is the face of the policy? What is the value of the factory?

13. A factory worth $33,000 is insured for $\frac{5}{6}$ of its value, at $2. In case of total loss, how much would the owner lose, including his premium paid?

14. A man insured his library for its full value, $2500, at $1.25. What was the premium? If a fire destroyed $\frac{3}{5}$ of the library, how much could he recover?

307. Life Insurance. If a man is careful as to the future and considers the fact that in later years there will probably be one or more persons dependent upon him, he very likely will wish to insure his life. The earlier this is done, the lower will be the premium, and the sooner he will cease to make payments if he takes out a 10-payment, 15-payment, or 20-payment policy.

Premiums in life insurance are stated at so much on $1000. They vary with the age of the person insured, reckoned at the time he takes out his policy.

The best plan is to take a policy in a company of unquestionable standing, and never forfeit it by nonpayment of premiums.

ORAL EXERCISE

State the premiums on the following policies, the premium on $1000 being given:

1. $2000, $26.
2. $3500, $25.
3. $2000, $21.
4. $4500, $30.
5. $3000, $22.
6. $10,000, $29.
7. $20,000, $28.
8. $25,000, $32.
9. $30,000, $32.
10. $20,000, $27.
11. $4000, $22.50.
12. $12,000, $25.
13. $5000, $28.40.
14. $2500, $22.20.
15. $1000, $27.65.

State the faces of the policies, given the following premiums and the premiums on $1000:

16. $131, $26.20.
17. $175, $25.
18. $55, $27.50.
19. $49, $24.50.
20. $96.60, $32.20.
21. $63.90, $21.30.

State the premiums on $1000, given the following policies and premiums:

22. $2000, $30.
23. $3000, $84.
24. $4000, $96.
25. $5000, $150.
26. $5000, $105.50.
27. $6000, $180.50.

308. Kinds of Policies. The following are four of the leading kinds of life-insurance policies:

Ordinary Life, the insured agreeing to pay during his life a certain premium, usually annually or semiannually.

The rate is always given as the cost of $1000 worth of insurance; that is, the rate $25.50 means that the annual premium on a policy for $1000 is $25.50. Some insurance companies allow *dividends* each year, thus reducing the premium slightly.

Limited Life, the premiums being payable for some fixed number of years, as twenty, after which the policy is called *paid up* (no more premiums being due), but the face is not paid until the death of the insured.

Naturally the premiums are higher on this form.

Endowment, the premiums being paid for some fixed number of years, as ten, fifteen, or twenty, at the end of which time the face of the policy will be paid to the insured.

Since this may be paid during the lifetime of the insured, the premiums are also higher than on an ordinary life policy.

Term Insurance, the premiums being paid for a specified length of time and the face of the policy being payable if the insured dies within the term of insurance.

Thus a man may insure his life or his health for thirty days, as in certain forms of accident insurance. He may also insure his life for a certain number of years only. The premium on this form of policy is low, since the company may not have to pay the face at all.

There are various other forms of policies, but most of them are modifications of the above types. Some are arranged so that the face will not be paid on the death of the insured, but a certain amount will be paid annually (an annuity) during the lifetime of the one for whose benefit the policy was written.

WRITTEN EXERCISE

1. What is the annual premium that a man must pay on a 10-year endowment policy for $4000 at $102.60? How much would he pay in the 10 years?

2. If a young man takes out a 20-payment policy for $5000 at $27.39, how much will he have paid when the policy matures; that is, after 20 payments?

3. What is the premium on a 20-payment policy for $5000, taken out by a man at such an age that the rate is $34.20? How much are the total premiums for the 20 years?

4. What is the premium on a 20-payment policy for $7500, taken out by a man at such an age that the rate is $32.50? How much are the total premiums for the 20 years?

5. If a man takes out a $2500 policy in one company at $26.40 and a $3500 policy in another at $23.50, how much are his annual premiums?

6. A young man takes out a $10,000 policy on the 25-payment plan, at $26.36. How much is the annual premium? How much are the premiums for the 25 years?

7. A man took out a $5000 policy at $26.40. He died just before the eighth annual payment was due. How much did his estate receive above what he paid to the company? How much would the man have saved in the 7 yr. if he had put the amount of the premium each year at simple interest at 5%?

8. Twenty years ago a young man took out a 20-payment policy for $5000, paying $136.95 a year. How much has he paid in the 20 years? Estimating that the insurance company has had the use of all this amount for the equivalent of 10 years, at 5%, what is the total amount received by the company?

309. Investments. After a man has saved some money either he will use it to pay for something he wishes to buy, or he will invest it. A man feels much more independent if he has saved money for use in case of long illness or for the purchase of a home.

The subject of investment in stocks and bonds is presented briefly on pages 437–445, but, if any pupils know about such investments, it is well to discuss the subject. The teacher may speak to the class along these lines: Having saved money by wise economy, it is essential that we should know how to take care of it. Formerly people hoarded their savings in hidden places about their homes. It is customary now to invest money that is not needed for immediate use, so that it will not only be safer than when kept at home but will also bring some income. Unfortunately some of the investments offered to the public are offered by dishonest men and are not desirable, but many others are perfectly safe. It is well to remember that the larger the income offered by any form of investment, the greater is likely to be the risk of losing the money put into it. Would it be better to put money into an investment which promised a large income, but which might not be safe, or into something which promised a very small income, but which was perfectly safe, like United States bonds? We should learn how to distinguish between good investments and bad investments so that we may invest wisely whatever money we may be able to save.

WRITTEN EXERCISE

1. If you have $75 to invest and deposit it in a savings bank that pays 2% interest semiannually, how much interest is due you at the end of 6 mo.?

2. If a man has $1250 to invest and can so invest it as to receive 4% interest, how much will it bring him each year?

3. A man has deposited $225 in a savings bank that pays 2% interest semiannually. How much interest is due him in 6 mo.?

4. A woman has saved $1750. She can deposit it in a savings bank at 4%, or invest it in a mining business which she knows nothing about and which the bankers in her town have never heard of but which advertises that it will probably pay 10% a year. How much income would she get from the bank? Which investment should she make?

5. A man has saved $3500. He can invest it in his city in a well-managed manufacturing business of which he is thoroughly informed, and can get 7% on his investment. Is this a good investment? How much income will he receive a year?

6. A man comes to town and offers investments in a fruit plantation in Mexico. No one has heard of the man or of his plantation. He says the business will pay 15%, and he asks Mr. Doe to invest $7500. How much income does he promise Mr. Doe? Mr. Doe can invest the money in his own town and get 6%. How much would this bring him per year? Which investment would you advise him to make? Write the reason.

7. How much better income will a man receive on an investment of $6500 at 6% than on one of the same amount at 4½%?

Find the incomes from the following investments:

8. $400, 5%.
9. $475, 5%.
10. $575, 4%.
11. $725, 4%.
12. $650, 6%.
13. $875, 6%.
14. $925, 5½%.
15. $875, 4½%.
16. $1250, 4%.
17. $1575, 4%.
18. $6250, 5%.
19. $7500, 5%.
20. $8750, 6%.
21. $8225, 6%.
22. $6500, 3%.
23. $7500, 4½%.
24. $8500, 7%.
25. $2500, 7%.

26. Mr. Sinclair has $7500 invested at 6% and $9000 at 5%. Which, if either, brings him the greater income, and how much greater? Which investment would you rather have? Why?

27. How does the income from $9000 at 8% compare with that from $12,000 at 6%.

28. Mrs. Radcliffe has inherited $2500, and has a chance to invest it at 5% in a first-class security or at 12% in a banana plantation that her banker tells her is very risky. How much would be the income from each? Which should she take?

310. Investing in Real Estate. Besides investing in banks and in promissory notes, by which an income is received in the form of interest, people who have saved some money often buy real estate; that is, land and buildings.

The following suggestions relating to this subject should be read and understood:

Real estate makes a particularly desirable form of investment because of its comparative safety, of the tendency of wisely selected property to increase in value, and of the possibility of using such property for a home for the owner and thus saving the expense of renting.

The disadvantages of this form of investment are first, the expense of maintaining the property, insurance, taxes, repairs, etc., together with the loss of interest on investment; second, the liability to depreciation in value, which is, of course, generally greater with houses than with land; and third, the difficulty of selling the property readily at its full value.

Let each pupil in the class select some piece of property and ascertain the price at which it was held five years ago and its present price. What has been the gain or loss per cent during the five years? If it can be ascertained, the amount expended for insurance, taxes, repairs, etc., and the interest on the original investment and on the annual expenses for the five years at 4% or 5% should be deducted from the gain or added to the loss.

WRITTEN EXERCISE

1. Robert's father paid $4000 for a house. He rents it for $30 a month. He pays $44 a year for taxes, $4 for insurance, and $32 for repairs. What per cent does he receive on his investment?

2. Mr. St. John bought a house for $4500 and rented it for $28 a month. He paid annually $50 for taxes, $4.50 for insurance, and $45 for repairs. At the end of 3 yr. from the time when he purchased the property he sold it for $5200. How much more did he gain than he would have gained by investing the money for 3 yr. at 6%?

3. Mary's father bought a house for $3800 and placed on it a mortgage of $2000. This means that some one else had a legal claim against the property for this amount, with a paper called a mortgage allowing him to sell the property in case his claim was not paid when due. The mortgage drew interest at 6%. Allowing $50 a year for taxes, insurance, and repairs, and supposing that Mary's father could have invested his $1800 in a bank at 4%, how much does the house cost him each year?

4. A man owns a house which he rents for $30 a month, but in 3 yr. the house is vacant for 2 mo. The house cost him $4500, and his annual expenses for taxes, repairs, and insurance are $87.50. What rate of income does he receive?

5. Mr. Jewett buys a house for $4500, mortgaging it for $2000 at 6%. His annual expenses for taxes, repairs, and insurance are $75, and he rents the property for $28 a month, paying a collector 5% for collecting the rent. After 3 yr. he sells the property for $5200 and pays the mortgage. Find the difference in his income and the income that he would have had for the 3 yr. if he had invested the money at 6%.

6. Mr. Davids rents a house in St. Louis for $45 a month. He pays $65 a year for taxes, $30 for insurance and repairs, and $130 which the city assesses him for paving the street. How much is his net income? This being 5% on the cost of the property, how much did the property cost?

7. Mr. Carson buys a house and lot for $4200, mortgaging the property for $1800 at 6%, and paying the balance in cash. His annual expenses for taxes, repairs, and insurance are $68, and he rents the property for $25 a month. After 2 yr. he sells it for $3000, the buyer agreeing to pay the mortgage. Find his gain, assuming that he could invest the money at 6%.

311. Business Messages. Formerly when one business man wished to communicate with another he had to go to see him or send a letter by a messenger. To-day he can send a letter to any part of our country, including our island possessions, to Panama, Cuba, Mexico, Canada, and certain parts of Europe for 2¢. For 5¢ he can send a letter to any part of the world. The telegraph and telephone are also extensively used.

The writing of business letters, including the proper way of addressing envelopes, should form part of the work in writing, but it is not part of arithmetic.

WRITTEN EXERCISE

1. A telegram to a certain place costs 25 and 2, meaning 25¢ for 10 words and 2¢ for each additional word. How much will a 36-word message cost?

2. If the rate is 50 and 4, what will a 45-word telegram cost?

3. A "night letter" sent by telegraph allows 50 words or less to be sent at the 10-word day rate. If the day rate is 75¢, how much will a night letter of 100 words cost?

4. A "day letter" sent by telegraph costs 50% more than a night letter. If the day rate is 50¢, how much will a day letter of 150 words cost?

5. A cable message from New York to Paris costs 25¢ a word. Find the cost of a 38-word message.

6. If a cable letter costs 75¢ for 12 words, and 5¢ for each additional word, find the cost of a 20-word message.

7. A telegraphic money order costs twice the rate for a 10-word message plus 1% premium on the face of the money order. A 10-word message from Helena to New York costs 75¢. What will a telegraphic money order for $550 cost?

If convenient, teachers should ask the pupils to ascertain telegraph and telephone rates in their vicinity. Such problems involve but little arithmetic.

312. Nature of a Corporation. Some boys in the eighth grade have organized a baseball club. There are fourteen boys and they pay \$100 for uniforms and \$12 for balls and bats. If each of the boys contributes $\frac{1}{14}$ of \$112, he will contribute \$8, and if they make some money from tickets to the games, each boy will have $\frac{1}{14}$ of the profits after the expenses are paid.

But some of the boys cannot afford to contribute as much as others, so they divide the \$112 into 224 shares of 50¢ each, and sell to each member as many shares as he cares to buy.

James is one of the chief promoters of the club, and he takes 40 shares, thus making his payment \$20. Fred takes 20 shares, and the others take various amounts.

The first three games draw large crowds, and the gate receipts are heavy. The boys divide the profits according to the number of shares they hold. Fred tries to buy some of the shares that James owns, so as to get more of the profits, but James will not sell for less than 60¢ a share. He says his stock is now above par.

The boys really formed a *corporation*. The *capital* was \$112. There were 224 *shares* of *stock*, the *par value* of each being 50¢. The profits they divided were *dividends*, and these dividends were so high that the stock went *above par*.

In practical life men form corporations in this way, only they play the game of business instead of the game of baseball.

313. Corporation. The laws of our various states permit a number of persons who wish to go into business together to organize as one body, called a *corporation*.

314. Capital. The total amount contributed by the members of a corporation to carry on the business is called *capital*.

The capital varies with different corporations. Some corporations have very large capital.

ORAL EXERCISE

1. Some boys organize a tennis club. They put in $150, and issue shares at 50¢ each. How many shares are there?

2. If William buys 30 shares, how much does he pay?

3. If Frank buys 20 shares, how much does he pay?

4. If there are enough profits from games and from renting the courts so that the boys who bought the shares of stock can sell each share for 55¢, what per cent has the stock increased in value?

5. If the stock goes still higher and becomes 20% above par, what is each share worth?

6. If a heavy storm injures the courts, and the boys have to raise more money, probably no one will want to buy the stock, although Frank offers some of his for 45¢ a share. This price is what per cent below par?

7. The boys appoint a few of their number to direct or manage the business. These are called directors, and they decide on a 10% assessment on all stockholders. How much has to be paid on each share of stock?

8. Why is stock sometimes above par and sometimes below?

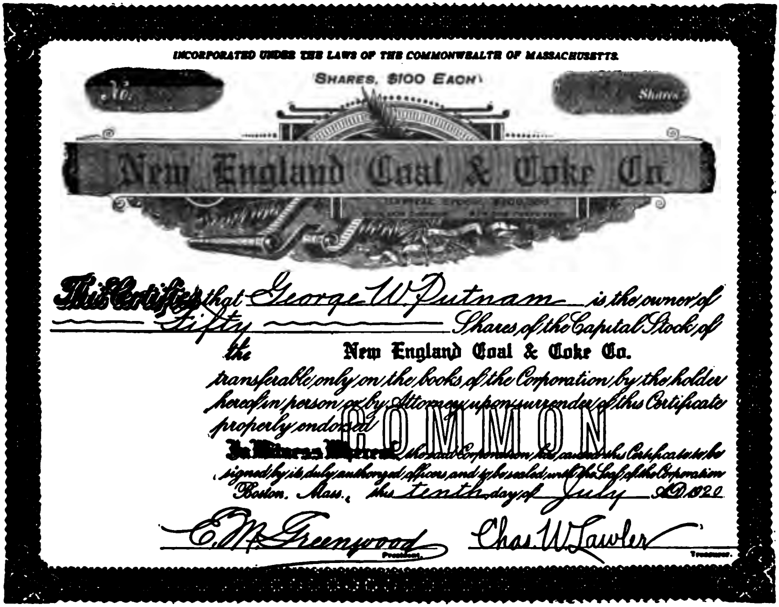
9. Why is it necessary in carrying on business to have stock companies, or corporations, instead of having one man furnish the capital?

10. Why is it necessary for the stockholders to select a few men to direct the business instead of managing it themselves?

11. If you own $5 of stock in a baseball club and the stock goes 10% above par, how much is it then worth?

12. If you own $5 of stock in a tennis club and the stock falls 10% below par, how much is it then worth?

315. Shares and Dividends. The capital is divided into *shares*, and a person who owns any shares is a *stockholder* and receives a *certificate of stock*. Railway shares are usually \$100 each, although sometimes \$50. Mining and other shares are often \$25, \$5, or \$1 each, but the amount varies. In our work the par value of each share is to be taken as \$100 unless otherwise stated.



INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS.

No. — SHARES, \$100 EACH — Shares

New England Coal & Coke Co.

This Certifies that George W. Putnam is the owner of Fifty Shares of the Capital Stock of the New England Coal & Coke Co. transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

COMMON

In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation

Boston, Mass., this tenth day of July A.D. 1920

E. M. Greenwood
President.

Chas. W. Lawler
Treasurer.

A CERTIFICATE OF STOCK

The net earnings of a company are divided among the stockholders in the form of *dividends*. There are often two kinds of stock: *preferred stock*, which pays a fixed rate of dividend if that amount is earned; and *common stock*, which takes whatever dividends may be paid after other claims have been satisfied.

316. Directors and Officers. The stockholders elect a few of their number to have general direction of the company. These are called *directors*, and they elect the *officers*. These officers include a president, a vice president, a secretary, and a treasurer.

For example, if the capital is $100,000 and the shares are $100 each, there are 1000 shares. If a man has 50 shares, he owns $5000 worth of stock, or 5% of the whole corporation, and at the election of directors he is entitled to a vote for each share. If this company earns $6000 beyond all expenses, the directors may declare a 6% dividend. Then a man who owns $5000 worth of stock will receive a check for $300.

The custom as to preferred and common stock and the par value of stock varies with different companies. The pupils should study these different forms of securities as issued by some local corporation, such as a traction company or a railroad.

Has your class formed an imaginary stock company?

WRITTEN EXERCISE

1. A company divides $50,000 in dividends, and the stockholders receive $4 per share. What is the capital?

2. A company with a capital of $3,000,000 declares a 5% dividend. How much does the holder of 100 shares receive?

3. A company with a capital of $500,000 divides $45,000 in dividends. How much does the holder of 30 shares receive?

4. How much does the holder of 30 shares of railway stock receive when a $4\frac{1}{2}$% dividend is declared?

5. A company with $50,000,000 capital declares a dividend of $2\frac{3}{4}$% every six months. How much money does it distribute annually among the stockholders?

6. A company with a capital of $250,000 has earned $15,000 this year above all expenses. It decides to save $2500 of this for emergencies, and to divide the rest in dividends. What is the rate of dividend?

317. Above Par and Below Par. If a stock is paying a high rate of dividend, that is, more than can be received from good ordinary investments, people will be so anxious to buy it that they will pay more than \$100 for a \$100 share. The stock is then said to be *above par*. If a \$100 share can be bought for just \$100, the dividends are about on a par with other investments, and the stock is said to be *at par*. If the dividends are low, the public will not care to buy the stock, and it will be *below par*.

318. Buying Stock. A purchaser usually buys and sells stock through a *broker*, generally in a place called a *stock exchange*, a kind of auction room for such business.

319. Brokerage. The broker charges *brokerage* or *commission*, usually $\frac{1}{8}\%$ of the par value of the stock.

320. Meaning of Stock Quotations. A quotation of $118\frac{3}{4}$ means that a share of stock, par value \$100, will cost $\$118.75 + \$\frac{1}{8}$ (or $12\frac{1}{2}$¢) brokerage. Thus the buyer will pay $\$118.87\frac{1}{2}$ per share, while the seller, who must also pay his broker, will receive $\$118.75 - \$0.12\frac{1}{2}$, or $\$118.62\frac{1}{2}$ per share.

In stock quotations fractions are usually expressed in halves, fourths, or eighths. Fractional parts of a share cannot be bought on the stock exchange.

1. What is the cost of 10 shares of stock quoted at $137\frac{1}{8}$, allowing the usual brokerage?

One share costs $\$137\frac{1}{8} + \$\frac{1}{8}$ brokerage, or $\$137\frac{1}{4}$.
Therefore 10 shares cost $10 \times \$137\frac{1}{4}$, or \$1372.50.

2. What is the amount received from the sale of 100 shares of stock quoted at $96\frac{5}{8}$, allowing the usual brokerage?

One share brings $\$96\frac{5}{8} - \$\frac{1}{8}$ brokerage, or $\$96\frac{1}{2}$.
Therefore 100 shares bring $100 \times \$96\frac{1}{2}$, or \$9650.

321. Bond. A written or printed promise to pay a certain sum at a specified time, signed by the maker and bearing his seal, is called a *bond.* When corporations wish to borrow any considerable amount of money, they issue bonds. These bonds usually have small notes called *coupons* annexed, agreeing to pay the interest as due. Bonds are usually secured by a *mortgage*, an agreement by which the holders of the bonds may sell the property if the bonds and the interest are not paid when due.

322. Difference between Bonds and Shares of Stock. Bonds differ from shares of stock in this way: the stockholders of a corporation are the owners, while the bondholders are owed money by the corporation. Bonds bear a fixed rate of interest, but the income on shares of stock depends on the earnings of the company after the running expenses and the interest on the bonds have been paid. A share of stock is like a deed to a farm; a bond is like a mortgage on a farm.

323. Computing Dividends. Stock dividends and bond incomes are always computed on the par value.

1. How much is the income on $5000 worth of 4% bonds?

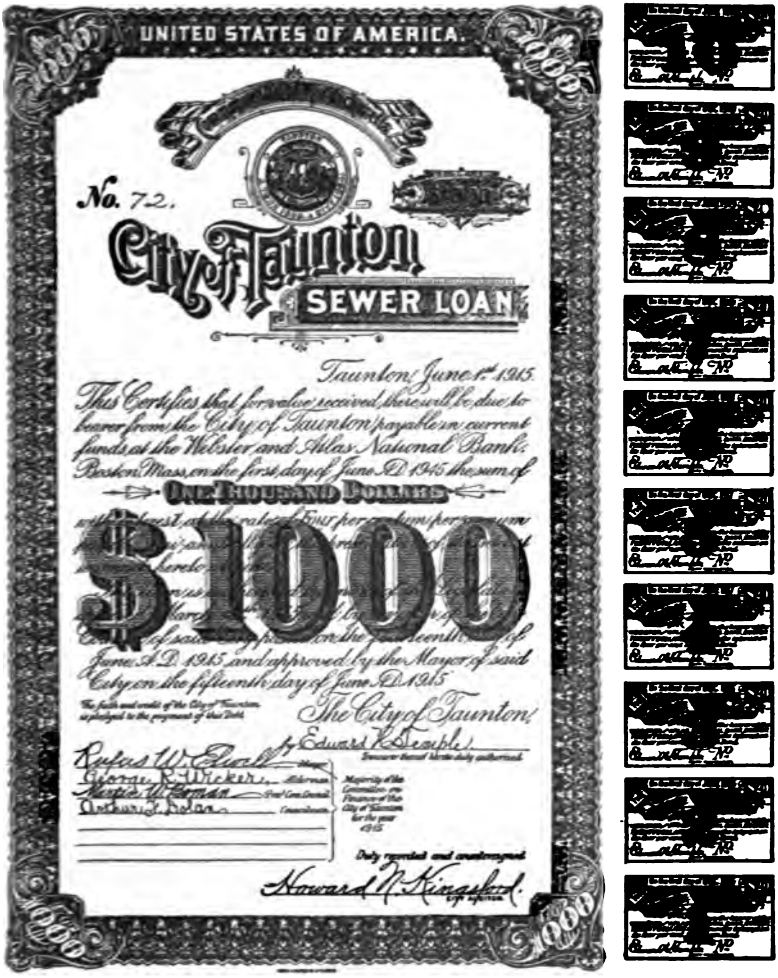
4% of $5000 = $200.

2. What is the income on $5000 of railway stock at 5%?

Since the dividend is 5%, we have 5% of $5000 = $250.

3. If a 5% bond is bought at $124\frac{7}{8}$, what is the rate of income on the investment?

The cost of $100 worth of bonds = $\$124\frac{7}{8} + \$\frac{1}{8}$ (brokerage) = $125.
The income on $100 worth of bonds is 5% of $100, or $5.
Since we have a product, $5, and one factor, $125, we divide.
Therefore $5 ÷ $125 = 0.04.
Hence the income is 4% on the *investment* (but 5% on the *par value*), not considering the date of maturity of the bond.



UNITED STATES OF AMERICA.

No. 72.

City of Taunton

SEWER LOAN

Taunton, June 1st 1915.

This Certifies that for value received there will be due to bearer from the City of Taunton payable in current funds at the Webster and Atlas National Bank, Boston, Mass. on the first day of June A.D. 1945 the sum of

ONE THOUSAND DOLLARS

with interest at the rate of Four per centum per annum [illegible] hereto [illegible] of said City passed on the fourteenth day of June A.D. 1915 and approved by the Mayor of said City on the fifteenth day of June A.D. 1915

$1000

The faith and credit of the City of Taunton is pledged to the payment of this Debt.

The City of Taunton,
by Edward K. Temple
Treasurer thereof hereto duly authorized.

Rufus W. Ewall, Mayor
George R. Wicker, Alderman
Martin W. Coman, Pres't Com. Council
Arthur J. Dolan, Councilman

Majority of the Committee on Finance of the City of Taunton for the year 1915

Duly recorded and countersigned
Howard N. Kingsford

A COUPON BOND

324. Newspaper Quotations. Daily newspapers give the prices of the leading stocks. The examples in the next exercise may be solved by using the newspaper quotations or the following short list:

At., Top., & S.F.	$93\frac{1}{2}$	Penn. R.R.	$104\frac{1}{2}$
Balt. & Ohio	67	Union Pac.	$118\frac{1}{2}$
N.Y. Central	$84\frac{1}{2}$	Western Un. Tel.	$62\frac{1}{2}$

The quotations of stocks are often, as in the above list, the same as if the par value were $100 per share, even though this is not the case. That is, the par value of a share of the Pennsylvania R.R. is $50, and when it is worth $52.25 the quotation is often given as $104.50 ($104\frac{1}{2}$). These are technicalities of the stock market which do not concern the school.

In the following examples add $\frac{1}{8}\%$ to the quotation if you are buying, and subtract $\frac{1}{8}\%$ if you are selling, to pay the broker.

If a man should buy 100 shares of Western Union Telegraph stock when it is quoted at $63\frac{1}{4}$ and should sell it when it is quoted at $62\frac{1}{2}$, how much does he lose on the transaction?

$$\begin{array}{r} \$63\frac{3}{8} \\ \underline{62\frac{3}{8}} \\ \$1 \\ \underline{100} \\ \$100 \end{array}$$

He buys the stock at $63\frac{1}{4} + \frac{1}{8}$ (brokerage), or $63\frac{3}{8}$.
He sells the stock at $62\frac{1}{2} - \frac{1}{8}$ (brokerage), or $62\frac{3}{8}$.
Therefore he loses $1 a share.
On 100 shares he loses 100 × $1, or $100.

WRITTEN EXERCISE

1. What is the par value of 50 shares of stock, the par value of a share being $100? the par value of a share being $50? the par value of a share being $25?

2. If a man has $7500 to invest, and he can buy some shares of stock at par value without paying any brokerage, how many shares can he buy if the par value of each share is $100? if the par value of each share is $25?

3. How much will 75 shares of stock cost when quoted at 96⅛ a share? at 102⅝? at 68½? at 99⅞?

4. A 6% bond is purchased at 149⅞. What is the rate of income on the money invested?

5. Which gives the better income, 5% stock bought when quoted at 139⅞ or 3½% bonds bought when quoted at 99⅞?

Compare \$5 ÷ \$140 with \$3.50 ÷ \$100.

6. Not considering brokerage, which gives the better rate of income, a 6% bond at 120 or a 5% promissory note?

Compare \$6 ÷ \$120 with \$5 on \$100.

7. A man buys, when quoted at 199⅞, stocks that pay 7% dividends. What is the rate of income on the money invested?

8. When United States 4% bonds are quoted at 116⅞, what rate of income does a purchaser receive on his investment, not considering the question of the date of maturity of the bonds?

Find the gain or loss on buying 50 shares, par value \$100, of the following stocks as quoted on page 444, and selling at these prices:

9. At., Top., & S.F., 92⅛.
10. Balt. & Ohio, 68½.
11. N.Y. Central, 82⅜.
12. N.Y. Central, 89¾.
13. Union Pac., 116.
14. Western Un. Tel., 62¾.

15. If you had some money to invest, which would you prefer, a stock that regularly pays 8%, quoted at 159⅞, a 5% bond at 109⅞, or a 5½% mortgage, the security being equally good?

Compare \$8 return on \$160 invested, \$5 return on \$110 invested, and \$5.50 return on \$100 invested. Use short methods of work.

16. If you had some money to invest, which would you prefer, a stock quoted at 149⅞, that regularly pays 7%, a 4% bond at 89⅞, or a 4½% note, the security being equally good?

From time to time recall the suggestion of estimating the result in advance.

LITTLE EXAMINATIONS

I. *At $5\frac{1}{2}$ mills find the taxes on:*

1. $3000. **2.** $5000. **3.** $7500. **4.** $15,000.

Find the taxes on $14,000 at these rates:

5. 6.2 mills. **6.** $7\frac{1}{2}$ mills. **7.** 8.1 mills. **8.** 8.3 mills.

Find the duties on goods valued as follows:

9. $17,500, 15%. **10.** $25,750, 25%.

II. *Find the premiums on the following insurance policies:*

1. $8500, $1.10. **2.** $12,500, $1.20. **3.** $37,000, $1.60.

4. Compute a dividend of $4\frac{1}{4}$% on $150,000.

5. If a mining company declares a dividend of $5\frac{1}{2}$%, and this amounts to $8250, what is the capital?

6. Which will bring the better income, if any, a 5% note or a 6% stock @ $119\frac{7}{8}$?

7. Which will bring the better income, a 5% note or a 7% stock quoted @ 150? how much better?

8. Which will bring the better income, a 5% note or an 8% stock quoted @ $149\frac{7}{8}$? how much better?

9. Which will bring the better income, a 6% stock quoted @ $139\frac{7}{8}$ or a 5% stock quoted @ $119\frac{7}{8}$? how much better?

10. If a man receives $24 on a share of stock, and this is 4% on what he paid for it, how much did it cost him?

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write in Roman numerals the numbers 75, 86, 93, and 39.

2. Write in ordinary numerals the numbers two hundred and five thousandths, and two hundred five thousandths.

Add, and also subtract these numbers:

3.	**4.**	**5.**	**6.**
1742.8	4183.42	3978.41	9002.07
396.9	968.75	699.56	361.56

Multiply, and also divide these numbers:

7. 172.8 by 1.44. **9.** 473.48 by 6.23. **11.** 47.595 by 5.01.
8. 409.53 by 5.1. **10.** 517.42 by 63.1. **12.** 161.463 by 32.1.

Add, and also subtract these numbers:

13.	**14.**	**15.**	**16.**	**17.**	**18.**
$3\frac{9}{10}$	$4\frac{7}{8}$	$7\frac{5}{8}$	$9\frac{1}{16}$	$8\frac{1}{32}$	$6\frac{4}{5}$
$2\frac{2}{3}$	$3\frac{1}{5}$	$2\frac{15}{16}$	$3\frac{5}{8}$	$2\frac{3}{4}$	$3\frac{3}{8}$

Multiply, and also divide these numbers:

19. $\frac{2}{3}$ by $\frac{4}{5}$. **22.** $\frac{7}{32}$ by $\frac{1}{5}$. **25.** $\frac{9}{10}$ by $\frac{2}{3}$. **28.** 37 ft. 8 in. by 2.
20. $\frac{5}{8}$ by $\frac{3}{4}$. **23.** $\frac{7}{16}$ by $\frac{3}{4}$. **26.** $\frac{3}{10}$ by $\frac{4}{5}$. **29.** 48 lb. 6 oz. by 2.
21. $\frac{5}{8}$ by $\frac{2}{3}$. **24.** $\frac{7}{16}$ by $\frac{2}{3}$. **27.** $\frac{3}{16}$ by $\frac{2}{5}$. **30.** 46 ft. 3 in. by 3.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you know the assessed valuation and the tax rate, how do you find the tax to be paid?

2. If you know the assessed valuation of a county and the amount of tax to be raised, how do you find the tax rate?

3. If you know the amount of tax to be raised by a village and the tax rate, how do you find the assessed valuation?

4. If you know the face of a fire-insurance policy and the premium per $100, how do you find the premium?

5. If you know the face of a life-insurance policy and the premium per $1000, how do you find the premium?

6. There are companies that insure employers against accidents to the workmen. If you know the annual pay roll of an employer, and the insurance company charges a certain per cent of the pay roll, how do you find the total premium?

7. If you know the newspaper quotation on some railroad stock, how do you find the cost of a share?

8. If you know the newspaper quotation on a certain stock, how do you find the amount you would receive for a share?

9. If some mining stock is quoted at a specified amount above par, how do you find the cost of a share?

10. What would have to be the rate of dividend on a first-class stock, in comparison with the average rate of income on good investments, to have it at par? to have it below par?

11. If you put some money in the postal savings bank, how can you find the amount of the principal and interest at the end of a given time?

III. INDUSTRIAL PROBLEMS

THE FAMILY BUDGET

1. On New Year's Day Fred's father said: "Let us figure up last year's expenses. I received an income of $1500. I set aside certain per cents that I would spend for the family. This is what I thought we ought to spend: rent, 17%; heat, 4%; light, $1\frac{1}{2}\%$; food, 28%; wages, $8\frac{1}{2}\%$; incidentals, 6%; other personal expenses, 15%; books, music, church, and pleasure, 15%. How much money did I allow for these different purposes?" How much did Fred find this to be in each case?

2. Fred's father really paid for rent, $240; for heat, $52.75; for light, $17.60; for food, $406.85; for wages, $120; for incidentals, $43.60; for other personal expenses, $248.65; and for books, music, church, and pleasure, $258.80. He asked Fred to see how much each expenditure differed from the estimate, and how much he saved during the year.

3. He also asked Fred to find what per cent of the amount spent for heat and light was spent for light alone, and what per cent was spent for heat alone. Find these per cents.

4. He told Fred that when he began to work he received only $840 a year, and on this income he was married. He said that the first year he paid for rent, $168; for heat and light, $42.50; for food, $340; for clothing, $85.50; for insurance, $32.50; and for other expenses, $145. He asked Fred to find how much he saved that year. How much was it?

5. He asked Fred to find what per cent of his income he paid for rent then. What was it? Was it more or less than he pays now? What was the per cent of his income spent for food then? Was it greater or less than the per cent spent now?

MONEY VALUE OF AN EDUCATION

1. Kate, who left school at the end of the seventh grade, went into a shop to work at $17.40 a month. Her sister Alice went on through the high school and got a position paying $24.70 a month. How much more did Alice earn in a year than Kate?

2. The second year that Alice worked she received an increase of 10% and Kate received an increase of 5%. How much more did Alice earn the second year than Kate?

3. Kate had a friend in the shop who was working for $16 a month, but who left and studied in a trade school. When the friend finished her course she got work at $6.25 a week. How much more did she now receive a year than she did in the shop?

4. A girl who is just leaving the public school finds she can enter a city shop at $4 a week the first year, with 10% more the second year, and a 15% increase the third year over the second. Instead of this she enters a trade school for a year, tuition free. She then receives $7 a week the first year and 20% more the second year. Counting 50 working weeks a year, how much more does she receive in three years by this plan?

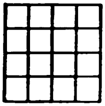
5. A college graduate earns on an average $1125 a year, while the average wages of a day laborer, who has no more than completed the elementary school, are $475. Find the difference in the total earnings of two such men after 25 yr. of work.

6. The college man mentioned in Ex. 5 has spent on an average 180 da. a year for 8 yr. on his course beyond the elementary school. He has also averaged $275 a year for his education. Deducting what he spent from the excess of his earnings over those of the day laborer, we have the net surplus. Divide this by the number of days he spent in the high school and in college, and thus find the approximate money value of each of those days.

CHAPTER XIII

SQUARE ROOT AND MENSURATION

325. Squares and Square Roots. If a square has a side of 4 units, it has an area of 16 square units. Therefore 16 is called the *square* of 4, and 4 is called the *square root* of 16.



326. Square Roots of Areas. Therefore, considering the abstract numbers representing the sides and area,

The side of a square equals the square root of the area.

327. Symbols. The square of 4 is written 4^2, and the square root of 16 is written $\sqrt{16}$.

328. Perfect Squares. Such a number as 16 is called a *perfect square*, but 10 is not a perfect square. We may say, however, that $\sqrt{10}$ equals 3.16 +, because 3.16^2 nearly equals 10.

329. Square Roots of Perfect Squares. Square roots of perfect squares may often be found by simply factoring the numbers.

For example,
$$\begin{aligned}\sqrt{441} &= \sqrt{3 \times 3 \times 7 \times 7}\\ &= \sqrt{3 \times 7 \times 3 \times 7}\\ &= \sqrt{21 \times 21} = 21.\end{aligned}$$

$$\boxed{\begin{array}{r} 3\overline{)441} \\ 3\overline{)147} \\ 7\overline{)49} \\ 7 \end{array}}$$

ORAL EXERCISE

State the square roots of the following:

1. 64. **3.** 81. **5.** 121. **7.** 400. **9.** 1.44. **11.** 1600.
2. 9. **4.** 49. **6.** 144. **8.** 900. **10.** 0.64. **12.** 4900.

WRITTEN EXERCISE

By factoring, find the square roots of the following:

1. 625.
2. 324.
3. 484.
4. 729.
5. 576.
6. 2025.
7. 1296.
8. 1089.
9. 65.61.
10. 12.25.
11. 40.96.
12. 14,641.

Find the sides of squares whose areas are:

13. 4.84 sq. in.
14. 1.96 sq. ft.
15. 2.25 sq. ft.
16. 4.41 sq. in.
17. 4356 sq. in.
18. 15,625 sq. in.
19. 5929 sq. yd.
20. 10.89 sq. yd.
21. 0.1225 sq. ft.

Find the perimeters of squares whose areas are:

22. 6561 sq. ft.
23. 12,100 sq. in.
24. 11,025 sq. ft.
25. 19,600 sq. ft.
26. 146.41 sq. in.
27. 20,736 sq. in.
28. 16,384 sq. in.
29. 16,900 sq. ft.
30. 129,600 sq. in.

31. From a corner of a square piece of land that contains 576 sq. rd. a small square lot containing 64 sq. rd. is cut. Draw a plan of the lots and find the perimeter of each.

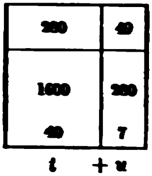
32. A square lot has an area of 169 sq. rd. How far is it around the lot? How far is it around a lot four times this area? The second perimeter is how many times the first?

33. A man has two adjacent building lots fronting on the street, each lot being square. The area of the two together is 89 sq. rd., that of the larger being 64 sq. rd. What is the frontage of each lot?

34. A square lot has an area of 289 sq. rd. How far is it around the lot? (Try the prime numbers between 10 and 20.) How far is it around a lot of nine times this area? The second perimeter is how many times the first?

330. Square of the Sum of Two Numbers. Since $47 = 40 + 7$, the square of 47 may be obtained as follows:

$$\begin{array}{lrr}
 & 40+7 & \\
 & 40+7 & \\ \hline
 & (40\times 7)+7^2 & \\
40^2+ & (40\times 7) & \\ \hline
40^2+2\times & (40\times 7)+7^2 & \\
=1600+2\times & 280 & +49\\
=1600+ & 560 & +49\\
=2209. & &
\end{array}$$



This relationship is seen in the annexed figure, where the side of the square is 40 + 7.

Every number consisting of two or more figures may be regarded as composed of tens and units. Therefore,

The square of a number contains the square of the tens, plus twice the product of the tens and units, plus the square of the units.

This is the most important principle in square root and should be clearly understood, both from the multiplication and from the illustration.

331. Separating into Periods. The first step in extracting the square root of a number is to separate the figures of the number into groups of two figures each, called *periods.*

Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10, and the square root of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number expressed by *one* figure or *two* figures is a number of *one* figure; the square root of any integral number expressed by *three* or *four* figures is a number of *two* figures; and so on.

Therefore, if an integral number is separated into periods of two figures each, from the right to the left, the number of figures in the square root will be equal to the number of the periods of figures. The last period at the left may have one figure or two figures; for example, 22 09, and 7 89 04 81.

332. Extracting the Square Root. The process of extracting the square root of a number will now be considered.

For example, required the square root of 2209.

Separating the figures of the number into periods (§ 331), we see that there will be two integral places in the root.

The first period, 22, contains the square of the tens' number of the root. Since the greatest square in 22 is 16, hence 4, the square root of 16, is the tens' figure of the root.

Subtracting the square of the tens, the remainder contains twice the tens × the units, plus the square of the units (§ 330). If we divide by twice the tens (that is, by 80, which is 2 × 4 tens), we shall find approximately the units. Dividing 609 by 80 (or 60 by 8), we have 7 as the units' figure.

$$\begin{array}{lr} & 22\ 09\,(47 \\ & 16 \\ \hline 80 & 6\ 09 \\ 87 & 6\ 09 \\ \hline \end{array}$$

Since twice the tens × the units, plus the square of the units, is equal to (twice the tens + the units) × the units, that is, since $2 \times 40 \times 7 + 7^2 = (2 \times 40 + 7) \times 7$, we add 7 to 80 and multiply the sum by 7. The product is 609, thus completing the square of 47.

Checking the work, $47^2 = 2209$.

WRITTEN EXERCISE

Find the square roots of the following:

1. $\sqrt{3249}$.
2. $\sqrt{1681}$.
3. $\sqrt{3721}$.
4. $\sqrt{1225}$.
5. $\sqrt{3969}$.
6. $\sqrt{2809}$.
7. $\sqrt{5041}$.
8. $\sqrt{3481}$.

Find the sides of the squares, given the following areas:

9. 6724 sq. ft.
10. 7569 sq. ft.
11. 9025 sq. ft.
12. 9409 sq. ft.
13. 7921 sq. rd.
14. 6889 sq. yd.

Find the square roots of the following by taking the square root of each term of the fractions:

15. $\sqrt{\frac{144}{529}}$.
16. $\sqrt{\frac{3025}{3249}}$.
17. $\sqrt{\frac{961}{1024}}$.
18. $\sqrt{\frac{361}{8281}}$.
19. $\sqrt{\frac{1089}{1156}}$.
20. $\sqrt{\frac{529}{9801}}$.
21. $\sqrt{\frac{1936}{5041}}$.
22. $\sqrt{\frac{289}{9409}}$.

333. Square Root with Decimal Fractions. Find the square root of 151.29.

The greatest square of the tens in 151.29 is 100, and $\sqrt{100} = 10$.

Then 51.29 contains 2 × 10 × the units' number of the root, plus the square of the units' number. Why is this?

Dividing by 2 × 10, or 20, we find that the next figure of the root is 2.

We have now found 12, the square being 100 + 44 = 144.

Then 7.29 contains 2 × 12 × the tenths' number of the root, plus the square of the tenths' number, because we have subtracted 144, which is the square of 12.

Dividing by 24, we find that the tenths' figure of the root is 3.

Hence the square root of 151.29 is 12.3.

If the number is not a perfect square, we may annex pairs of zeros at the right of the decimal point and find the root to as many decimal places as we choose.

	1 51.29(12.3
	1
20	51
22	44
240	729
243	729

334. Summary of Square Root. We now see that the following are the steps to be taken in finding square root:

Separate the number into periods of two figures each, beginning at the decimal point.

Find the greatest square in the left-hand period and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

Divide the new dividend thus obtained by twice the part of the root already found.

Annex the figure thus found to this divisor and multiply by the same figure.

Subtract this result, bring down the next period, and proceed as before until all the periods have been thus annexed.

The result is the square root required.

WRITTEN EXERCISE

Find the square roots of the following:

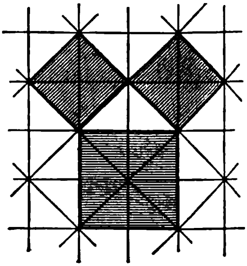
1. 12,321.
2. 54,756.
3. 110.25.
4. 8046.09.
5. 19.4481.
6. 0.2809.
7. 1176.49.
8. 82.2649.
9. 63,001.
10. 21,224,449.
11. 49,112,064.
12. 96,275,344.

In Exs. 13–18 carry the square roots to two decimal places only.

13. 2. **14.** 5. **15.** 7. **16.** 8. **17.** 11.

18. Find the number of rods in the side of a square containing 1 acre (160 sq. rd.).

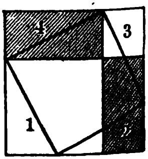
335. Square on the Hypotenuse. In a right triangle the side opposite the right angle is called the *hypotenuse* of the triangle.



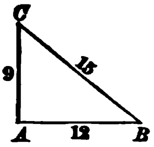
If a floor is made up of triangular tiles like this, it is easy to mark out a right triangle. In the figure it is seen that the square on the hypotenuse contains eight small triangles, while each square on a side contains four such triangles. Hence

The square on the hypotenuse equals the sum of the squares on the other two sides.

This is also true of any right triangle, for we see that if the four triangles, 1 + 2 + 3 + 4, are taken away from this figure, there remains the square on the hypotenuse. But if we take away the two shaded rectangles, which together have exactly the same area as the four triangles, the squares on the two sides remain. Therefore the square on the hypotenuse must equal the sum of these two squares.



336. Illustrative Problem. If $AB = 12$ and $AC = 9$, find BC.



Since $\overline{AB}^2 + \overline{AC}^2 = \overline{BC}^2$,

therefore $12^2 + 9^2 = \overline{BC}^2$,

or $144 + 81 = 225 = \overline{BC}^2$,

and $\sqrt{225} = BC$. Therefore $BC = 15$.

WRITTEN EXERCISE

1. How long is the diagonal of a hall 51 ft. by 68 ft.?

2. Find to two decimal places the length of the diagonal of a square that contains 4 sq. ft.

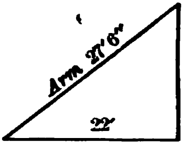
3. The two sides of a right triangle are 20 in. and 30 in. Find to two decimal places the length of the hypotenuse.

4. The two sides of a right triangle are 57 in. and 76 in. Find the length of the hypotenuse.

5. What is the length of a wire drawn taut from the cornice of a 100-foot building to a spot 75 ft. from the foot?

6. Find the length of the hypotenuse of a right triangle when the sides are 321 in. and 428 in.; 40 in. and 75 in.; 72 ft. and 135 ft.; 36 rd. and $67\frac{1}{2}$ rd.; 20 yd. and 37 yd. 18 in.

7. A telegraph pole is set perpendicular to the ground, and a wire is fastened to it 18 ft. from the ground, and then to a stake 13 ft. 6 in. from the foot of the pole, so as to hold it in place. How long is the wire?



8. A derrick for hoisting coal has its arm 27 ft. 6 in. long. It swings over an opening 22 ft. from the base of the arm. How far is the top of the derrick above the opening?

Reversing the procedure in § 335, the square on either side equals the difference of what squares?

Find to two decimal places the hypotenuse of each of the right triangles of which the sides are here given:

9. 35 ft., 26 ft.

10. 81 ft., 35 ft.

11. 10 rd., 13 rd.

12. $42\frac{1}{2}$ ft., $63\frac{1}{4}$ ft.

13. 4.5 in., 7.2 in.

14. 6.25 in., 7.5 in.

15. $3\frac{3}{4}$ in., $8\frac{1}{2}$ in.

16. 23 in., 34 in.

17. 3.4 in., 2.7 in.

18. A room is 16 ft. long, 12 ft. wide, and 9 ft. high. Find to two decimal places the distance from an upper corner through the room to the opposite lower corner.

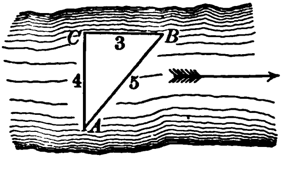
First find the hypotenuse on the floor and then the hypotenuse required. This hypotenuse is called the *diagonal* of the room.

19. Find to two decimal places the length of the diagonal of a cube whose volume is 8 cu. in.

First find the edge. Then proceed as in Ex. 18.

20. A school flag pole is broken by the wind 16 ft. from the ground. The two pieces hold together, and the top of the pole touches the ground 30 ft. from the foot of the pole. Find the length of the flag pole.

21. If I start to row directly across a stream, in the direction *AC*, at the rate of 4 mi. an hour, and if the stream carries me in the direction *CB*, at the rate of 3 mi. an hour, my course will really be *AB*, the result of these two motions. Suppose I row at the rate of 4.5 mi. per hour, and the stream flows 6 mi., what is my rate of progress?



22. The captain is walking at the rate of 4 mi. per hour directly across the deck of a steamer, while the boat is moving at the rate of 8 mi. per hour. Find to two decimal places the rate at which the captain is moving. Draw a plan to scale.

It is a good exercise to make up problems like those in Exs. 18–22.

23. A square lot has an area of 27,556 sq. ft. How many feet in the length of a side?

24. A square piece of oilcloth has an area of 148 sq. ft. 4 sq. in. How many inches in the length of a side?

25. The bottom of a square box has an area of 1936 sq. in. How long is the box?

26. A square lot has an area of 62,001 sq. ft. How many feet in the length of a side?

27. Determine the number of rods in the perimeter of a square field the diagonal of which is 1520 rd.

28. The diagonal of a square is 15 ft. Find to two decimal places the length of a side.

29. The side of a square is 12.5 ft. Find to two decimal places the length of the diagonal.

30. The area of a square is $3\frac{1}{2}$ sq. ft. Find to two decimal places the length of a side.

31. The foot of a 45-foot ladder is 27 ft. from the wall of a building against which the top rests. How high does the ladder reach on the wall?

32. How far from the wall of a house must the foot of a 45-foot ladder be placed in order that the top of the ladder may touch a window sill 40 ft. from the ground?

33. A rope stretched from the top of a 30-foot pole just reaches the ground 18 ft. from the foot of the pole. Assuming the rope to be straight, how long is it?

In practice, tables showing the square and cube roots of numbers are used. The extraction of cube root by the old method of arithmetic is obsolete as a working plan, and is therefore omitted from the essentials which make up this series. A short table of square and cube roots is given in the Supplement on page 519 and, if desired, may be used for many of the above examples.

337. Circumference, Diameter, and Radius. The entire distance around a circle is called the *circumference.* A line drawn through a circle, and passing through the center, is called a *diameter*, and half a diameter is called a *radius.*

A circumference is like the length of a tire of a wheel; a radius is like a spoke of a wheel; a diameter is like two opposite spokes. The word "circumference" is used for the line which bounds the circle as well as for the length of the line.

338. Ratio of Circumference to Diameter. By measuring several circles and dividing each circumference by its diameter, the circumference will be found to be about $3\frac{1}{7}$ times the diameter.

It is proved in geometry that this ratio of the circumference to the diameter is more nearly 3.1416. For our work use $3\frac{1}{7}$, or $\frac{22}{7}$, instead of 3.1416, unless the contrary is stated.

1. Required the circumference when the radius is 7 in.

Since the radius is 7 in., the diameter is 2 × 7 in. Therefore the circumference is

$$3\tfrac{1}{7} \times 2 \times 7 \text{ in.} = 44 \text{ in.}$$

2. Required the diameter when the circumference is 28.6 in.

Since the circumference is the product of the diameter and $\frac{22}{7}$, we divide the circumference by $\frac{22}{7}$ to find the diameter.

Therefore $$\frac{28.6}{\frac{22}{7}} = \frac{7 \times \overset{1.3}{\cancel{28.6}}}{\cancel{22}} = 9.1.$$

Hence the diameter is 9.1 in.

ORAL EXERCISE

State the circumferences of circles of the following diameters:

1. 7 in. **2.** 21 in. **3.** 35 in. **4.** 42 in.

State the circumferences of circles of the following radii:

5. 7 in. **6.** 14 in. **7.** 21 in. **8.** 28 in.

WRITTEN EXERCISE

Find the circumferences, given the diameters:

1. 68.2 in.
2. 48.3 ft.
3. 423 in.
4. 5.11 ft.
5. 53.9 ft.
6. 6.37 in.
7. 4.69 ft.
8. 58.1 in.
9. 6.02 ft.
10. 13 ft. 6 in.
11. 17 ft. 8 in.
12. 64.4 ft.

Find the diameters, given the circumferences:

13. 132 ft.
14. $58\frac{2}{7}$ ft.
15. $97\frac{3}{7}$ ft.
16. 176 in.
17. 770 ft.
18. $96\frac{4}{5}$ ft.
19. 68.2 in.
20. 3.96 ft.
21. 0.484 ft.
22. 198 in.
23. 15.4 in.
24. 0.11 mi.

Using 3.1416 instead of $\frac{22}{7}$, find the circumferences, given the diameters:

25. 17 in.
26. 13 in.
27. 2.8 in.
28. 4.37 in.
29. 4.2 in.
30. $3\frac{1}{2}$ ft.
31. 2.25 ft.
32. 6 ft. 2 in.

Using 3.1416, find the diameters, given the circumferences:

33. 53.4072 in.
34. 84.8232 in.
35. 97.3896 in.
36. 94.2478 ft.
37. 182.2228 ft.
38. 138.2304 ft.

Using $\frac{22}{7}$, find the circumferences, given the radii:

39. 49 in.
40. 77 in.
41. 91 in.
42. 8.4 in.
43. 9.8 in.
44. 105 in.
45. 15.4 in.
46. $1\frac{3}{4}$ in.

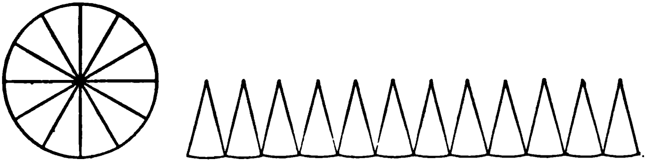
47. What is the diameter when the circumference is 74.8 ft.?

48. What is the circumference of a rod whose diameter as measured by calipers is 2.1 in.?

49. What is the circumference of a wire whose diameter as measured by calipers is 0.63 in.?

50. What radius must be used in making a pattern for a wheel that is to be $91\frac{1}{7}$ in. in circumference?

339. Area of a Circle. A circle can be separated into figures which are nearly triangles. The height of each triangle is the radius, and the sum of the bases is the circumference. If these were exact triangles, the area would be $\frac{1}{2}$ × radius × circumference. It is proved in geometry that this is the true area.



340. Given the Radius, to find the Area. Because the circumference is $\frac{22}{7}$ × diameter, the area must be

$$\tfrac{1}{2} \times \text{radius} \times \tfrac{22}{7} \times \text{diameter},$$

or

$$\tfrac{1}{2} \times \text{radius} \times \tfrac{22}{7} \times 2 \times \text{radius},$$

or

$$\tfrac{22}{7} \times \text{the square of radius}.$$

The area of a circle is $\frac{22}{7}$ times the square of the radius.

Required the area of a circle whose radius is 5 in.

Since $5^2 = 25$,

we have area = $\frac{22}{7}$ × 25 sq. in. = $78\frac{4}{7}$ sq. in.

ORAL EXERCISE

State the areas, given the radii as follows:

1. 7 in.
2. 70 in.
3. 14 in.
4. 28 in.
5. 21 in.
6. 42 in.
7. $\frac{1}{22}$ in.
8. $\frac{7}{22}$ in.
9. 1.4 in.
10. 2.8 in.

11. If you take a radius half as long as a given radius, the area of the circle will be what part as great?

12. If you double the length of the radius, the area of the circle will be how many times as large?

341. Illustrative Problem. Required the radius of the circle whose area is $50\frac{2}{7}$ sq. in.

The area is the product of $\frac{22}{7}$ by the square of the radius.

Therefore, dividing by $\frac{22}{7}$ we have the square of the radius.

$$50\tfrac{2}{7} \div \tfrac{22}{7} = \tfrac{7}{22} \times \tfrac{352}{7} = 16.$$

Since $\sqrt{16} = 4$, the radius is 4 in.

WRITTEN EXERCISE

1. What is the area of the cross section of a water pipe that is 2 in. in diameter?

2. How many square feet in the base of a water tank that is 42 ft. in diameter?

3. A horse tethered by a rope 21 ft. long can graze over how many square feet of ground?

4. A horse tethered by a rope can graze over 3850 sq. ft. of ground. How long is the rope?

5. How long is the equator on a globe of diameter 12.67 in.? What is the area of the equator circle cut from such a globe?

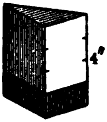
6. A school flag pole has a circumference of 24.2 in. at the base. What is the diameter? What is the radius? What is the area of a cross section?

7. What is the area of the cross section of a circular iron beam whose circumference is 31.416 in.? (Use 3.1416.)

8. A boy has a ball tied to a string 1 yd. 6 in. long. As he swings it around, how long is the circumference traveled by the ball? What is the area of the circle inclosed?

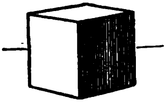
9. A tinsmith wishes to make a pattern for the bottom of a pail, the area being 154 sq. in., and to allow $\frac{1}{4}$ in. all around for soldering. What radius should he use in drawing the circle?

342. Prism. A solid having two parallel faces exactly alike and all the rest of the faces parallelograms is called a *prism.*



The solid shown in the picture is a prism, the parallel faces in this case being triangles. The *base* of a prism is one of the two parallel faces, and the *altitude* of the prism is the distance between these faces. If all the faces of a prism are rectangles, the prism is called a *rectangular prism.* An ordinary crayon box is a rectangular prism.

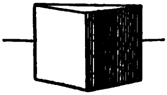
343. Volume of a Prism. Contractors, builders, and mechanics frequently have to find the volume of a prism of some kind. For example, a contractor computing the amount of earth to be removed in excavating a canal uses the rule for the volume of a prism. To find this rule first consider the following questions:



If this kind of prism, a cube, is 1 in. on an edge, what is its volume?

If a cube is 2 in. on an edge, what is its volume? If a cube is 3 in. on an edge, what is its volume?

This figure represents half of the above cube. If the cube is 1 in. on an edge, what is the volume of this half?



If the cube is 4 in. on an edge, what is the volume of this half?

If the area of the base of the first figure on this page is 5 sq. in., what is the volume of the lower shaded part which is 1 in. high? What is the total volume?

We have now found the volume of a special kind of prism, a cube; of another special kind, half a cube; and of a triangular prism. The volume of any prism is found in the same way as that of a triangular prism.

We see from the above that

The volume of a prism equals the area of the base multiplied by the altitude.

WRITTEN EXERCISE

Find the volumes of prisms with bases and altitudes as follows:

1. 375 sq. in., 29 in.

2. 67.9 sq. in., 4.8 in.

3. 61.3 sq. ft., 2.91 ft.

4. 1.28 sq. ft., 3.2 in.

5. 42.6 sq. in., 7.3 in.

6. 62.8 sq. in., 3.17 in.

Find the altitudes of prisms with volumes and bases as follows:

7. 243 cu. in., 29 sq. in.

8. 140 cu. in., $17\frac{1}{2}$ sq. in.

9. 783 cu. in., 87 sq. in.

10. $178\frac{1}{4}$ cu. in., $19\frac{1}{4}$ sq. in.

Find to the nearest tenth the volumes of rectangular prisms with dimensions as follows:

11. 42 in. long by 6.8 in. wide by 3.5 in. high.

12. 2.9 in. wide by 3.8 in. long by 6.4 in. high.

13. 3.91 ft. wide by 4.27 ft. long by 6.8 ft. deep.

14. 2.8 ft. deep by 3.25 ft. wide by 4.75 ft. long.

15. 6.5 ft. long by 4.75 ft. wide by 3.25 ft. high.

16. 21.3 in. wide by 29.2 in. long by 3.7 in. deep.

17. The area of one face of a cube is 3721 sq. in. What is the volume of the cube?

18. A prism with a square base of 1156 sq. in. is twice as high as it is wide. Find the volume of the prism.

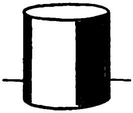
19. A prism has a square base, a volume of 4851 cu. in., and a height of 11 in. Required the lengths of the edges.

20. A prism has a square base, a volume of $1687\frac{1}{2}$ cu. in., and a height of $7\frac{1}{2}$ in. Find the area of the base.

21. In Ex. 20, find the edge of the base of the prism.

Although pupils at this stage do not need to have an artificial motive for work, as in the earlier grades, teachers sometimes find it helpful to consider such local measures as those relating to a water tank or standpipe before beginning page 466.

344. Cylinder. A solid bounded by two parallel circular faces and a curved surface is called a *cylinder.*



The solid shown in the picture is a cylinder.

The two parallel circular faces are called the *bases* of the cylinder, and the distance between the bases is called the *altitude* of the cylinder.

The above definition is sufficient for the purposes of arithmetic. There are other cylinders than those having circular bases, but we do not need to study them at this time.

345. Volume of a Cylinder. It is often necessary to find the volume of a cylinder, as in finding the volume of a tank or pipe.

In the same way that we found the volume of a prism (§ 343) we may find the volume of a cylinder.

The volume of a cylinder equals the area of the base multiplied by the altitude.

Since the area of the base of a cylinder is $\frac{22}{7}$ times the square of the radius, we may say that *the volume of a cylinder equals the product of the altitude multiplied by* $\frac{22}{7}$ *times the square of the radius of the base.*

WRITTEN EXERCISE

1. Find the volume of a cylindrical water tank 40 ft. high and 40 ft. in diameter.

2. How many cubic feet in a section of water pipe 16 ft. long, the diameter being 1 ft. 11 in.?

3. How many gallons (231 cu. in.) in a cylindrical tank 32 ft. high and 30 ft. in diameter?

4. Find the number of cubic feet in a boiler 15 ft. long and 3 ft. 6 in. in diameter, internal measure.

5. Find the number of cubic inches in a pipe 18 ft. long, 4 in. in external diameter, the metal being $\frac{1}{4}$ in. thick.

346. Curved Surface of a Cylinder. If we slit the curved surface of a cylinder and spread it out flat, what will it become? How may we find the area of this surface? Try it.

What is the area of the curved surface of a cylinder 6 in. high and 8 in. in circumference?

The area is the same as that of a rectangle 6 in. by 8 in., or 48 sq. in.

We see that *the area of the curved surface of a cylinder equals the circumference multiplied by the altitude.*

That is, area = circumference × altitude,

or area = $\frac{22}{7}$ × diameter × altitude.

ORAL EXERCISE

State the areas of the curved surfaces of cylinders with altitudes and circumferences as follows:

1. 25 in., 80 in.
2. $33\frac{1}{3}$ in., 69 in.
3. 75 in., 40 in.
4. 50 in., 106 in.
5. $66\frac{2}{3}$ in., 48 in.
6. $12\frac{1}{2}$ in., 24 in.
7. $16\frac{2}{3}$ in., 36 in.
8. $33\frac{1}{3}$ in., 33 in.

9. How many square inches in the curved surface of a wire of circumference 1 in. and length 200 ft.?

10. How many square feet of tin are needed for a pipe 8 ft. long and 6 in. in circumference, allowing 1 sq. ft. for overlapping?

11. A tin cup is 7 in. in circumference and 3 in. high, both measures including allowance for soldering. How many square inches of tin are needed for the curved surface?

12. A tin water pipe has a circumference of 9 in. and a length of 10 ft., both measures including allowance for soldering. How many square inches of tin are needed?

WRITTEN EXERCISE

1. How many square inches of surface on a wire 10 ft. long and $\frac{7}{8}$ in. in circumference?

2. If the height and diameter of a solid cylinder are both 7 in., what is the total area of surface?

To the cylindrical surface add the areas of the circles forming the top and bottom.

3. How many square inches of tin are needed for a cylindrical cup 4 in. high and 4 in. in diameter?

4. How many square feet in the curved surface of a water tank that is 40 ft. high and 127.3 ft. in circumference?

5. Write a rule for finding the circumference of a circle, given the radius; also for finding the area of the circle.

6. How many square feet of surface on the outside of a smokestack 30 ft. high and 2 ft. in exterior diameter?

7. The curved surface of a granite cylindrical shaft 3 ft. 6 in. in diameter and 22 ft. 3 in. long is to be polished. How many square feet in the surface to be polished?

8. In a certain factory there is a room heated by 210 ft. of steam pipe, 2 in. in diameter. Required the radiating surface; that is, the area of the curved surface which radiates the heat.

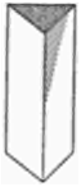
9. A large suspension bridge has 4 cables, each 1872 ft. long and 1 ft. 2 in. in diameter. In letting the contract for painting these cables it is necessary to know their surface. Compute it.

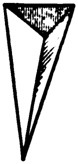
10. In making metal bedsteads some iron rods 0.75 in. in diameter are covered with thin rolled brass. Suppose a shop needs 6000 ft. of such rods, how many square feet of rolled brass will be needed, not allowing for waste?

Teachers should not fail to keep in mind the importance of estimating results in advance.

347. Pyramid. A solid of this shape, in which the base is any polygon and the other faces are triangles meeting at a point, is called a *pyramid.* The point at which the triangular faces meet is called the *vertex* of the pyramid. The distance from the vertex to the base is called the *altitude* of the pyramid.

348. Volume of a Pyramid. A contractor or builder occasionally needs to find the volume of a pyramid. The rule for finding the volume is easily seen by taking a hollow prism and a hollow pyramid of the same base and the same height, as here shown, filling the pyramid with water and pouring the water into the prism. It will be found that the prism can be exactly filled with three times the amount that fills the pyramid. That is,





The volume of a pyramid equals one third the product of the base and altitude.

WRITTEN EXERCISE

Find the volumes of pyramids with bases and altitudes as follows:

1. 129.3 sq. in., 3.72 in.
2. 202.11 sq. ft., 8.92 ft.
3. 702.39 sq. in., 16.8 in.
4. 187.02 sq. ft., 10.7 ft.

Find the bases of pyramids with volumes and altitudes as follows:

5. 89.6 cu. in., 12.8 in.
6. 178.2 cu. in., 19.8 in.
7. 123.2 cu. in., 17.6 in.
8. 140.4 cu. in., 23.4 in.

9. The Great Pyramid in Egypt is 480$\frac{3}{4}$ ft. high, and has a square base, 764 ft. on a side. What is the volume? If it weighs 168 lb. per cubic foot, what is the weight?

LITTLE EXAMINATIONS

I. *Find the square roots of the following:*

1. $\frac{49}{144}$. **2.** $\frac{225}{361}$. **3.** $\frac{529}{729}$. **4.** $\frac{1369}{1681}$. **5.** $\frac{2209}{2601}$.

Find to two decimal places the square roots of the following:

6. 13. **7.** 17. **8.** 30. **9.** 125. **10.** $\frac{5}{9}$.

II. *Find the circumferences, given the diameters:*

1. 34.1 in. **2.** 32.2 ft. **3.** 3 in. **4.** $4\frac{1}{2}$ in. **5.** 9 in.

Find to two decimal places the square roots of the following:

6. 19. **7.** 15. **8.** 50. **9.** 650. **10.** $\frac{7}{12}$.

III. *Find the hypotenuses, given the other sides as follows:*

1. 39 ft., 52 ft. **2.** 21 ft., 72 ft. **3.** 51 ft., 68 ft.

Find the volumes of cylinders whose altitudes and areas of base are as follows:

4. 48 in., 135 sq. in. **5.** 9 ft. 6 in., 5 sq. ft. 36 sq. in.

Find the volumes of cylinders whose altitudes and radii of base are as follows:

6. 37 in., 5 in. **7.** 35 in., 9 in. **8.** 3 ft. 7 in., 3 in.

9. Find the volume of a pyramid of base 87 sq. in. and height $14\frac{1}{2}$ in.

10. Find the volume of a pyramid of base 4 sq. ft. 9 sq. in. and height 1 ft. 3 in.

REVIEW AND DRILL

I. DRILL SECTION

1. Write in ordinary numerals:

MDCCCCXIX MCMXXI LXXXIX

2. Find the value of 48,998 + 76,554 + 83,456 + 96,387.

3. Find the value of 82,021,322 − 67,137,487.

4. Multiply the sum of $27.63 and $99.75 by 346.

5. Divide the difference between $910.07 and $756.59 by 1.37, and carry the result to the nearest cent.

Add the following:

6. $\frac{1}{2} + \frac{7}{8}$. **8.** $4\frac{1}{2} + 5\frac{3}{4}$. **10.** $6\frac{2}{3} + 2\frac{1}{2}$. **12.** $3\frac{1}{8} + 2\frac{3}{5}$.
7. $\frac{5}{8} + \frac{2}{3}$. **9.** $2\frac{3}{4} + 6\frac{3}{8}$. **11.** $5\frac{1}{2} + 3\frac{3}{5}$. **13.** $4\frac{7}{8} + 3\frac{1}{3}$.

Subtract the following:

14. $\frac{5}{8} - \frac{1}{2}$. **15.** $2\frac{1}{2} - \frac{7}{8}$. **16.** $6\frac{1}{2} - 2\frac{7}{8}$. **17.** $9\frac{1}{3} - 2\frac{1}{2}$.

Multiply the following:

18. $\frac{7}{8}$ of $\frac{4}{5}$. **19.** $\frac{1}{2}$ of $2\frac{7}{8}$. **20.** $2\frac{1}{2} \times 3\frac{1}{2}$. **21.** $3\frac{1}{4} \times 5\frac{3}{8}$.

Divide the following:

22. $\frac{7}{8} \div \frac{1}{2}$. **23.** $2\frac{1}{2} \div 1\frac{1}{4}$. **24.** $9\frac{3}{4} \div 3\frac{1}{4}$. **25.** $18\frac{3}{4} \div 3\frac{3}{4}$.

Find the values of the following to two decimal places:

26. 25% of $17,256.75. **27.** $5.293 \div 0.971$. **28.** $1 \div 0.7854$.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. How do you find the square of a number?

2. If you know the side of a square, how do you find the area? Illustrate by an example.

3. If you know the area of a square, how do you find the side? Illustrate by an example.

4. How do you find by factoring the square root of a number that is a perfect square? Illustrate.

5. If you know three terms of a proportion, how do you find the fourth term?

6. How do you extract the square root of an integer that is a perfect square?

7. How do you extract the square root of a common fraction? How do you extract the square root of a decimal?

8. How do you find the hypotenuse of a right triangle when you know the other two sides?

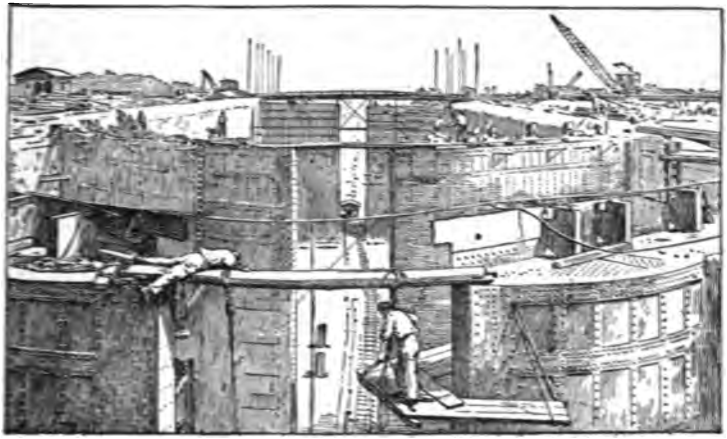
9. Suppose that a fly starts from the upper northeast corner of this room and flies straight to the lower southwest corner. How can you find the distance he flies, knowing the three dimensions of the room?

10. How would you find the number of cubic feet in a cylindrical water tank?

11. If you know the base and altitude of a pyramid, what else can you find, and how do you find it?

12. If you know the altitude of a cylinder, and the diameter of the base, what else can you find, and how do you find them?

III. INDUSTRIAL PROBLEMS



WHY WE BUILT THE PANAMA CANAL

1. The Panama Canal cost about $375,000,000. Taking our population as 100,000,000, this means that every man, woman, and child must pay how much on an average?

2. If the government has borrowed the money at 2%, what are the interest charges every year that all of us must help pay?

3. Goods can be shipped just as well by water as by rail, unless there is necessity for haste. It is cheaper to ship by water. The rate on a certain class of freight from Boston to Chicago wholly by rail is 30¢ per 100 lb., but by rail and lake it is $5.54 per ton. What is the per cent of saving by rail and lake?

4. The rate on a certain class of goods from New York to San Francisco is $2 per 100 lb. wholly by rail, or $1.20 per 100 lb. by water and the Panama Canal. What is the per cent of saving by the latter route?

5. If the rate by the Panama Canal is 40% cheaper than by the rail route, how much would a shipper save on 40 tons of the class of goods mentioned in Ex. 4, over the all-rail rate?

6. It is estimated that on a certain class of goods there is a saving of 46% by shipping by the Panama Canal over shipping by rail from the Atlantic coast to the Pacific. If $23,500,000 represents the all-rail freight charges on this class of goods for a given period, and it is just as well to ship by water as by rail, what is the saving by sending the freight through the Canal?

7. Before the Panama Canal was opened, the western coast of Central and South America had an annual foreign trade of $600,000,000. It was estimated that this would be increased 55% as a result of the opening of the Canal. What would this make the value of the annual foreign trade?

8. It costs us about $15,000,000 for a battleship, to defend our country in case of war. The Panama Canal allows our ships to be sent from the Atlantic to the Pacific and back again, with very little delay. If we need build one less battleship a year on this account, how much will be saved in twenty-five years?

9. If the trade of the Atlantic coast of this country with Australasia was $22,000,000 a year before the Panama Canal was opened, and increased 32% as a result of the opening of this route, what did it amount to after that?

10. Suppose England's trade with the west coast of South America was $86,000,000 a year before this route was opened, and increased 15% as a result of the opening of the Canal, what did it amount to after that?

11. The toll on merchant vessels is $1.20 per net ton of actual earning capacity. If a steamer has a net capacity of 5275 tons, how much toll must be paid in passing through the Canal?

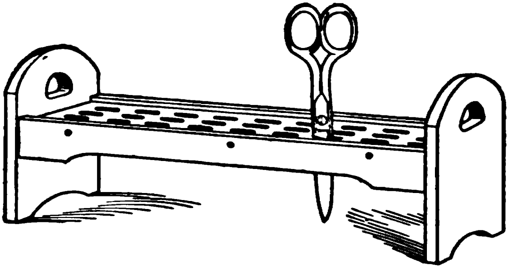
CHAPTER XIV

GENERAL REVIEW

349. General Review. We have now studied the subjects that are regarded as the essentials of arithmetic. In this chapter we shall consider further illustrations of the applications of arithmetic to the affairs of life, and these will also serve as a general review of the entire subject. The latter part of the chapter comprises a carefully arranged series of examinations.

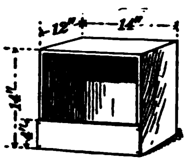
PROBLEMS IN WOODWORKING

1. The class is making this scissors rack for the sewing room. There are four strips of wood with holes for the scissors, the side piece hiding one of the strips in the picture. If each of the four strips is $\frac{5}{8}''$ wide, and each side piece is $\frac{1}{2}''$ wide, what is the width of the body of the rack?



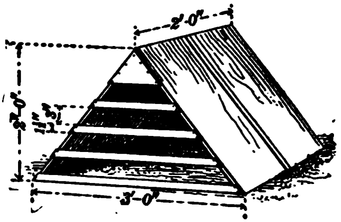
2. There are nine openings in each of the four strips, the distance from the center of each opening to the center of the next being $1\frac{1}{2}''$. The distance from the centers of the first and last openings to the ends of the strips is also $1\frac{1}{2}''$. What is the length of the rack?

3. Edward helped his father make some nests and coops. This is a picture of one of the nests for a sitting hen. Edward figured out the number of feet of lumber for each nest. How many feet were required?



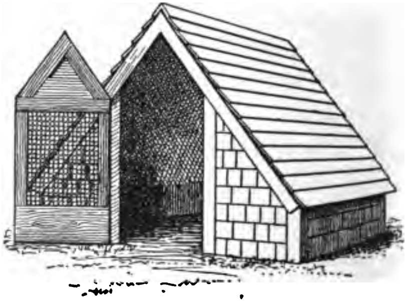
4. Edward and his father made eight of these nests, using lumber that cost $40 per M. How much did the lumber cost?

5. For the hens and chickens Edward and his father made eight coops like the one pictured below. Edward drew the figure to the scale of 1 in. to 1 ft., and then measured the size of each part of the coop. Do this, and find the number of feet of lumber required and its value at $40 per M.



Some approximation is allowed here, but this is exactly what happens in practical work, and is very desirable. The pupils may compare their answers and thus check one another's errors.

6. Edward's father built a sufficient number of poultry houses like this for his hens. The length of each was 9 ft., the width 7 ft., the height in the center 6 ft., and the height at the eaves 18 in. Allowing 20 cu. ft. for each hen, how many hens will this house accommodate?



If Edward gets an answer like 11.8 hens, he will say that the house will accommodate a dozen. In other words, he will use his common sense with such an answer, just as you always should.

BUSINESS STRATEGY

1. A druggist is considering whether to move his store to a more central location. He asks his son George to do some figuring with him, to see whether it will pay to move. The new store would cost him $330 more per year. How much is this a month?

2. If 20 more customers will enter the store each day, and each will buy 45¢ worth of goods on which the profit is 12% above all expenses, these figures all being averages and the drug store being open 30 days each month, does George find that the change of location is wise or not? Give his reason.

3. After moving to the new store, George's father decides to do some advertising. He has a cough medicine which he manufactures at a cost of 15¢ a bottle, and his annual sales average 200 bottles at 50¢ a bottle. If $250 spent in advertising enables him to sell 1250 more bottles during the year, is this advertising a good investment? Find his profit on this medicine before he advertised and also after deducting the amount spent for advertising. What was his per cent of profit in each case?

4. George thinks that the $250 spent in advertising was not as good an investment as the money spent in manufacturing. What was the per cent of returns on each? Was George right?

5. It occurs to George that instead of idling away his time after school and on Saturdays, he would do better to help his father in the store. His father offers him 5% on such money as he may take in. If he takes in $2.50 on each of the five school days and $9.50 on Saturday, how much does he make for himself in a week?

6. If George needs $225 to carry him through the trade school, beginning 5 yr. hence, how much of his money must he save each year, not considering interest, and how much may he spend?

350. Problem Data. The following price list may be used in solving the problems on page 479, and similar problems.

The data may also be secured by the pupils by inquiry at home or at some grocery. This may be made the basis of practical problems in simple domestic bookkeeping.

Coffee, Maracaibo, 20¢ per pound; 5 lb. for 85¢.
Java and Mocha, 31¢ per pound; 5 lb. for $1.45.
Old Government Java, green, 27¢ per pound; 5 lb. for $1.30.
Tea, Black India, 50¢ per pound.
English Breakfast, 40¢ per pound.
Ceylon, finest, 80¢ per pound.
Cocoa, half-pound cans, 33¢; $3.75 per dozen cans.
Sugar sirup, half-gallon cans, 50¢; 5-gallon cans, $4.
Maple sirup, pints, 25¢; gallon cans, $1.45; $16.50 per dozen cans.
Honey, 8-ounce bottles, 40¢; $4.50 per dozen bottles.
Flour, buckwheat, 5¢ per pound; a bag of 24½ lb., $1.10.
Self-raising, 3 lb. for 19¢; 6 lb. for 35¢.
Wheat, 4¢ per pound; $5 per barrel of 196 lb.; 70¢ per sack of 24½ lb.
Breakfast cereal, 14¢ per package; $1.60 per dozen packages.
Macaroni, 12¢ per package; 25 packages for $2.75.
Crackers, Alberts, 23¢ per tin; $2.65 per dozen tins.
Soda, 5¢ per carton; 55¢ per dozen cartons.
Soups, half-pint cans, 10¢; pint cans, 16¢; quart cans, 28¢; $3.25 per dozen quart cans.
Bacon, American, 25¢ per pound.
Sliced, in jars, 26¢ per pound; $3.10 per dozen jars.
Herrings, 15¢ per can; $1.75 per dozen cans.
Asparagus, 50¢ per can; $5.75 per dozen cans.
Domino sugar, 5 lb. for 50¢.
Loaf sugar, 7½¢ per pound.
Granulated sugar, 5¢ per pound.
Olives, 32¢ per bottle; $3.75 per dozen bottles.
Olive oil, 65¢ per bottle; 38¢ per half bottle; $7.75 per dozen bottles.
Allspice, 15¢ per can.
Cinnamon, 40¢ per pound.
Pepper, 22¢ per pound.
Cloves, 40¢ per pound.
Nutmegs, 55¢ per pound.
Ginger, 20¢ per pound.

INDUSTRY OF THE KITCHEN

1. If a family needs a dozen cans of cocoa, how much is saved by purchasing at the dozen rate?

2. If a family needs 5 gal. of sugar sirup, how much is saved by purchasing a 5-gallon can instead of buying ten half-gallon cans? What per cent is saved?

3. How much would a hotel save by purchasing 120 gal. of maple sirup by the dozen gallon cans instead of buying single cans? What per cent would the hotel save?

4. In which is the per cent of saving greater, in buying honey by the dozen bottles instead of by the bottle, or in buying maple sirup by the dozen cans instead of by the can?

5. How much is buckwheat flour a pound? At this rate, what will $24\frac{1}{2}$ lb. cost? How much is saved in buying a bag?

6. What is the per cent of saving in buying self-raising flour by the 6-pound package instead of by the 3-pound package?

7. By inquiry at home, make out a grocery list for a week, from page 230. Make two pages of a home account, the left-hand page showing the amount received, and the right-hand page showing the amounts spent for groceries.

8. Make out a bill for six items of groceries, making the proper extensions and footing. Receipt the bill.

9. Make out a bill for ten items of groceries, as in Ex. 8.

10. Make out a bill for twelve items of groceries, as in Ex. 8.

11. If gas costs $1 per 1000 cu. ft., and your neighbor uses 2320 cu. ft. in May, how much was his gas bill for that month?

12. At the beginning of the month the gas meter registers 14,260, and at the end of the month 17,140. How much is the gas bill for the month, at $1 per 1000 cu. ft.?

THE DAIRY INDUSTRY

1. A farmer sells 26,250 lb. of milk to a creamery in a certain month. The milk averages 4.2% butter fat. With how many pounds of butter fat does the creamery credit him in that month?

2. If 6 lb. of butter fat are needed in making 7 lb. of butter, what is the value of the butter produced from 1236 lb. of butter fat, the butter being worth 24¢ a pound?

3. A certain dairy during a certain month sells to a creamery milk averaging 3.75% of butter fat. The butter fat weighs 630 lb. How many pounds of milk does the dairy sell?

4. A farmer has two cows, one supplying 986 lb. of milk testing 3.1% butter fat in a certain month, and the other 812 lb. testing 4.2%. If the creamery allows him 27¢ a pound for butter fat, which cow pays him the more for that month, and how much more, the feed costing the same?

5. A creamery uses 7500 lb. of milk in a week. The skim milk amounts to 80% of the whole milk and contains 0.6% butter fat. How many pounds of butter fat are lost in the skim milk?

PROBLEMS IN IRRIGATION

1. An acre-inch of water is the amount of water 1 in. deep that it will take to cover a perfectly flat acre. What is an acre-foot of water, how many acre-inches does it contain, and how many cubic feet of water does it contain?

These terms are used in irrigation, particularly in the mountain states. The exercise may be omitted in those parts of the country in which irrigation is not common. The pupils should bring local problems to class. The technical terms on this page need not be learned unless needed in the locality, and they are sufficiently explained in the problems to make formal definitions unnecessary.

2. A second-foot of water is a cubic foot passing a particular point in 1 sec. If 7 cu. ft. of water pass in 1 sec., the irrigation ditch is delivering 7 second-feet of water. By timing a floating stick in a flume 4 ft. wide, the water being 6 in. deep, it was found that the water was flowing 10 ft. per second. How many second-feet of water was the flume carrying?

Allowance has to be made for friction of flowing water, but this need not be considered in an elementary treatment of the subject.

3. If a flume is delivering 20 second-feet of water, how long will it take to supply the water to irrigate 2 A. of land, the soil requiring 8 acre-inches per acre?

4. A farmer has 80 A. of beets which he wishes to irrigate from a reservoir. How many cubic feet should he store in the reservoir if each acre requires 2 acre-feet for the season?

There will have to be an allowance for evaporation and other waste, but this problem is supposed to require the net amount after such allowance is made.

5. A company is organized to furnish 25 sections (or 25 sq. mi.) of land with 1½ acre-feet of water per acre annually. If 25% is allowed for loss by seepage and evaporation, so that only 75% of the water is available, and if the reservoir has an average depth of 40 ft., and must store all the water necessary for a year at one time, how many acres will the reservoir cover?

PROBLEMS OF THE WESTERN FARM

1. A farmer has 200 A. on which he rotates his crops, with 3 yr. given entirely to alfalfa and 2 yr. entirely to sugar beets. The beets require 2 acre-feet of water per acre each season, and the alfalfa requires four fifths as much. How many acre-feet of water will the farmer need during the five-year period?

2. In a recent year the reclamation service of the government expended $86,500,000 in reclaiming about 2,500,000 acres of land. What was the average expense per acre?

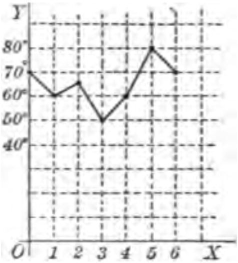
3. An experiment was made in raising alfalfa. A farmer rented 50 A. of irrigated land for 5 yr. at $5 per acre per year. The seed cost 40¢ per acre per year, the preparation of the land and the seeding cost $1 per acre per year, and the fertilization 80¢ per acre per year. It cost $3 per acre every time the alfalfa was cut, and it was cut twelve times in the 5 yr. The yield was 2.8 T. per acre per year. Find the total cost for 5 yr., the amount of alfalfa grown, and the cost per ton.

The crops as well as the industries vary in different parts of the country. The teacher may omit pages which relate to industries with which the pupils are not familiar. Thus, the raising of alfalfa may mean little in certain regions.

4. Some experiments in Utah show that alfalfa grows best with 1.6 acre-feet of water per season, wheat and oats with 1.2 acre-feet, barley with 1.7 acre-feet, and sugar beets with 1.9 acre-feet. A Utah farmer had 100 A. of alfalfa, 50 A. of wheat, 75 A. of oats, 40 A. of barley, and 50 A. of sugar beets. He used uniformly 2 acre-feet of water per acre on all this land. Disregarding any damage to the crops, how much water did he waste?

5. Some farmers use this rule for finding the number of bushels of corn in a bin: Take eight times the product of the three dimensions of the bin, and divide by ten. Using this rule, find the number of bushels in a bin 10 ft. by 6 ft. by 4 ft.

351. Graph. In this figure the successive days of the week are represented on the line OX, and the temperature at noon on the different days is represented on the lines parallel to OY. The broken line shows that the temperature one day at noon was +70°, the next day, 60°; then 65°, 50°, 60°, 80°, 70°. This broken line is called a *graph*. In this particular case it does not show the temperature at any time except noon.



WRITTEN EXERCISE

1. On ten successive January days in Duluth the thermometer at noon registered 40°, 60°, 55°, 50°, 20°, 17°, 12°, 8°, 10°, 30°. Trace the graph.

2. The number of ounces of silver produced in Colorado, stated in millions, was 0.5 in 1870; 13 in 1880; 19 in 1890; 20 in 1900; and 8 in 1910. Trace the graph.

3. The production of oats in the United States, in hundred million bushels, for eight consecutive years was 8, 9, 10, 10, 8, 8, 10, 11. Using any convenient scale, trace the graph.

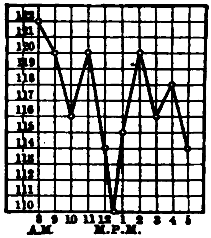
4. The production of cotton in the United States, in million bales, for seven consecutive years was 14, 11, 14, 12, 14, 11, 12. Using any convenient scale, trace the graph.

5. The number of tons of copper produced in this country in 1850 was 650; in 1880 it was 27,000; in 1900 it was 271,000; in 1914 it was 555,000. Trace the graph.

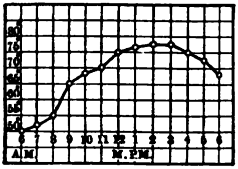
6. The value of the silver produced in this country in 1850 was $51,000; in 1880 it was $35,000,000; in 1900 it was $36,000,000; in 1914 it was $41,000,000. Trace the graph.

The graph is often found to be very helpful in the study of geography.

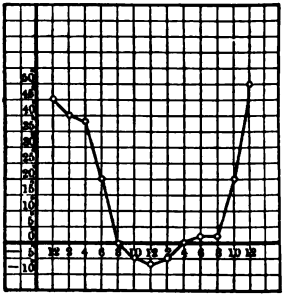
7. In a certain mill the steam gauge on a boiler showed a pressure of 122 lb. to the square inch at 8 A.M. The pressure varied through the day as shown by this graph. Write the pressures for every hour from 8 A.M. to 5 P.M., and also for 12.30 P.M. Why did the pressure go so low between 12 M. and 1 P.M.?



8. The temperature between 6 A.M. and 6 P.M. on the first of last May, at our school, was as shown by this graph. Write the approximate temperature for each hour and half hour of the day.

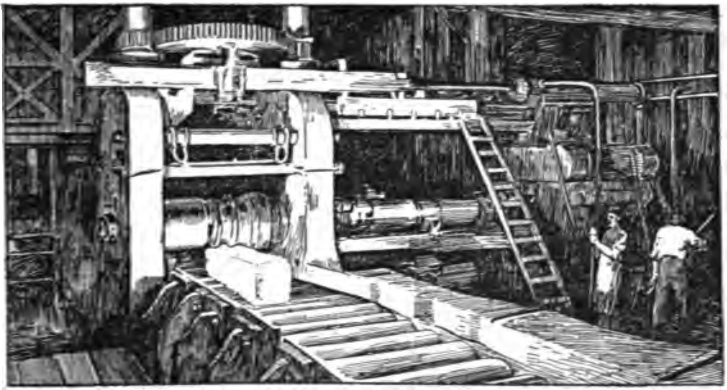


9. When the temperature is below zero this fact is indicated by a minus sign. That is, – 5° means 5° below zero. This graph shows the changes in temperature from noon on a day in January to noon on the following day. Write the approximate temperatures for every hour of this period, and a statement telling when the temperature fell the most rapidly and when it rose the most rapidly.



10. Draw a graph showing approximately, according to your estimate, the changes in temperature from 9 A.M. to 4 P.M. yesterday.

11. The average farm price of hay per ton in this country was $8.52 in 1905, $10.37 in 1906, $11.68 in 1907, $8.98 in 1908, $10.50 in 1909, $12.14 in 1910, $14.29 in 1911, $11.79 in 1912, and $12.43 in 1913. Trace the graph on the scale of 1 in. to a dollar.



THE IRON INDUSTRY

1. What is the weight of a steel girder rolled out at this mill, the girder being 26′ 6″ long, weighing $42\frac{1}{2}$ lb. to the foot?

2. The mill also rolls out iron rods. What is the cost of 12′ 8″ of iron rod, $2\frac{1}{8}$ lb. to the foot, at $1\frac{3}{4}$¢ a pound?

3. The wooden pattern from which an iron casting is made weighs $6\frac{1}{4}$% as much as the iron. The pattern weighs $45\frac{5}{8}$ lb. How much does the casting weigh?

4. If steel rails weighing 120 lb. to the yard are used between New York and Chicago, a distance of 980 mi., how many tons will be required for a double-track road between these cities?

5. An iron tire expands $1\frac{9}{16}$% on being heated for shrinking on a wheel. A wooden wheel needs a tire 4′ 8″ in diameter. How much longer will the tire be when thus heated?

6. In a certain blast furnace the casting machine turns out 20 pigs of iron per minute, averaging in weight 110 lb. each. If this machine runs at this rate for 308 days, 16 hours a day, how many long tons of pig iron will it turn out?

THE SHIPPING INDUSTRY

1. One of the fastest day's runs recorded for a steamer is 676 knots. How many statute miles is this?

2. The tonnage of the *Bismarck* is 60,000 tons, which is 10% more than that of the *Vaterland.* What is the tonnage of the *Vaterland*?

3. If a steamer's hull is 65 ft. deep, 955 ft. long, and 100 ft. broad, the depth is what per cent of the length? The breadth is what per cent of the length?

4. One of the steamship lines landed in New York on an average 1206 passengers per ship in a certain year, which was 90% of the average the year before. What was the average then?

5. One of the largest steamers will carry 80 barge loads of grain, at 8000 bu. to a barge. A freight car 40 ft. long will carry 1280 bu. What is the total length of a train carrying enough grain to load this steamer?

6. One of our American steamers pays $4760 in monthly wages to the 136 stokers who keep up the fires for its great engines. A German steamer pays $2864 to its 179 men who do this work. Each American stoker receives what per cent more monthly wages than the German?

TRANSPORTATION

1. If a freight car is 36 ft. long, 8 ft. 3 in. wide, and 7 ft. 6 in. high, inside measure, how many cubic feet does it contain?

2. If the width of the car as given in Ex. 1 were increased 1 in., what would be the per cent of increase in capacity?

3. If a wooden coal car can carry 80,000 lb., and a steel car 100,000 lb., the steel car can carry what per cent more than the wooden car? The wooden car can carry what per cent less than the steel car?

4. A car 36 ft. long and 8 ft. 4 in. wide is loaded with wheat to a depth of 4 ft. 6 in. Reckoning a bushel as containing $1\frac{1}{4}$ cu. ft., and a bushel of wheat as weighing 60 lb., how many pounds does the wheat weigh?

5. If the car of Ex. 4 has a carrying capacity of 80,000 lb., to what depth must it be filled with wheat to contain exactly this amount?

6. If a locomotive weighing $115\frac{1}{2}$ tons can exert a pull of $22\frac{1}{4}\%$ of its weight in starting a train, how many pounds of pull does it exert?

7. If a locomotive and coal tender together weigh 95 tons, and there are seven cars to the train, averaging 21 tons each, and a horizontal pull equal to 0.5% of the total weight of the train is required to maintain the necessary speed on a level track, what is this pull in pounds?

8. A 20,000-ton ship carries 12 times as much freight as the old-style ocean steamers, makes 25% better time, and the expenses per year are only four times as much. In a year such a modern boat will do how many times the work of the old kind? At the same cost, it will carry how many times as much freight?

PROBLEMS OF THE MACHINIST

1. If the diameter of a wheel is 7 in., find the circumference.

2. What is the circumference of a fly wheel of which the diameter is 7 ft. 6 in.?

3. The diameter of the piston of an engine is 8 in. Required the circumference of the piston.

4. If the pressure of steam in a boiler is 140 lb. to the square inch, find the pressure on a valve $2\frac{7}{8}$ in. in diameter.

5. A boiler contains 480 smoke tubes, each $2\frac{3}{8}$ in. in internal diameter. Find the total area for draft through them.

6. If a pressure of 130 lb. to the square inch is applied to the circular plunger of a cylinder 2 ft. in diameter, what is the total pressure?

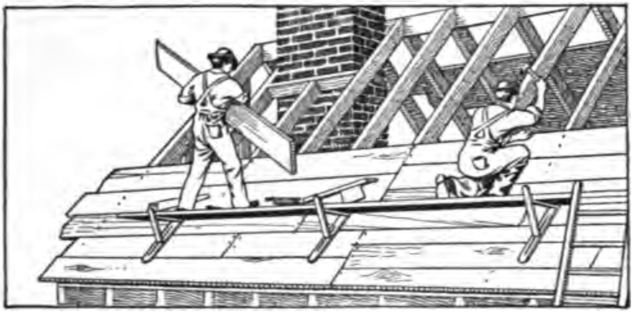
7. If an emery wheel of which the diameter is 10 in. makes 2000 revolutions per minute, its grinding speed per minute is 2000 times the circumference. What is the grinding speed?

8. A fly wheel of which the diameter is 6 ft. makes 100 revolutions per minute. At what rate per minute does a point on the rim travel?

9. How many revolutions per minute must be made by a circular saw of which the diameter is 28 in., in order that the saw may have a cutting speed of 8800 ft. per minute?

10. Two cogwheels are so geared that for every revolution of the larger wheel the smaller revolves 12 times. The larger wheel has 204 cogs. How many cogs has the smaller?

11. The fly wheel of an engine 8 ft. in diameter makes 250 revolutions per minute. It is directly connected by a belt with a wheel 2 ft. in diameter. How many revolutions per minute does the smaller wheel make?



PROBLEMS OF THE BUILDER

1. From the eaves to the ridge of this roof is 13 ft., and the span is 24 ft. Find the rise of the roof; that is, the height of the peak above the level of the eaves.

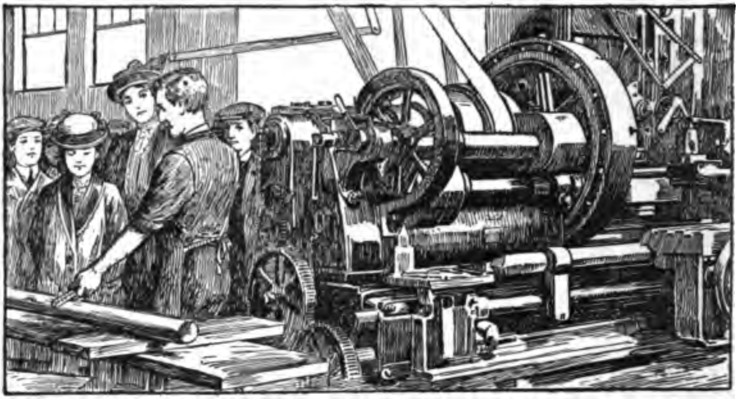
2. The span of a common A-shaped roof is 26 ft., and the rise of the roof is 6 ft. What is the distance, to two decimal places, from the eaves to the ridge?

3. A semicircular arch over a door has a diameter of 4′ 1″. Required the length of this semicircumference.

4. A builder is putting in a steam pipe 165 ft. long and 2 in. in diameter. What is the radiating surface, that is, the curved surface that gives out heat?

5. A contractor places on the top of a city apartment house a cylindric water tank 20 ft. high and 10 ft. in diameter. How many cubic feet of water will the tank hold?

6. The excavation needed for the cellar of a city apartment house is to be 92 ft. long, 60 ft. wide, and 7 ft. deep. In addition to this the excavation is to be extended 8 in. beyond each of the four sides, so as to allow space for working. How many loads (cubic yards) of earth must be removed?



VISIT TO THE MACHINE SHOP

1. Miss Adams took her class to visit a shop, and the foreman gave them some problems. He showed them a casting that was 3 ft. 7 in. long and explained that iron expands $\frac{3}{16}$ in. to the foot when heated to a red heat, and he asked them to compute the length of this casting when red hot. How long would it be?

2. Miss Adams asked if all metals expanded at the same rate, and the foreman told her that they did not. He said that in casting brass hinges they had to allow for a shrinkage of $\frac{1}{64}$ in length when the brass cooled. The boys were told to figure out the length of the mold for a hinge that was to be $5\frac{7}{8}$ in. long when cooled. How long was it?

3. The foreman showed them a piece of steel shafting that the men were just setting up. He told them that it was 3 in. in diameter, 20 ft. long, and weighed 490 lb. to the cubic foot. He said they had to know the weight of the shafting in order to be sure to give it ample support. The girls were told to figure it out. How much should they find that it weighed?

PROBLEMS IN MANUFACTURING

1. How much will a manufacturer receive for 8 rockers at \$7.25, 9 dressers at \$14.50, and 16 bedsteads at \$18.75, discounted at 8%?

2. A manufacturer sells the following bill of goods: 8 doz. plates @ \$1.42, 9 doz. cups @ \$2.35, $\frac{3}{4}$ doz. covered dishes @ \$9.75, 16 doz. plates @ \$1.65, $\frac{5}{6}$ doz. pitchers @ \$6.75. The bill is discounted at 3%, and he gains 20% on the goods. Required the cost of manufacturing the lot.

3. A manufacturing company has a capital of \$350,000, its annual business is $3\frac{1}{4}$ times its capital, and its profits are $7\frac{1}{2}$% of its business. How much are its annual profits?

4. A manufacturer sells brass lamps to a dealer at such a price that the dealer can sell them at \$6.60 and make a profit of 32%. The manufacturer himself makes a profit of 25%. What is the actual cost of manufacture per lamp?

5. On which will a manufacturer make the greater per cent of profit: an article which costs \$2.90 to make and sells for \$3.33$\frac{1}{3}$ less 3%, or one which costs \$8.70 to make and sells for \$12 less $\frac{1}{6}$? How much greater is this per cent of profit?

6. A manufacturer has a water tank 8 ft. 3 in. wide, 12 ft. 6 in. long, and 9 ft. 8 in. deep. If a cubic foot of water weighs $62\frac{1}{2}$ lb., what is the weight of the water required to fill the tank?

7. A factory pays \$3782.40 for materials, labor, and overhead charges, and sells the product at $12\frac{1}{2}$% profit. Find the receipts.

8. A manufacturer pays a salesman a salary of \$1250 a year and a commission of $1\frac{1}{2}$% on all sales, and allows him \$1000 a year for expenses. If the salesman sells goods to the value of \$172,500, how much does the manufacturer pay him, including salary, commission, and expenses?

THE PEARL BUTTON INDUSTRY

1. A manufacturer bought 980 lb. of white pearl shell at 78¢ per pound less 10%. Find the cost of this raw material.

2. A Chicago manufacturer bought 1225 lb. of white pearl shell at 82¢ per pound less 8% and 2%. How much did this raw material cost him?

3. A manufacturer bought 40 cases of black pearl shell at 45¢ per pound less 8%. The average weight of each case being 65 lb., how much did this shipment of raw material cost him?

4. A manufacturer put up pearl buttons in boxes of 6 gross each. He sold 64 boxes to Roberts & Jameson and 256 boxes to Peabody & Jones. How many gross of buttons did he sell to each firm? how many buttons?

5. The manufacturer received $1.90 per gross for the buttons mentioned in Ex. 4, but allowed a discount of 12%. How much was due him from Roberts & Jameson? How much was due him from Peabody & Jones?

6. A button company received from London a consignment of shells billed as follows: 440 lb. fancy white @ 43 d. (that is, 43 English pence), 1560 lb. dull white @ 31 d., 440 lb. black @ 21 d., 740 lb. medium gray @ 18 d., and 880 lb. sea snail @ 10½ d. Taking 2¢ to the penny, find the value of this consignment in our money.

The English bill would give 3 shillings and 7 pence (3 s. 7 d.) instead of 43 d., there being 12 pence in a shilling. This is the plan followed in Ex. 7.

7. A London exporter bills to a certain manufacturer a consignment of shells as follows: 760 lb. fancy white @ 3 s. 5 d., 380 lb. medium gray @ 1 s. 8 d., and 1275 lb. dull white @ 2 s. 8 d. Find the value of this consignment in our money.

PROBLEMS ON SPRAYING

1. To lessen potato scab some seed potatoes are soaked in a solution of $\frac{1}{2}$ pt. of formalin to 15 gal. of water. What is the per cent of formalin in the mixture?

In Exs. 1 and 2 answer to the nearest tenth per cent.

2. In Ex. 1, what would be the per cent of formalin in the mixture if $\frac{5}{8}$ pt. had been used to 15 gal. of water?

3. Bordeaux mixture, for spraying, contains $1\frac{1}{4}\%$ by weight of copper sulphate, $1\frac{1}{4}\%$ of unslaked lime, and the rest water. How many pounds of each in a barrel containing 250 lb. of the mixture?

4. A cubic foot of Bordeaux mixture weighs 63 lb. From the statements made in Ex. 3, find the number of pounds of copper sulphate and unslaked lime in a tank that is 6 ft. long, 4 ft. wide, and 1 ft. 6 in. deep.

5. Half of a peach tree was sprayed with Bordeaux mixture and the other half was not. From the sprayed half 284.8 lb. of fruit were taken, and from the other half 95% less. How many pounds were taken from the tree?

6. When copper sulphate costs 10¢ a pound, lime 1¢ a pound, and arsenate of lead 20¢ a pound, and we need for a spray 3 lb. of arsenate of lead and 4 lb. each of lime and copper sulphate to 50 gal. of water, and using $2\frac{1}{2}$ gal. of the spray to a tree, what will it cost for material to spray 125 trees?

7. For spraying potato plants a man prepares a solution of Paris green, using $\frac{1}{2}$ lb. of Paris green costing 30¢ a pound to 100 gal. of water. He uses 300 gal. to an acre, and sprays a field of $3\frac{1}{2}$ acres. What is the cost of the material? If the labor costs $7.25, what is the total cost? If he sells his product for $28 more than if he had not sprayed, what is his gain?

PROBLEMS OF THE CONTRACTOR

1. If a contractor makes $\frac{1}{4}$ of the cost on a \$4500 job, how much does he make?

2. If a contractor makes $\frac{1}{6}$ of the cost on a \$15,000 job, how much does he make?

3. If a contractor makes \$675 on a \$3375 job, how much does he make on each dollar of the cost?

4. If a contractor makes $\frac{1}{10}$ of the cost on one \$4500 job, and loses $\frac{1}{10}$ of the cost on another \$4500 job, has he gained or lost on the two together?

5. A contractor builds a brick wall 68′ 8″ long, 5′ 8″ high, and 2 bricks thick. Allowing 14 bricks per square foot of outside surface for a 2-brick wall, how much will the bricks cost at \$8.50 per M?

6. A man contracted to paper a house. The rooms are 14′ × 16′, 8′ × 12′, 15′ × 18′, 10′ × 12′, 13′ × 15′, 16′ × 16′, 6′ × 10′, and 12′ × 12′. He deducts 50′ from the combined perimeters of the rooms and allows one roll of paper for every $3\frac{1}{2}$ ft. of perimeter. How many rolls does he need?

7. A man has a contract to put up the four walls of a building which is to be 80′ long, 35′ wide, and 78′ high. The walls are to be 18″ thick, and 750 cu. ft. is to be allowed for openings. Allowing 22 bricks to a cubic foot, how much will the bricks cost at \$9.25 per M?

8. A contractor wishes to build a concrete wall 8′ × 66′, and 18″ thick. Concrete is $\frac{1}{9}$ Portland cement, which costs \$1.90 a barrel of 400 lb., and the rest is gravel costing 88¢ a load (cubic yard), delivered. A cubic foot of dry cement weighs 150 lb. How much will the materials cost?

352. Review Examinations. Pages 495–506 are devoted to oral and written review examinations, ten examples to a set.

WRITTEN EXAMINATION

1. Find the interest on $428.50 for 3 mo. 5 da. at 6%. Solve in two ways, stating the operations in steps.

2. From a lot 12 rd. by 16 rd. a path 4 ft. wide is cut on all four sides. How many square yards are left?

3. A man sold a lot for $500, gaining 20%. He sold another lot for $500, losing 20%. Did he gain or lose on the two transactions together, and how much?

4. Which is the better investment of $1500 for a year: a note bearing $4\frac{1}{2}$% interest, or a savings-bank account at 4% per year, the interest being payable semiannually and no interest being withdrawn until the end of the year?

5. If $x : 17\frac{1}{2} = 9\frac{1}{2} : 35$, what is the value of x?

6. A woman living in a city earned $819 last year by shopping for her friends. On half of her purchases she received 4% commission, and on half she received 5%. How much money did she spend in shopping?

7. Multiply 7 ft. 6 in. by 4, and divide the product by 5.

8. A wholesale jeweler advertised his stock to be sold at 50% and 25% off. I bought a watch for $15. What was the list price?

9. A man owns a house valued at $12,800. He rents it for $768, paying taxes at the rate of 2 mills, $24.20 for insurance, and $144 for other expenses. Find his net rate of income.

10. How many yards of carpet, $\frac{1}{2}$ yd. pattern and $\frac{3}{4}$ yd. wide, will it take to cover a room 16 ft. 8 in. long and 12 ft. wide, cutting so as to match patterns and running the strips lengthwise?

WRITTEN EXAMINATION

1. Find to the nearest hundredth of a foot the side of a square whose area is 50 sq. ft.

2. The time from one new moon to the next (a lunar month) is 29 da. 12 hr. 44 min. 3 sec. Express 6 lunar months in days and the decimal of a day.

3. Find the difference in the number of acres in two fields, one of which contains 120 sq. rd., and the other of which is 120 rd. square.

4. If a certain gun metal is composed of 84% copper and the remainder tin, how many pounds of tin will be needed with 638.4 lb. of copper to make the gun metal?

5. A company that has a capital of $1,250,000 earned net $103,750.75 last year. It paid a dividend of 6% and carried the balance to surplus. How much did the company pay in dividends, and how much did it carry to surplus?

6. A farmer has a bin 18 ft. long, 5 ft. 3 in. wide, and 5 ft. deep which holds 378 bu. How many bushels in a bin that is 4 ft. longer, 1 ft. 9 in. narrower, and has the same depth?

7. Water is flowing into a tank whose base is 9 ft. 4 in. long and 6 ft. 9 in. wide, at the rate of 3 cu. ft. in 2 min. How long will it take the water to fill the tank to a depth of 4 ft. 6 in.?

8. One face of a cube has an area of 3 sq. in. What is the volume of the cube?

9. A woman pays taxes at the rate of $1.09 per $100 on property assessed at $7500. Find the amount of her taxes.

10. How high is a tree that casts a shadow 85 ft. long at the same time that an upright post 3 ft. high casts a shadow 4 ft. 3 in. long?

ORAL EXAMINATION

1. How long must a man have $250 invested at 6% for the amount of principal and interest to be $265? to be $280? to be $295? to be $310? to be $325?

2. A man borrowed $8000 in the East at 5% and invested it in the West at 8%. How much did he make per year by the transaction? Why does not every one who has money to invest follow this plan?

3. What sum must be invested at 6% for the interest at the end of 2 yr. to be $180?

4. At what rate must $1000 be invested for the amount of principal and interest to be $1120 at the end of 3 yr.? for the amount to be $1180 at the end of 3 yr.?

5. A dealer sells $1000 worth of goods discounted at 20% and 10%. What is the net price?

6. A man having $10,000 invested at 4½% reinvests half of it at 5% and leaves the rest as before. What is the increase in income?

7. During the year a merchant borrows at various times $100, $200, $1200, $1500, each time for 60 da. at 6%. How much interest does he pay?

8. A bank makes a loan of $800 at 6% and receives $12 interest (bank discount). Required the time.

9. A bank lends $800 for 60 da. and receives $8 interest. Required the rate.

10. A bank lends some money for 60 da. at 6% and receives $10 interest. How much is the amount of the loan? How much is the loan on which a bank receives $20 interest in 60 da. at 6%?

WRITTEN EXAMINATION

1. The rates of the express and local trains are 45.6 mi. an hour and 28.4 mi. an hour respectively. How much time is saved by taking the express train for a distance of 161.88 mi.?

2. From a tank full of water $\frac{3}{5}$ of the water was drawn off. The tank then lacked 70 gal. of being half full. What is the capacity of the tank?

3. A tank of capacity $166\frac{1}{2}$ gal. is $\frac{2}{5}$ full of water. How long will it take a pipe supplying water at the rate of 3.7 gal. a minute to fill the rest of the tank?

4. A man who owned some shares of stock in a mill sold $\frac{1}{3}$ of his shares to A, $\frac{1}{5}$ to B, and $\frac{1}{6}$ to C. He then had 90 shares left. How many shares had he at first?

5. A boy lost $\frac{1}{6}$ of his kite string in a tree and 30 ft. of it in some wires. If he had $\frac{3}{4}$ of it left, how long was it at first?

6. How many bricks will be required for the walls of a house 42 ft. long, 28 ft. wide, and 18 ft. high, deducting 3 doors 7 ft. 6 in. by 4 ft., and 9 windows 5 ft. by 3 ft., the walls to be 1 ft. thick, allowing 22 bricks to 1 cu. ft.?

7. The sugar beet contains by weight $14\frac{1}{2}\%$ of sugar. In a recent year the United States produced 2,884,750 tons of sugar beets. How much sugar did this represent?

8. A man has a salary of $1750 a year. He pays 15% of his salary for rent, 25% for groceries, 12% for meats, 13% for clothing, and $112.50 for other expenses. What per cent of his salary does he save?

9. Find the square root of 7 to three figures.

10. How many tons of coal can be placed in a bin 16 ft. 6 in. long, 7 ft. wide, and 6 ft. deep, allowing 33 cu. ft. to the ton?

WRITTEN EXAMINATION

1. Find the difference in acres between two fields, one containing 100 sq. rd., and the other being 100 rd. square.

2. Find the cost of a stair carpet at $1.50 a yard for a straight flight of 16 steps 12 in. wide and 8 in. rise, allowing 20 in. extra at the bottom of the stairs and the same amount, besides the 12 in. for the top step, at the top.

3. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 650 lb. of flint glass.

4. A bell weighing 875 lb. is composed by weight of 77% copper, 2% zinc, and the rest tin. How many pounds of each metal does the bell contain?

5. Gun metal is composed by weight of $85\frac{1}{2}$% copper and the rest tin. How many pounds of copper must be put with $159\frac{1}{2}$ lb. of tin to make gun metal?

6. Bronze is composed by weight of 18% tin and the rest copper. How many pounds of tin must be put with 1066 lb. of copper to make this quality of bronze?

7. The driving wheels of a locomotive have a diameter of 6 ft. 5 in. How many revolutions a minute will each wheel make when the locomotive is traveling at the rate of 50 mi. an hour?

8. The front wheels of a wagon are 3 ft. 6 in. in diameter and the rear wheels are 4 ft. 1 in. How many more revolutions will a front wheel make than a rear wheel in going a mile?

9. Multiply 17 ft. 9 in. by 6.

10. It is required to make a rectangular tin box that shall contain exactly 1 gal. Required to know the three dimensions such that they shall each be in exact inches.

WRITTEN EXAMINATION

1. A boy computing the cost of 56 articles made the mistake of multiplying by 65 instead of by 56, and obtained $487.50. What was the cost of the lot?

2. Two masts of a ship are 52 ft. apart, one being 80 ft. high and the other 70 ft. high. Supposing them perpendicular to the deck, how far is it from the top of one to the top of the other?

3. A merchant asked 25% less than the cost of a certain article, but at an auction he received $33\frac{1}{3}$% more than the asked price. Did he gain or lose on the cost, and what per cent?

4. Water is flowing at the rate of 1 cu. ft. every 90 sec. into a tank whose base measure is 3′ 6″ by 4′ 2″. How long will it take to fill the tank to a depth of 1′ 6″?

5. A 12-inch gun can fire a shell weighing 850 lb. every 30 sec. At this rate how many pounds of shell could a battleship fire from four 12-inch guns in 2 min. 30 sec.?

6. A train leaves a city at 10.45 A.M. and reaches another city $127\frac{1}{2}$ mi. distant at 5 min. past 2 P.M. Allowing 20 min. for stops, what is the rate per hour?

7. A farmer finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels in a bin that is half as long, 6 in. wider, and 1 ft. less in depth?

8. If 12 men can lay the asphalt blocks in a certain length of a city street in 16 da., how many men must be added to the force to complete the work 4 da. sooner?

9. Divide 124 ft. 3 in. by 7, and check the result.

10. A village has a water tank 42 ft. in diameter. In 3 hr. 45 min., when no water is being pumped in, the water is lowered 3 ft. How many gallons per hour are drawn out on an average?

ORAL EXAMINATION

1. If you have a field in the shape of a triangle, and can make all the measurements you wish, and know the value of the land per acre, how do you find the value of the field?

2. If you know the assessed valuation of a man's property and the tax rate, how do you find the amount of his taxes?

3. If you know the face of a draft and the rate of premium, how do you find the cost of the draft?

4. If you have the first, third, and fourth terms of a proportion given, how do you find the second term? Given any three terms, how do you find the other term?

5. If you know the number of times a boy is at bat in a series of baseball games and the number of base hits, how do you find the per cent of base hits?

6. If you send to a city for a book, and wish to send the money with the order, how do you send the money? Why do you take this method? How much does it cost?

7. If you know the height and the diameter of a cylindrical silo, how do you find the number of cubic feet of ensilage that the silo will contain?

8. If you know the length of a strip of matting in feet, how do you find the length in yards? If you know the length in yards, how do you find the length in feet?

9. If you know the number of rods in one side of a square field, how do you find the number of square rods in the field? the number of acres?

10. In dividing one decimal fraction by another, where do you place the decimal point in the quotient? Give the reason for your answer.

WRITTEN EXAMINATION

1. The amount of water that flows over Niagara Falls averages 264,000 cu. ft. a second. Find the average number of gallons carried over the falls in a minute.

2. If $\frac{7}{8}$ in. on a map represents an actual distance of $35\frac{1}{2}$ mi., what distance on the map represents 28.4 mi.?

3. How many loads of gravel averaging 1 cu. yd. will be required to grade $2\frac{3}{4}$ mi. of road, the gravel to be laid 14 ft. wide and 6 in. deep?

4. A dealer pays $9737.50 for some wool, including the purchasing agent's commission of $2\frac{1}{2}\%$. Find the commission.

5. The sugar contained in the sugar beet is $6\frac{1}{4}\%$ of the weight of the beet. How many pounds of beets will be required to produce $122\frac{1}{2}$ lb. of sugar?

6. How many boards each 12 ft. long will be required to build a straight fence 4 boards high about a rectangular field 40 rd. wide containing 20 acres?

7. Find the circumference of a circle whose diameter is 22.6 in., using 3.1416 as the ratio of circumference to diameter; also using $3\frac{1}{7}$; also using the ratio 355 : 113.

8. Find the diameter of a circle whose circumference is 471.24 in., using 3.1416 as the ratio of circumference to diameter; also using $3\frac{1}{7}$; also using the ratio 355 : 113.

9. If 24 burners, each consuming $7\frac{1}{2}$ cu. ft. of gas per hour, are used on an average $3\frac{1}{2}$ hr. per day for 365 da., what will be the gas bill at 85¢ per thousand?

10. A bell weighing 1110 lb. is composed of 76.5% copper, 21.5% tin, and the rest zinc. How many pounds of each metal does the bell contain?

ORAL EXAMINATION

1. Last year a gardener had 3 A. of potatoes and raised 200 bu. to the acre. How many bushels did he raise in all? If he sold these potatoes at 60¢ a bushel, how much did he get for them all? Potatoes weigh 60 lb. to the bushel. What was the weight of his potato crop?

2. For spraying his potato plants a gardener uses 1 lb. of Paris green to 100 gal. of water. Since 1 gal. of water weighs 8.3 lb., the weight of the Paris green is what part of the weight of water?

3. A gardener soaked his seed potatoes in a solution of 1 pt. of formalin to 25 gal. of water, to keep off the potato scab. What part of the solution is formalin?

4. If $\frac{3}{4}$ of the weight of potatoes is water, and if a bushel of potatoes weighs 60 lb., what is the weight of water in a bushel of potatoes?

5. A farmer had 2 A. of celery, and the yield was 1550 doz. heads to the acre. How many dozen did he have in all?

6. A farmer bought seed for 1000 celery plants at 24¢ an ounce. Allowing an ounce of seed to 2000 plants, how much did the seed cost?

7. When wheat costs \$1.12$\frac{1}{2}$ a bushel, how much will 100 bu. cost? 1000 bu.? 2000 bu.?

8. If wheat is worth \$1 a bushel, how much will the yield of 100 acres be worth if the average yield is 25 bu. per acre?

9. If wheat is worth \$1.10 a bushel, how much will the yield of 50 acres be worth if the average yield is 20 bu. per acre?

10. If wheat is worth \$1.12$\frac{1}{2}$ a bushel, how much will the yield of 80 acres be worth if the average yield is 25 bu. per acre?

WRITTEN EXAMINATION

1. The following numbers are the given per cents more than what numbers and the same per cents less than what other numbers? That is, 390 is 62½% more than what number and 62½% less than what other number?

390, 62½%	175, 66⅔%	2.4, 20%	9.1, 30%
150, 87½%	350, 16⅔%	42.3, 50%	17.6, 60%
640, 33⅓%	143, 8⅓%	210, 40%	56.1, 70%

2. If a man spends 5% of his salary for clothes and then has $855 left, what is his salary? How much does he spend for clothes?

3. If you have read all but 40% of a book and have read 234 pages, how many pages has the book?

4. If the width of a schoolhouse is 10% less than the length, and the width is 46.8 ft., what is the length?

5. If you lose $\frac{1}{10}$ of a kite string and have 585 ft. left, how long was the string? How many feet did you lose? What per cent of the string was left?

6. A man sold some damaged goods for $1547, at a loss of 9%. How much did the goods cost him?

7. A village has a population of 3384, having lost 6% in the last ten years. What was the population ten years ago? What was the number lost?

8. A man sold a car for $1000, at a loss of 20%. How much did the car cost him?

9. A man sold some property and gained $360, which was 12% on the money he invested. How much did he invest?

10. A man sold some property for $3360, at a profit of 24%. How much did the property cost him?

ORAL EXAMINATION

1. A ruler is marked in thirty-seconds of an inch. A piece of veneer is measured and found to be $\frac{6}{32}$ in. thick. Express this thickness in lower terms.

2. A surveyor uses a chain 66 ft. long, made of 100 links. He measures a certain distance and finds it to be 1 chain and 50 links. Express this as a chain and a fraction; as links; as yards; as feet.

3. How much picture molding is needed for a frame $5\frac{3}{8}$ in. long and $3\frac{1}{8}$ in. wide?

4. Four packages weigh $12\frac{1}{2}$ lb., $13\frac{1}{4}$ lb., $10\frac{3}{4}$ lb., and $10\frac{3}{4}$ lb. respectively. What is the total weight?

5. The page of a book is $4\frac{3}{8}$ in. wide and $7\frac{1}{16}$ in. long. What is the sum of the two sides? What is the perimeter? What is the difference between the length and width?

6. A picture is $3\frac{1}{4}$ in. wide and the mat is $1\frac{1}{8}$ in. wide on each side of the picture. How wide are the picture and mat together?

7. A plank $1\frac{7}{8}$ in. thick is planed down $\frac{1}{16}$ in. on one side. How thick is the plank then? How thick would it be if it were planed down $\frac{1}{16}$ in. on both sides?

8. From a piece of picture molding 7 ft. $2\frac{1}{8}$ in. long there is cut a piece 2 ft. $1\frac{1}{4}$ in. long. How long is the remainder?

9. A piece of marble $8\frac{3}{32}$ in. thick is polished down $\frac{1}{16}$ in. What is then the thickness? How much more will it have to be polished down to be exactly 8 in. thick? to be $7\frac{15}{16}$ in. thick? to be $7\frac{7}{8}$ in. thick?

10. A piece of cloth $7\frac{1}{8}$ in. wide is folded so as to be $\frac{3}{4}$ in. less in width. How wide is it then? How much more will have to be folded in order that it may be $6\frac{1}{4}$ in. wide?

WRITTEN EXAMINATION

1. A family with an income of $1200 spent 0.25 of it on food. How much did it spend on food?

2. A family with an income of $1200 spent $300 on food. Express the amount spent on food as a common fraction of the income; as a decimal fraction. Tell why the answers in Exs. 1 and 2 check each other.

3. A family with an income of $2000 spent 0.23 of it on food. How much did it spend on food? A family with an income of $2000 spent $460 on food. What fractional part of the income was spent on food?

4. A family with an income of $800 spent 0.45 of it on food. How much did it spend on food? Make a problem that shall check Ex. 4, as Ex. 2 checked Ex. 1, and as the second part of Ex. 3 checked the first part. Solve the problem.

5. With an income of $1100 a family spent $275 for food. This was what decimal fraction of the income?

6. With an income of $900 a family spent $252 for food. This was what decimal fraction of the income?

7. A man whose income is $2800 a year spends 0.26 of it for rent. How much does he spend for rent?

8. A man whose income is $2800 a year spends $728 for rent. This is what decimal fraction of the income?

9. A man spends 0.23 of his income for food, 0.26 of it for rent, and 0.1 of it for clothing. If his income is $2400 a year, how much does he spend for each? for all three?

10. A man earns $200 a month and spends 0.22 of it for rent. How much does he pay for rent each month? How much does he pay for rent in a year?

SUPPLEMENT

353. Nature of this Supplement. As frequently stated in this series, there is a tendency at present to limit arithmetic to the absolute essentials. This has been the policy of the authors in preparing the series. There are certain topics which are essential in some places and which should be where the teacher can use them if necessary. Such are the topics treated in this supplement.

354. Land Measure. In surveying land the following measures are used:

LENGTH

7.92 inches (in.) = 1 link (li.)
100 links, or 4 rods (rd.) = 1 chain (ch.)
80 chains, or 5280 feet (ft.) = 1 mile (mi.)

City property is usually measured by feet and decimals of a foot.

AREA

16 square rods (sq. rd.) = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
36 square miles = 1 township (T.)

The square rod is sometimes called a *perch*.

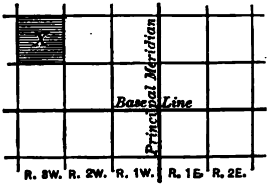
How many acres in a field 32 ch. long and 6 ch. 30 li. wide?

6 ch. 30 li. = 6.30 ch.
32 × 6.30 sq. ch. = 32 × 6.30 × 0.1 A. = 20.16 A.

Actual measurements of this kind are valuable in connection with the school ground and other pieces of land. The pupils should have a clear conception of an acre, by being shown a field containing one acre or two acres or a half acre.

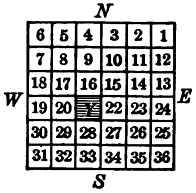
355. Laying out Public Lands. In the more recently settled parts of the country, land is laid out as here described.

356. Principal Meridian and Base Line. Through a given tract a meridian is chosen as the *principal meridian.* An east and west line is chosen as the *base line.* The principal meridian and base line are here shown.



357. Township. Lines are run parallel to the principal meridian and base line at intervals of 6 mi. This divides the land into *townships.*

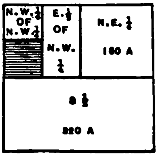
358. Range. The north and south rows of townships are called *ranges.* On the first map X is numbered T. 2 N., R. 3 W.; that is, the second township north of the base line, in the third range west of the meridian.



359. Section. A township is divided into *sections,* each 1 mi. square. This map shows the method of numbering these sections.

Each section is then divided as shown in the third map.

If this map represents the shaded part Y of the second map, and the second map represents the shaded part X of the first map, the shaded part here shown would be thus described: S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 21, T. 2 N., R. 3 W. This means the southwest quarter of the northwest quarter of section 21, second township north, third range west.



In sections of the country where land is not laid out in this way little attention should be given to this subject. In rural schools, however, some attention should always be given to the actual measuring of land, particularly by dividing fields of irregular shape into triangles.

WRITTEN EXERCISE

Write the description, plot, and find the area:

1. S.E. $\frac{1}{4}$, Sec. 5, T. 3 S., R. 3 W.

2. N.E. $\frac{1}{4}$, Sec. 8, T. 2 N., R. 2 W.

3. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 2, T. 2 N., R. 3 E.

4. S. $\frac{1}{2}$ of S.E. $\frac{1}{4}$, Sec. 20, T. 2 S., R. 3 W.

5. N. $\frac{1}{2}$ of S.W. $\frac{1}{4}$, Sec. 10, T. 3 S., R. 2 E.

6. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 30, T. 2 S., R. 3 E.

7. N.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$, Sec. 5, T. 1 N., R. 1 W.

8. S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 32, T. 1 S., R. 3 E.

9. How much is this farm worth at $65 an acre? W. $\frac{1}{2}$ of S. $\frac{1}{2}$, Sec. 3, T. 2 N., R. 2 W.

10. How much is this farm worth at $75 an acre? N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, Sec. 5, T. 2 S., R. 2 E.

11. Find the area of this farm: S.W. $\frac{1}{4}$, Sec. 10, T. 2 S., R. 2 E. Draw a plan of the township and locate the farm.

12. Find the area of this farm: N. $\frac{1}{2}$, Sec. 6, T. 1 N., R. 1 E. Draw a plan of the township and locate the farm.

13. Mr. Simmons owns the S. $\frac{1}{2}$, N.E. $\frac{1}{4}$, Sec. 3, T. 2 N., R. 3 E. How many rods of fence are needed to inclose his land?

14. How far is it from Mr. Taylor's farm, N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 16, T. 1 N., R. 3 W., to Mr. Hunt's farm, S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 28, T. 1 N., R. 3 W.? Draw the map.

15. A road runs straight through a farm and is $\frac{1}{2}$ mi. long and 3 rd. wide. How many acres in the road? If hay can be cut from the sides, averaging $\frac{1}{4}$ of the area of the road, and the amount of hay averages 2 tons to the acre and is worth $9.50 a ton, what is gained by attending to this crop?

360. Table of English Money. The English table of money is as follows:

12 pence (d.) = 1 shilling (s.)
20 shillings = 1 pound (£)

The pound is worth $4.8665, or about $5; the shilling is worth $0.243, or about 25¢; and the penny is worth about 2¢.

361. Table of French Money. The French table of money, used in several other European countries also, is as follows:

100 centimes (c.) = 1 franc (fr.)

The franc is worth $0.193, or about 20¢.

362. Table of German Money. The German table of money is as follows:

100 pfennigs (pf.) = 1 mark (M.)

The mark is worth $0.238, or about 25¢; and 4 pf., about 1¢.

WRITTEN EXERCISE

Taking £1 as equal to $4.87, express in our money:

1. £75. **2.** £96. **3.** £68. **4.** £16 10s.

Taking £1 as equal to $4.87, express in English money:

5. $38.96. **6.** $48.70. **7.** $24.35. **8.** $43.83.

Taking 1 M. as equal to 23.8¢, express in our money:

9. 75 M. **10.** 96 M. **11.** 125 M. **12.** 3750 M.

Taking 1 M. as equal to 23.8¢, express in German money:

13. $9.52. **14.** $11.90. **15.** $14.28. **16.** $71.40.

Taking 1 fr. as equal to 19.3¢, express in our money:

17. 85 fr. **18.** 250 fr. **19.** 230 fr. **20.** 750 fr.

363. Longitude and Time. In geography you learned about the longitude of places, and you know that longitude is reckoned in degrees, minutes, and seconds from the prime meridian, which passes through Greenwich, England.

364. Correspondence of Longitude to Time. Since the earth turns about on its axis once every 24 hours, the place in which we live will pass through 360° between now and this time to-morrow. That is, to the time 24 hours will correspond the longitude 360°. Therefore

360° of longitude corresponds to 24 hr.
1° of longitude corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.
1′ of longitude corresponds to $\frac{1}{60}$ of 4 min., or $\frac{1}{15}$ min., or 4 sec.
1″ of longitude corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

365. Earlier West, Later East. Because the earth revolves 360° in 24 hours, the sun appears to pass through 15° an hour. Therefore, when it is noon here it is an hour later, or 1 P.M., 15° east of here; an hour earlier, or 11 A.M., 15° west of here; and 6 hours earlier, or 6 A.M., 90° west of here.

Since this work belongs partly to geography, no illustrations are here introduced. The teacher should use the globe as needed.

ORAL EXERCISE

1. When it is 9 A.M. here, what time is it 15° east of here? 30° west of here? directly south of here?

2. When it is noon here, what time is it 30° east of here? 90° west of here? 150° east of here? directly north of here?

3. What difference of time corresponds to a difference of longitude of $7\frac{1}{2}$°? 45°? 60°? 105°? 120°? 165°? 180°?

4. Since 1′ of longitude corresponds to 4 sec. of time, to what does 7′ correspond? 11′? 15′? 30′? 45′? 60′? 120′?

366. Illustrative Problem. Two ships at sea are 65° 7′ 30″ of longitude apart. What is their difference in time?

Since 1° corresponds to $\frac{1}{15}$ hr., 65° corresponds to $65 \times \frac{1}{15}$ hr., or $4\frac{1}{3}$ hr., or 4 hr. 20 min.

Since 1′ corresponds to $\frac{1}{15}$ min., 7′ corresponds to $7 \times \frac{1}{15}$ min., or $\frac{7}{15}$ min., or 28 sec.

Since 1″ corresponds to $\frac{1}{15}$ sec., 30″ corresponds to $30 \times \frac{1}{15}$ sec., or 2 sec.

Therefore the difference in time between the ships is 4 hr. 20 min. 30 sec.

It is apparent that the mere figures of the answer could be obtained by this division, although the reason would not be so clear.

$$\begin{array}{r|rrr} 15 & 65 & 7 & 30 \\ \hline & 4 & 20 & 30 \end{array}$$

WRITTEN EXERCISE

1. A difference of 27° 4′ 15″ in longitude corresponds to what difference in time?

2. Two ships are 75° 30′ 30″ of longitude apart. What is their difference in time?

3. A ship 62° 3′ 40″ W. receives at 11 A.M. a wireless telegram from one 60° 1′ 10″ W. When was it sent?

4. For time purposes the longitude of Berlin is 15° E., and that of Chicago is 90° W. What is the difference in longitude? Illustrate by a rough map. When it is 2 P.M. in Chicago, what time is it in Berlin?

5. For time purposes the longitude of San Francisco is taken as 120° W. When it is noon in London (on the prime meridian), what time is it in San Francisco?

6. A steamer 68° 10′ 30″ W. sends a wireless telegram to the Nantucket lightship, 70° W., at 8 hr. 30 min. A.M. At what time is it received? If transmitted to New York, 75° W., without loss of time, when is it received there?

367. Correspondence of Time to Longitude. Since 24 hours of time corresponds to 360° of longitude, we have the following:

24 hr. corresponds to 360° of longitude.
1 hr. corresponds to $\frac{1}{24}$ of 360°, or 15°, of longitude.
1 min. corresponds to $\frac{1}{60}$ of 15°, or $\frac{1}{4}$°, or 15′, of longitude.
1 sec. corresponds to $\frac{1}{60}$ of 15′, or $\frac{1}{4}$′, or 15″, of longitude.

The difference in time between two ships is 3 hr. 7 min. 3 sec. What is the difference in longitude?

Since 1 hr. corresponds to 15°, 3 hr. corresponds to 3 × 15°, or 45°.

Since 1 min. corresponds to 15′, 7 min. corresponds to 7 × 15′, or 105′, or 1° 45′.

Since 1 sec. corresponds to 15″, 3 sec. corresponds to 3 × 15″, or 45″.

Therefore the difference in longitude is 46° 45′ 45″.

Evidently the mere figures of the answer could be obtained by calling 3 hr. 7 min. 3 sec. 3° 7′ 3″ and multiplying by 15, although the reason for the work would not be so clear.

$$\begin{array}{r} 3^\circ \quad 7' \quad 3'' \\ 15 \\ \hline 46^\circ \; 45' \; 45'' \end{array}$$

WRITTEN EXERCISE

1. The difference of time between two ships is 2 hr. 3 min. 10 sec. What is the difference in longitude?

2. When it is noon at Denver, it is 7 P.M. at Greenwich (0°). What longitude does Denver use for its time purposes?

3. When it is noon at Greenwich, it is 6 A.M. in Iowa. What longitude does Iowa use for its time purposes?

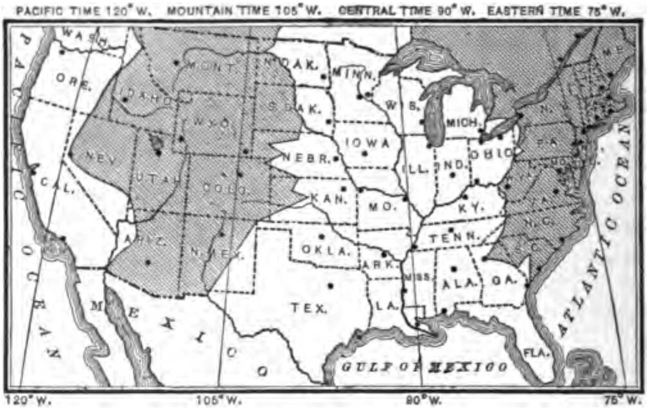
4. The difference in time between the Harvard and Columbia university observatories is 8 min. 22.7 sec. What is the difference in longitude?

5. At 10 hr. 4 min. A.M. a steamer in longitude 26° 30′ W. sends a wireless message to another steamer. It is received at 10 hr. 19 min. A.M. What is the longitude of the second steamer?

368. Standard Time. For practical purposes most of the civilized world is now divided into sections, and all places in the same section use the ***standard time*** of a certain meridian.

The details of this plan, like the study of railway time-tables, are matters of geography and have no value in arithmetic.

The following is the standard-time map of the United States:



When it is noon by Eastern time (75° W.), it is 11 A.M. by Central time (90° W.), 10 A.M. by Mountain time (105° W.), 9 A.M. by Pacific time (120° W.), and 5 P.M. by Greenwich time (0°).

ORAL EXERCISE

1. When it is 10 A.M. in Iowa, what time is it in New York?

2. What time is it now in your school? in Boston?

3. When it is 10 A.M. in San Francisco, what time is it in Denver? in St. Louis? in Cleveland? in Albany?

4. When it is 9 A.M. in Denver, what time is it in Los Angeles?

5. When it is noon in California, what time is it in Illinois?

6. When it is 3 P.M. in Milwaukee, what time is it in Oregon?

369. Six Per Cent Method of Finding Interest. This method, known as the Six Per Cent Method, will often be convenient.

Required the interest on $420 for 5 mo. 10 da. at 6%.

Since 2 mo. = $\frac{1}{6}$ yr., the rate for 2 mo. is $\frac{1}{6}$ of 6%, or 1%.

The interest for 2 mo. is 1% of $420 = $4.20
The interest for 2 mo. more = 4.20
The interest for 1 mo. more is $\frac{1}{2}$ of $4.20 = 2.10
The interest for 10 da. more is $\frac{1}{3}$ of $2.10 = .70
The interest for 5 mo. 10 da. = $11.20

Hence *the interest at 6% for 60 days is 0.01 of the principal, and for 6 days 0.001 of the principal.* For other periods the interest can be found from the interest for these periods.

The method is sometimes stated as follows:

To find the interest for 600 da. at 6%, move the decimal point one place to the left; for 60 da., two places to the left; for 6 da., three places to the left.

After finding the interest at 6%, the interest for other rates is easily found. Thus the interest at 5% is $\frac{5}{6}$ of that at 6%.

ORAL EXERCISE

Find the interest at 6% on the following:

1. $750, for 60 da.
2. $300, for 30 da.
3. $200, for 90 da.
4. $275, for 60 da.
5. $250, for 30 da.
6. $400, for 90 da.
7. $500, for 60 da.
8. $500, for 30 da.
9. $500, for 600 da.
10. $1200, for 720 da.

Find the interest at 5% on the following:

11. $600, for 60 da.
12. $1200, for 30 da.
13. $120, for 30 da.
14. $720, for 90 da.

WRITTEN EXERCISE

1. Find the interest on $1750 for 60 da., first using the Six Per Cent Method, then using cancellation, and finally using the ordinary method of finding the interest for 1 yr. and then for the fractional part of a year. Write a statement telling the advantage of the Six Per Cent Method over the other methods.

2. Using the three methods mentioned in Ex. 1, find the interest on $675 for 90 da. at 6%; on $1425 for 30 da. at 6%.

3. A note for $750 is discounted for 60 da. at 6%. Find the discount and the proceeds.

4. A note for $650 is discounted for 30 da. at 6%. Find the discount and the proceeds.

5. A note for $2400 is discounted for 90 da. at 6%. Find the discount and the proceeds.

Since bank notes usually run for 30 da., 60 da., or 90 da., since 6% is the most common rate, and since we can tell the discount for 60 da. by simply glancing at the face of the note, we can often find the discount on bank notes for the usual periods without using pencil and paper.

Find the discounts at 6% on notes for the following amounts:

6. $1200, for 90 da.

7. $1270, for 90 da.

8. $1525, for 30 da.

9. $2575, for 90 da.

10. $3625, for 90 da.

11. $4250, for 30 da.

12. $450, for 3 mo. 10 da.

13. $725, for 2 mo. 15 da.

14. $850, for 1 mo. 20 da.

15. $550, for 2 mo. 20 da.

16. $1275, for 3 mo. 15 da.

17. $6250, for 1 mo. 18 da.

Find the interest at 5% on notes for the following amounts:

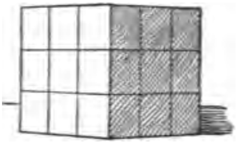
18. $4200, for 60 da.

19. $3750, for 90 da.

20. $500, for 1 mo. 10 da.

21. $750, for 2 mo. 15 da.

370. Cubes and Cube Roots. If a cube has an edge of 3 units, it has a volume of 27 cubic units. Therefore 27 is called the *cube* of 3, and 3 is called the *cube root* of 27.



The cube of 3 is written 3^3; the cube root of 27 is written $\sqrt[3]{27}$.

371. Cube Roots of Volumes. Therefore, considering the *abstract* numbers representing the measures,

The edge of a cube is the cube root of its volume.

372. Powers. Squares and cubes are called *powers.* We also have higher powers, like the fourth, fifth, and so on. Raising to powers is sometimes called *involution.* Extracting roots is sometimes called *evolution.*

373. Cube Roots of Perfect Cubes. Cube roots of perfect cubes may often be found by factoring.

For example, $\sqrt[3]{216}=\sqrt[3]{2\times2\times2\times3\times3\times3}$

$=\sqrt[3]{(2\times3)\times(2\times3)\times(2\times3)}$

$=\sqrt[3]{6\times6\times6}=6.$

2)216
2)108
2)54
3)27
3)9
3

The subject of cube root is now so commonly postponed until algebra is studied that extended explanation is not felt to be necessary. A brief treatment will therefore suffice. Teachers are advised to omit the subject unless the local course of study calls for it.

WRITTEN EXERCISE

Find the following cube roots by factoring:

1. $\sqrt[3]{729}$. 3. $\sqrt[3]{512}$. 5. $\sqrt[3]{2744}$. 7. $\sqrt[3]{4096}$.
2. $\sqrt[3]{1331}$. 4. $\sqrt[3]{1728}$. 6. $\sqrt[3]{15{,}625}$. 8. $\sqrt[3]{5832}$.

9. What is the edge of a cube of volume 13.824 cu. in.?

10. What is the edge of a cube of volume 10,648 cu. in.?

374. Finding Square and Cube Roots. We have seen (§ 336) that we may need to find the square roots of numbers. We shall also see that we may need to find the cube roots of numbers, particularly in measuring solid figures.

There are methods of finding the square root and the cube root of any number. These methods are seldom used in ordinary computations. When we wish to find squares, cubes, square roots, or cube roots, we generally use tables. For example, on page 519 is a convenient table giving the squares, cubes, square roots, and cube roots of all numbers from 1 to 100.

1. Find the square root of 39.

From the table, $\sqrt{39} = 6.245$. This means that 6.245^2 is approximately 39. By squaring 6.245 we see that the result is 39.000025. That is, $\sqrt{39}$ is 6.245, to three places of decimals.

2. Find the cube root of 88.

From the table, $\sqrt[3]{88} = 4.448$. That is, 88 is not an exact cube, and its cube root is between 4.447 and 4.448.

3. Find the square root of 841.

The column of numbers (under No.) goes only to 100. We therefore look to see if 841 is in the column of squares. We find that it is, being the square of 29. Therefore $\sqrt{841} = 29$.

4. Find the cube root of 884,736.

The column of cubes shows that $884{,}736 = 96^3$. Hence, $\sqrt[3]{884{,}736} = 96$.

5. Find the square root of 2000.

Since $2000 = 20 \times 100$, we may find the square roots of 20 and 100 and take their product. We see that $\sqrt{20} = 4.472$, and $\sqrt{100} = 10$. We therefore have $\sqrt{2000} = 10 \times 4.472 = 44.72$.

6. Find the square root of 1765.

The table shows that $\sqrt{1764} = 42$, so that $\sqrt{1765} = 42 +$ some fraction; that is, this table gives us only the two integral figures.

375. Table of Powers and Roots. This table will be helpful:

No.	Squares	Cubes	Square Roots	Cube Roots	No.	Squares	Cubes	Square Roots	Cube Roots
1	1	1	1.000	1.000	51	2,601	132,651	7.141	3.708
2	4	8	1.414	1.260	52	2,704	140,608	7.211	3.733
3	9	27	1.732	1.442	53	2,809	148,877	7.280	3.756
4	16	64	2.000	1.587	54	2,916	157,464	7.348	3.780
5	25	125	2.236	1.710	55	3,025	166,375	7.416	3.803
6	36	216	2.449	1.817	56	3,136	175,616	7.483	3.826
7	49	343	2.646	1.913	57	3,249	185,193	7.550	3.849
8	64	512	2.828	2.000	58	3,364	195,112	7.616	3.871
9	81	729	3.000	2.080	59	3,481	205,379	7.681	3.893
10	100	1,000	3.162	2.154	60	3,600	216,000	7.746	3.915
11	121	1,331	3.317	2.224	61	3,721	226,981	7.810	3.936
12	144	1,728	3.464	2.289	62	3,844	238,328	7.874	3.958
13	169	2,197	3.606	2.351	63	3,969	250,047	7.937	3.979
14	196	2,744	3.742	2.410	64	4,096	262,144	8.000	4.000
15	225	3,375	3.873	2.466	65	4,225	274,625	8.062	4.021
16	256	4,096	4.000	2.520	66	4,356	287,496	8.124	4.041
17	289	4,913	4.123	2.571	67	4,489	300,763	8.185	4.062
18	324	5,832	4.243	2.621	68	4,624	314,432	8.246	4.062
19	361	6,859	4.359	2.668	69	4,761	328,509	8.307	4.102
20	400	8,000	4.472	2.714	70	4,900	343,000	8.367	4.121
21	441	9,261	4.583	2.759	71	5,041	357,911	8.426	4.141
22	484	10,648	4.690	2.802	72	5,184	373,248	8.485	4.160
23	529	12,167	4.796	2.844	73	5,329	389,017	8.544	4.179
24	576	13,824	4.899	2.884	74	5,476	405,224	8.602	4.198
25	625	15,625	5.000	2.924	75	5,625	421,875	8.660	4.217
26	676	17,576	5.099	2.962	76	5,776	438,976	8.718	4.236
27	729	19,683	5.196	3.000	77	5,929	456,533	8.775	4.254
28	784	21,952	5.292	3.037	78	6,084	474,552	8.832	4.273
29	841	24,389	5.385	3.072	79	6,241	493,039	8.888	4.291
30	900	27,000	5.477	3.107	80	6,400	512,000	8.944	4.309
31	961	29,791	5.568	3.141	81	6,561	531,441	9.000	4.327
32	1,024	32,768	5.657	3.175	82	6,724	551,368	9.055	4.344
33	1,089	35,937	5.745	3.208	83	6,889	571,787	9.110	4.362
34	1,156	39,304	5.831	3.240	84	7,056	592,704	9.165	4.380
35	1,225	42,875	5.916	3.271	85	7,225	614,125	9.220	4.397
36	1,296	46,656	6.000	3.302	86	7,396	636,056	9.274	4.414
37	1,369	50,653	6.083	3.332	87	7,569	658,503	9.327	4.431
38	1,444	54,872	6.164	3.362	88	7,744	681,472	9.381	4.448
39	1,521	59,319	6.245	3.391	89	7,921	704,969	9.434	4.465
40	1,600	64,000	6.325	3.420	90	8,100	729,000	9.487	4.481
41	1,681	68,921	6.403	3.448	91	8,281	753,571	9.539	4.498
42	1,764	74,088	6.481	3.476	92	8,464	778,688	9.592	4.514
43	1,849	79,507	6.557	3.503	93	8,649	804,357	9.644	4.531
44	1,936	85,184	6.633	3.530	94	8,836	830,584	9.695	4.547
45	2,025	91,125	6.708	3.557	95	9,025	857,375	9.747	4.563
46	2,116	97,336	6.782	3.583	96	9,216	884,736	9.798	4.579
47	2,209	103,823	6.856	3.609	97	9,409	912,673	9.849	4.595
48	2,304	110,592	6.928	3.634	98	9,604	941,192	9.899	4.610
49	2,401	117,649	7.000	3.659	99	9,801	970,299	9.950	4.626
50	2,500	125,000	7.071	3.684	100	10,000	1,000,000	10.000	4.641

WRITTEN EXERCISE

Using the table on page 519, find the square roots of the following:

1. 25.
2. 49.
3. 64.
4. 81.
5. 7.
6. 13.
7. 29.
8. 47.
9. 67.
10. 91.

Find the cube roots of the following:

11. 8.
12. 27.
13. 64.
14. 13.
15. 17.
16. 19.
17. 28.
18. 46.
19. 51.
20. 99.

Find the square roots of the following:

21. 225.
22. 529.
23. 961.
24. 2116.
25. 5184.
26. 6889.
27. 7744.
28. 8649.
29. 9025.
30. 9604.

Find the cube roots of the following:

31. 5832.
32. 9261.
33. 19,683.
34. 42,875.
35. 85,184.
36. 97,336.
37. 531,441.
38. 804,357.

Find the integral parts of the square roots of the following:

39. 230.
40. 375.
41. 509.
42. 850.
43. 1475.
44. 1695.
45. 3850.
46. 4775.
47. 6730.
48. 8675.

Find the integral parts of the cube roots of the following:

49. 2775.
50. 22,325.
51. 75,750.
52. 675,750.

53. The volume of a cube is 2197 cu. in. Find the edge.

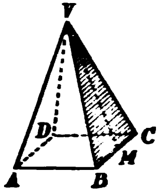
54. The volume of a cube is 39 cu. ft. Find the edge.

55. The area of one face of a cube is 78 sq. in. Find the volume of the cube.

56. The area of all six faces of a cube is 36 sq. in. Find the volume of the cube.

376. Lateral Area of a Pyramid. We have already learned how to find the volume of a pyramid, a problem more often found in practical life than the one of finding the surface.

The only kind of pyramid considered in arithmetic is one in which the vertex is exactly over the center of the base, the so-called *regular pyramid* here illustrated.



The altitude of any triangular face of a pyramid is called the *slant height* of the pyramid.

For example, *VM* is the slant height of the pyramid here shown.

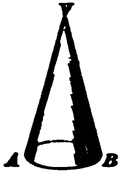
The sum of the areas of the triangular faces of a pyramid is called the *lateral area* of the pyramid.

Since the area of each triangular face of a pyramid equals half the product of the base and altitude,

The lateral area of a pyramid equals the perimeter of the base multiplied by half the slant height.

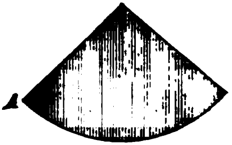
377. Lateral Area of a Cone. If we should slit the surface of a cone and flatten it out as shown below, we would have part of a circle.

The only kind of cone considered in arithmetic is the so-called *right circular cone* here illustrated. The cone is so familiar as to require no formal definition. The terms "lateral area" and "slant height" will be understood from the study of the pyramid.



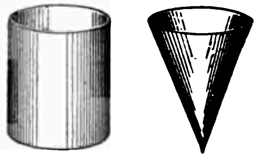
In the way that we found the area of a circle we find that

The lateral area of a cone equals the circumference of the base multiplied by half the slant height.



In the exercise on page 522 it is understood that regular pyramids and right circular cones are given, these being the ones usually encountered.

378. Volume of a Cone. In the way that we found the volume of a pyramid we may find the volume of a cone. Then



The volume of a cone equals one third the product of the base and height.

What is the volume of a cone 5 in. high, the radius of the base being 2 in.?

Area of base = $\frac{22}{7} \times 4$ sq. in.

Volume = $\frac{1}{3} \times 5 \times \frac{22}{7} \times 4$ cu. in. = 20.95 cu. in.

That is, the volume is a little greater than 20.95 cu. in. Common sense should be used as to rejecting small fractions. The result may be given as $20.95\frac{5}{21}$ cu. in., but we would never measure so closely as this.

WRITTEN EXERCISE

Find the lateral areas of pyramids with the following perimeters of bases and slant heights:

1. 24 in., 16 in. **2.** 4 ft. 3 in., 9 in. **3.** 4 ft. 9 in., 12 in.

Find the lateral areas of cones with the following circumferences of bases and slant heights:

4. $15\frac{3}{4}$ in., $4\frac{3}{4}$ in. **5.** 7 ft. 6 in., 4 ft. **6.** 3 ft. 6 in., 15 in.

Find the volumes of cones with the following radii of bases and heights:

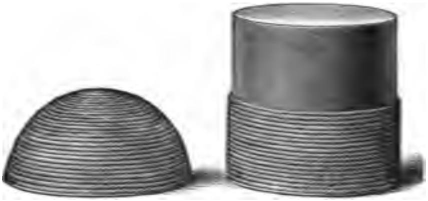
7. 15.4 in., 6 in. **8.** 6.3 in., 15 in. **9.** 42 in., 18 in.

10. The Great Pyramid of Cheops has a slant height of 500 ft. and a square base 764 ft. on a side. Find the lateral area.

11. How many square feet of slating on a steeple in the form of a pyramid of slant height 40 ft. and perimeter of base 48 ft.?

12. A dome is surmounted by a gilded cone whose slant height is 16 ft. 8 in., the radius of the base being 3 ft. 6 in. How many square feet must be allowed for gilding?

379. Surface of a Sphere. If we wind half of the surface of a sphere with a cord as here shown, and then wind with exactly



the same length of the cord the surface of a cylinder whose radius equals the radius of the sphere and height equals the diameter, we find that the cord covers half the curved surface of the cylinder.

Therefore, *the surface of a sphere equals the curved surface of a cylinder of the same radius and height.*

Since the curved surface of the cylinder is $\frac{22}{7}$ × diameter × height, and the height is 2 × radius,

$$\begin{aligned}\text{area of sphere} &= \tfrac{22}{7} \times 2 \times \text{radius} \times 2 \times \text{radius}\\ &= 4 \times \tfrac{22}{7} \times \text{square of radius.}\end{aligned}$$

380. Volume of a Sphere. It is sometimes desirable to be able to find the volume of a sphere, although this is not one of the essentials of arithmetic. It is shown in geometry that

The volume of a sphere is equal to $\frac{4}{3}$ times $\frac{22}{7}$ times the cube of the radius.

1. If the earth is a sphere of 4000 mi. radius, find the surface.

$$4 \times \tfrac{22}{7} \times 4000^2 = 4 \times \tfrac{22}{7} \times 16{,}000{,}000 = 201{,}142{,}857\tfrac{1}{7}.$$

Therefore the surface is about 201,143,000 sq. mi.

2. If a ball is 4 ft. in diameter, find the volume.

The radius is $\frac{1}{2}$ of 4 ft., or 2 ft.

The volume is $\frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2$ cu. ft., or 33.52 cu. ft.

WRITTEN EXERCISE

1. If a ball has a radius of $2\frac{1}{4}$ in., what is the surface?

2. The sun's diameter is 866,500 mi. Find the surface.

3. If a tennis ball is $2\frac{5}{8}$ in. in diameter, what is the surface?

4. A gilded ball is to be put on top of a tower. The diameter of the ball is 2′ 6″. How many square inches are to be gilded?

5. If a cubic foot of granite weighs 165 lb., find the weight of a sphere of granite 1 ft. in diameter.

6. A bowl is in the form of a hemisphere 4.2 in. in diameter. How many cubic inches does it contain?

7. What is the weight of a sphere of marble 3 ft. in circumference, marble being 2.7 times as heavy as water and 1 cu. ft. of water weighing 1000 oz.?

8. Taking the radius of the earth as 4000 mi. and the earth as an exact sphere, find the volume to the nearest 1000 cu. mi.

9. A pyramid has a lateral area of 200 sq. in. The slant height is 8 in. Find the perimeter of the base.

10. A conical spire has a slant height of 68 ft. The perimeter of the base is 60 ft. What is the lateral surface?

11. What is the entire surface of a cone whose slant height is 7 ft. and the diameter of whose base is 7 ft.?

12. If a cubic foot of marble weighs 173 lb., what is the weight of a cylindrical marble column that is 10 ft. high and 16 in. in diameter?

13. How many cubic yards of earth must be removed in digging a canal 2 mi. 1740 ft. long, 200 ft. wide, and 16 ft. deep?

14. A marble 1 in. in diameter is dropped into a cylindrical jar 4 in. high and 3 in. in diameter, half full of water. How much does the marble cause the water to rise?

381. Metric System. There is a system of measures used in many countries which is known as the *Metric System.*

The unit of length is the *meter.* The meter is 39.37 in. nearly.

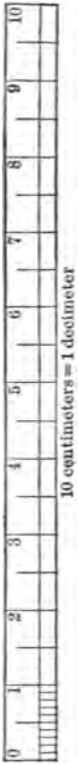
The unit of capacity is the *liter* (lētẽr), a cube 0.1 of a meter on an edge. The liter is 1 qt. nearly.

The unit of weight is the *gram.* It is practically the weight of a cube of water 0.01 of a meter on an edge. The gram is 15.4 grains nearly.

382. Prefixes. Learn the following prefixes:

Just as 1 mill = 0.001 of a dollar,
so 1 millimeter = 0.001 of a meter.
Just as 1 cent = 0.01 of a dollar,
so 1 centimeter = 0.01 of a meter.
Just as decimal means tenths,
so 1 decimeter = 0.1 of a meter.

PREFIX	MEANS	AS IN	WHICH MEANS	
myria-	10,000	myriameter	10,000	meters.
kilo-	1000	kilogram	1000	grams.
hekto-	100	hektoliter	100	liters.
deka-	10	dekameter	10	meters.
deci-	0.1	decimeter	0.1	of a meter.
centi-	0.01	centigram	0.01	of a gram.
milli-	0.001	millimeter	0.001	of a meter.



This system is as much easier than our common one as the system of United States money is easier than the English system. This is the reason why it is so extensively used on the continent of Europe and in Central America and South America.

Unless the pupil is going to use the metric system at once in some of his measurements in science, the study of this subject may be omitted. It is easily learned when the necessity for its use arises. The system is used almost everywhere in scientific work.

383. Table of Length. This table is as follows:

A myriameter = 10,000 meters
A **kilometer** (km.) = 1000 meters
A hektometer = 100 meters
A dekameter = 10 meters
Meter (m.)
A decimeter (dm.) = 0.1 of a meter
A **centimeter** (cm.) = 0.01 of a meter
A **millimeter** (mm.) = 0.001 of a meter

The meter is about 39.37 in., $3\frac{1}{4}$ ft., or a little over a yard; the kilometer is about 0.6 of a mile.

The abbreviations in this book are recommended by various scientific associations. Some, however, use Km., Dm., dm., for kilometer, dekameter, and decimeter.

WRITTEN EXERCISE

Express as inches, taking 39.37 in. as 1 m.:

1. 47 m.
2. 34.5 m.
3. 52 m.
4. 67.5 m.
5. 324 cm.
6. 2.83 cm.
7. 4680 mm.
8. 3000 mm.

9. Express the diameter of a 7-centimeter gun in inches.

10. A certain hill is 203 m. high. Express this in feet.

11. A tower is 37.5 m. high. Express this in feet.

12. A wheel is 1.3 m. in diameter. Express this in inches.

13. The distance from Dieppe (dē-ep′) to Paris is 209 km. Express this in miles.

14. The distance between two places in Germany is 178 km. Express this in miles.

15. The distance from Paris to Cologne is 306 mi. What is the railway fare at 12 centimes per kilometer?

16. The distance from Paris to Brussels is 326 km., and it takes 7 hours to make the trip by railway. What is the average rate in miles per hour?

384. Table of Square Measure. This table is as follows:

A square myriameter = 100,000,000 square meters
A **square kilometer** (km.^{2}) = 1,000,000 square meters
A square hektometer = 10,000 square meters
A square dekameter = 100 square meters
Square meter (m.^{2})
A **square decimeter** (dm.^{2}) = 0.01 of a square meter
A **square centimeter** (cm.^{2}) = 0.0001 of a square meter
A square millimeter (mm.^{2}) = 0.000001 of a square meter

The abbreviation sq. m. is often used for m.^{2}, and similarly for sq. cm., etc.

The square dekameter is also called an *are* (ăr); and since there are 100 square dekameters in 1 hm.^{2}, a square hektometer is called a *hektare* (ha.). The hektare is 2.47 acres, or nearly 2½ acres.

385. Table of Cubic Measure. This table is as follows:

A cubic myriameter = 1,000,000,000,000 cubic meters
A cubic kilometer = 1,000,000,000 cubic meters
A cubic hektometer = 1,000,000 cubic meters
A cubic dekameter = 1000 cubic meters
Cubic meter (m.^{3})
A **cubic decimeter** (dm.^{3}) = 0.001 of a cubic meter
A **cubic centimeter** (cm.^{3}) = 0.000001 of a cubic meter
A cubic millimeter (mm.^{3}) = 0.000000001 of a cubic meter

The abbreviation cu. m. is often used for m.^{3}, and similarly for cu. cm., etc.

In measuring wood a cubic meter is called a *stere* (st.).

WRITTEN EXERCISE

Express the following as square centimeters:

1. 750 km.^{2}
2. 37 m.^{2}
3. 4296 mm.^{2}
4. 6.25 m.^{2}

Express the following as cubic meters:

5. 19.75 cubic dekameters.
6. 427,653.84 mm.^{3}
7. 7 st.
8. 3.5 st.
9. 937 cm.^{3}
10. 486 dm.^{3}

386. Table of Weight. The table of weight is as follows:

A **metric ton** (t.) = 1,000,000 grams
A quintal (q.) = 100,000 grams
A myriagram = 10,000 grams
A **kilogram** (kg.) = 1000 grams
A hektogram = 100 grams
A dekagram = 10 grams
Gram (g.)
A decigram = 0.1 of a gram
A centigram (cg.) = 0.01 of a gram
A milligram (mg.) = 0.001 of a gram

A kilogram, commonly called a *kilo,* is about 2½ lb. A 5-cent piece weighs 5 g. A metric ton is nearly 2204.6 lb.

387. Table of Capacity. The table of capacity is as follows:

A **hektoliter** (hl.) = 100 liters
A dekaliter = 10 liters
Liter (l.)
A deciliter = 0.1 of a liter
A **centiliter** (cl.) = 0.01 of a liter
A milliliter (ml.) = 0.001 of a liter

A liter is the same as 1 dm.^{3} A liter is about one quart.

WRITTEN EXERCISE

Express as kilos:

1. 374 lb.
2. 428 lb.
3. 204 lb.
4. 352 oz.
5. 48.4 lb.
6. 300 T.
7. 3275 g.
8. 7275 g.
9. 173.8 lb.
10. 7386 g.
11. 4984 g.
12. 6249 g.

Express as liters:

13. 3 hl.
14. 7.5 hl.
15. 8 pt.
16. 17 hl.
17. 2.25 hl.
18. 12½ gal.
19. 5000 ml.
20. 2500 ml.
21. 500 gal.
22. 15 hl.
23. 724 hl.
24. 6.83 hl.

388. Letters used in Solving Problems. Teachers who may wish to use a little algebra in arithmetic will find the material in this section.

For example, I am thinking of a number. When it is multiplied by 2, and 7 is added to the result, the sum is 33. What is the number?

Solution using a letter

If I am thinking of n, then, letting $2n$ mean $2 \times n$,

$2n$ is twice the number,

$2n + 7$ is 7 added to $2n$,

and $2n + 7 = 33$, as stated in the problem.

Then $2n = 26$, by subtracting 7 from equals.

Therefore $n = 13$, by dividing these equals by 2.

Check or Proof. $2 \times 13 + 7 = 33$.

Solution without using letters

Because twice the number added to 7 equals 33, therefore if 7 is taken away from 33 there will remain twice the number. Therefore 26 is twice the number. Therefore once the number is half of 26, or 13.

The solutions compared

$2n + 7 = 33.$	33
Subtracting 7,	7
$2n = 26.$	$2\overline{)26}$
Therefore $n = 13.$	13

We therefore see that the two solutions are the same, but that the letters make the reasoning clearer. Therefore in solving a problem by algebra we proceed as follows:

Write a letter for the number sought.
Use this letter in the statement of the problem.
This will give an equation, as shown above.
Solve this equation.

389. Applications of the Equation. The following are illustrations of some of the applications of the equation:

1. Find what per cent 25 is of 225.

Let $x\% =$ the required rate per cent.

Then $x\% \times 225 = 25.$

Dividing by 225, $x\% = \frac{25}{225}$

$= \frac{1}{9}$, or $11\frac{1}{9}\%$.

2. Find the number of which $8.40 is 12%.

Let $x =$ the required number.

Then $0.12\,x = \$8.40.$

Dividing by 0.12, $x = \$8.40 \div 0.12$

$= \$70.$

WRITTEN EXERCISE

Find what per cent the first number is of the second:

1. 25, 750.
2. 5 ft., $37\frac{1}{2}$ ft.
3. 528 ft., 2 mi.
4. $1\frac{1}{2}$, $7\frac{1}{2}$.
5. 7 ft., 7 yd.
6. $4.90, $29.40.

7. A man's income is $1650 a year, and he spends $693. What per cent of his income does he save?

8. In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys? What per cent are girls?

9. Of what length is 126 ft. 6 in. exactly 50%?

10. A baseball team won 65% of the games played, and won 13 games. How many games did it play?

11. A baseball team lost $33\frac{1}{3}\%$ of the games played, and lost 16 games. How many games did it play?

The use of the algebraic equation is not essential to the study of arithmetic. It simplifies the study of formulas, but only a few of the practical problems of daily life are rendered more simple by its use.

TABLES FOR REFERENCE

Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards, or 16½ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)
2150.42 cu. in. (approximately 1¼ cu. ft.) = 1 bushel

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)
365 days = 1 common year
366 days = 1 leap year

Value

10 mills = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)

Angles and Arcs

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)

Surveyors' Table of Length

7.92 inches = 1 link (li.)
100 links = 4 rods = 1 chain (ch.)
80 chains = 5280 feet = 1 mile

Counting

12 units = 1 dozen (doz.)
12 dozen, or 144 = 1 gross (gro.)

Paper

24 sheets = 1 quire
20 quires, or 480 sheets = 1 ream

For convenience in counting, 500 sheets are now more often called a *ream*; and the word "quire" is used only for folded note paper, other paper being usually sold by the pound.

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